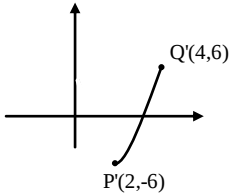
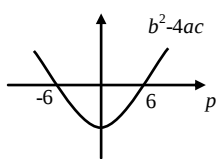


Higher Prelim Revision 3

Marking Scheme - Paper I

	Give 1 mark for each •	Illustration(s) for awarding each mark
1.	(a) ans: $k = 2$ 1 mark •1 sub. to answer (b) ans: $y = 3x - 7$ (or equiv.) 3 marks •1 for gradient of line •2 for gradient of AB •3 for equation (c) ans: $A(0, -7)$ 1 mark •1 answer (y intercept) (d) ans: Area = 15 square units 4 marks •1 y intercept of top line •2 length of base •3 perpendicular length •4 calculation to answer	(a) •1 $3 + 3k = 9 \therefore k = 2$ (b) •1 $m = -\frac{1}{3}$ •2 $m_{AB} = 3$ •3 $y - 2 = 3(x - 3)$ (c) •1 $A(0, -7)$ (d) •1 $(0, 3)$ •2 distance between y intercepts = 10 •3 y-axis to B ... 3 units $A = \frac{1}{2}bh$ •4 $= \frac{1}{2} \times 10 \times 3 = 15 \text{ units}^2$
2.	(a) ans: $a = 2$ 4 marks •1 knowing to solve deriv. to zero •2 setting up synthetic division •3 completing synth. div. •4 solving equ. to zero and answer (b) ans: zero, another stat. point 2 marks •1 substituting for a and -2 •2 calculation finds zero + conclusion ** pupils may complete part (a) by substitution.	(a) •1 at s.p. $f'(x) = 0$ (stated or implied) •2 $\begin{array}{r rrrr} 4 & 1 & -a & -4a & 0 \\ & & & & \\ \hline & & & & \end{array}$ •3 $\begin{array}{r rrrr} 4 & 1 & -a & -4a & 0 \\ & & 4 & 16-4a & 64-32a \\ \hline & 1 & 4-a & 16-8a & 0 \end{array}$ •4 $64 - 32a = 0, a = 2$ (b) •1 $f'(-2) = (-2^3) - 2(2)(-2^2) - 4(2)(-2)$ •2 $f'(-2) = 0 \therefore$ another stat. point
3.	(a) ans: see sketch 3 marks •1 for reflecting •2 for translation up 6 •3 for annotating, coordinates (b) ans: $g(x) = 3x^2 - 12x + 6$ 2 marks •1 for strategy •2 for answer (any equivalent form)	(a) •1  •2 •3 (b) •1 strategy •2 $g(x) = -(12x - 3x^2) + 6$

	Give 1 mark for each •	Illustration(s) for awarding each mark
4.	ans: $a = -6$ 5 marks •1 integrating first term •2 integrating 2nd term •3 sub. in limits •4 simplifying to quad. equation •5 solving and choosing answer	•1 $4x \dots\dots$ •2 $\dots\dots + \frac{2x^2}{2}$ •3 $(8+4) - (4a + a^2) = 0$ •4 $a^2 + 4a - 12 = 0$ •5 $(a-2)(a+6) = 0$ $a = 2$ or $a = -6$
5.	ans: $\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\}$ 6 marks •1 knowing to solve to zero •2 replacement •3 factorising and 2 solutions •4 1st angle from one solution •5 1st angle from other solution •6 for remaining two angles	•1 $\sin 2x + \cos x = 0$ •2 $2\sin x \cos x + \cos x = 0$ •3 $\cos x(2\sin x + 1) = 0$ $\therefore \cos x = 0$ or $\sin x = -\frac{1}{2}$ •4 $x = \frac{\pi}{2}$ •5 $x = \frac{7\pi}{6}$ •6 $x = \frac{3\pi}{2}, \frac{11\pi}{6}$
6.	ans: $h(x) = (x+4)^2 + 4$, $h_{\min} = 4$ @ $x = -4$ 6 marks •1 for f into g •2 for expansion and simplifying •3 bracket term •4 number term •5 for minimum value •6 for x	•1 $g(f(x)) = (x+1)^2 + 6(x+1) + 13$ •2 $h(x) = x^2 + 8x + 20$ •3 $[(x+4)^2 \dots\dots$ •4 $\dots\dots -16] + 20$ •5 $\min = 4$ •6 @ $x = -4$

	Give 1 mark for each •	Illustration(s) for awarding each mark
7.	(a) ans: proof 5 marks •1 know to solve a system •2 combining equations •3 simplifying to quad. •4 for 1st coordinate •5 for 2nd coordinate (b) ans: B(9,-3) , $(x-9)^2 + (y+3)^2 = 45$ 6 marks •1 knowing T mid-pt between centres •2 drawing out centre of top circle •3 finding B •4 knowing r the same •5 finding r^2 •6 writing down equation of lower circle	(a) •1 set up a system •2 $(2y)^2 + y^2 - 6(2y) - 18y + 45 = 0$ •3 $5y^2 - 30y + 45 = 0$ •4 $5(y-3)(y-3) = 0 \therefore y = 3$ •5 $x = 2(3) = 6$ (or equivalent) (b) •1 strategy •2 A(3,9) •3 $A(3,9) \rightarrow T(6,3) \rightarrow B(9,-3)$ •4 stated or implied $r_1 = r_2$ •5 $r^2 = \sqrt{9+81} = 45$ •6 $(x-9)^2 + (y+3)^2 = 45$
8.	ans: $f'(x) = 1 + \frac{1}{x^{\frac{3}{2}}}$, 9 6 marks •1 preparing to differentiate •2 diff. 1st term •3 diff. 2nd term •4 writing with positive indices •5 substituting •6 answer	•1 $f(x) = x^{-1}(x^2 - 2x^{\frac{1}{2}})$ $= x - 2x^{-\frac{1}{2}}$ •2 1..... •3 $+1x^{-\frac{3}{2}}$ •4 $f'(x) = 1 + \frac{1}{x^{\frac{3}{2}}}$ •5 $f'(\frac{1}{4}) = 1 + \frac{1}{(\frac{1}{4})^{\frac{3}{2}}}$ •6 $f'(\frac{1}{4}) = 1 + \frac{1}{\frac{1}{8}} = 9$
9.	ans: $-6 < p < 6$ (or equivalent) 6 marks •1 dealing with the fractions •2 manipulation to quad. form •3 discriminant statement •4 for a, b and c •5 finding discriminant •6 solution from quad. inequat.	•1 strategy $\times px$ (or equiv.) •2 $x^2 + 9 = px$ $x^2 - px + 9 = 0$ •3 for no real roots $b^2 - 4ac < 0$ •4 $a = 1, b = -p, c = 9$ •5 $p^2 - 36 < 0$ •6 $-6 < p < 6$ 

Total 60 marks

Higher Prelim Revision 3

Marking Scheme - Paper II

	Give 1 mark for each •	Illustration(s) for awarding each mark
1.	(a) ans: $k = 2$ 4 marks •1 gradient of AB •2 gradient of AP •3 equating gradients •4 finding k (b) ans: $y = 2x$ 2 marks •1 gradient of PQ •2 equation (c) ans: R(2,4) 5 marks •1 coordinates of M •2 equation of median •3 setting up a system •4 finding first coordinate •5 finding 2nd coordinate	(a) •1 $m_{AB} = \frac{-3-5}{11+5} = -\frac{1}{2}$ •2 $m_{AP} = \frac{k-5}{6}$ •3 $\frac{k-5}{6} = -\frac{1}{2}$ •4 $k = 2$ (b) •1 $m_{PQ} = \frac{8-2}{4-1} = 2$ •2 $y - 2 = 2(x - 1)$ (c) •1 M(9,3) no mark given for gradient of AM ... $m_{AM} = -\frac{1}{7}$ •2 $y - 3 = -\frac{1}{7}(x - 9)$ •3 $7y = -x + 30$ $y = 2x$ •4 $x = 2$ •5 $y = 4$
2.	(a) ans: 1011.33 bats (ignore rounding) 3 marks •1 first two lines of calculation •2 lines 3 and 4 of calculations •3 answer (b) ans: Colony is in danger. 600 prior to breeding week is less than 700 bats 4 marks •1 knows to calculate limit + knows formula •2 calculates limit correctly •3 knows to subtract 200 •4 explanation	(a) •1 Low High $U_1 = 0.75(2100) = 1575 + 200 = 1775$ $U_2 = 0.75(1775) = 1331.25 + 200 = 1531.25$ •2 $U_3 = 0.75(1531.25) = 1148.44 + 200 = 1348.44$ $U_4 = 0.75(1348.44) = 1011.33$ •3 1011.33 (b) •1 $L = \frac{b}{1-a}$ •2 $L = \frac{200}{1-0.75} = 800$ •3 low population $800 - 200 = 600$ •4 600 prior to breeding week is less than 700 bats so colony in danger

	Give 1 mark for each •	Illustration(s) for awarding each mark																																
3.	(a) ans: proof 4 marks •1 area of rectangle •2 area of triangle •3 subtracting areas •4 tidy up and common factor (b) ans: $p = -\frac{1}{4}$ 6 marks •1 equating to zero •2 discriminant statement •3 a, b and c •4 substitution •5 to quadratic form •6 answer (c) ans: $x = 1$ 2 marks •1 substitution •2 solving to answer	(a) •1 $A_{rec} = (2x + 2)(x - 4p)$ $= 2x^2 + 2x - 8px - 8p$ •2 $A_{tri} = \frac{1}{2}(x + 6) \times 2x = x^2 + 6x$ •3 $A_1 - A_2 =$ $= 2x^2 + 2x - 8px - 8p - (x^2 + 6x)$ •4 $A_1 - A_2 = x^2 - (8p + 4)x - 8p$ (b) •1 $x^2 - (8p + 4)x - 8p - 1 = 0$ •2 $b^2 - 4ac = 0$ for equal roots •3 $a = 1, b = -(8p + 4), c = -8p - 1$ •4 $(8p + 4)^2 - 4(-8p - 1) = 0$ •5 $64p^2 + 96p + 20 = 0$ •6 $4(4p + 5)(4p + 1) = 0$ $p = -\frac{5}{4}$ or $p = -\frac{1}{4}$ (c) •1 $x^2 - (8(-\frac{1}{4}) + 4)x - 8(-\frac{1}{4}) - 1 = 0$ •2 $(x - 1)(x - 1) = 0, x = 1$																																
4.	(a) ans: A(2,6) 5 marks •1 knowing and preparing to differentiate •2 differentiating •3 solving to zero •4 x coordinate •5 y coordinate (b) ans: B(4,10) 6 marks •1 know to form a system •2 combining equations •3 manipulation to polynomial form •4 sets up synthetic division •5 finds x coordinate •6 for y coordinate	(a) •1 $y = \frac{1}{2}x^2 + 8x^{-1}$ •2 $\frac{dy}{dx} = x - 8x^{-2} = x - \frac{8}{x^2}$ •3 $x - \frac{8}{x^2} = 0$ •4 $x^3 - 8 = 0 \therefore x = 2$ •5 $y = 6$ (b) •1 $y = \frac{x^2}{2} + \frac{8}{x}$ $2y = x^2 + \frac{16}{x}$ •2 $7x - 8 = x^2 + \frac{16}{x}$ •3 $x^3 - 7x^2 + 8x + 16 = 0$ •4 <table><tr><td>1</td><td>-7</td><td>8</td><td>16</td></tr><tr><td colspan="4"> </td></tr><tr><td></td><td></td><td></td><td>0</td></tr></table> •5 <table><tr><td>1</td><td>-7</td><td>8</td><td>16</td></tr><tr><td colspan="4"> </td></tr><tr><td>4</td><td>4</td><td>-12</td><td>-16</td></tr><tr><td colspan="4"> </td></tr><tr><td>1</td><td>-3</td><td>-4</td><td>0</td></tr></table> $x = 4$ •6 $2y = 7(4) - 8 \therefore y = 10$	1	-7	8	16								0	1	-7	8	16					4	4	-12	-16					1	-3	-4	0
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4	4	-12	-16																															
1	-3	-4	0																															

	Give 1 mark for each •	Illustration(s) for awarding each mark
5.	(a) ans: $h(x) = x^4 - 2ax^2 + a^2 - a$ 2 marks •1 for dealing with composition •2 for formula (any equivalent form) (b) ans: $a = 3$ 3 marks •1 for differentiating •2 substituting and solving to 8 •3 answer	(a) •1 $f(x^2 - a) = \dots$ •2 $h(x) = (x^2 - a)^2 - a$ $= x^4 - 2ax^2 + a^2 - a$ (b) •1 $h'(x) = 4x^3 - 4ax$ •2 $4(2^3) - 4(2)a = 8$ •3 $a = 3$
6.	(a) ans: proof 2 marks •1 for solving to 4 •2 for answer (b) ans: $7\frac{1}{3} \text{ cm}^2$ 6 marks •1 setting up integral •2 integrating •3 substituting limits •4 area under curve •5 area of rectangle •6 subtraction to answer	(a) •1 $x^2 - 6x + 12 = 4$ •2 $(x - 4)(x - 2) = 0 \therefore x = 2 \text{ or } 4$ (b) •1 $A = \int_2^4 x^2 - 6x + 12 \, dx$ •2 $= \left[\frac{x^3}{3} - 3x^2 + 12x \right]_2^4$ •3 $= \left(\frac{64}{3} - 3(16) + 12(4) \right) - \left(\frac{8}{3} - 12 + 24 \right)$ •4 $= 6\frac{2}{3} \text{ cm}^2$ •5 $A_{\text{rec}} = 2 \times 7 = 14$ •6 $A = 14 - 6\frac{2}{3} = 7\frac{1}{3} \text{ cm}^2$

	Give 1 mark for each •	Illustration(s) for awarding each mark
7.	(a) ans: proof 4 marks •1 length of MR •2 length of PR •3 value of $\sin x$ •4 required form (b) ans: proof 4 marks •1 knowing angle is equiv. to $\sin 2x$ •2 use replacement •3 for value of $\cos x$ and substitution •4 required answer	(a) •1 $MR = \sqrt{2}$ or $\frac{\sqrt{8}}{2}$ •2 $PR^2 = 2^2 + \sqrt{2}^2 = 6 \therefore PR = \sqrt{6}$ •3 $\sin x = \frac{\sqrt{2}}{\sqrt{6}}$ •4 $\sin x = \frac{\sqrt{2}}{\sqrt{6}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{3}\sqrt{3}$ (b) •1 $\sin RPQ = \sin 2x$ •2 $= 2\sin x \cos x$ •3 $= 2 \times \frac{\sqrt{3}}{3} \times \frac{2}{\sqrt{6}}$ •4 $\frac{4\sqrt{3}}{3\sqrt{6}} = \frac{4}{3\sqrt{2}} = \frac{2}{3}\sqrt{2}$
8.	(a) ans: $r^2 = 144 - h^2$ 1 mark •1 answer (b) ans: proof 2 marks •1 substituting for r^2 •2 required form (c) ans: $h = \sqrt{48} = 4\sqrt{3}$ ft 5 marks •1 know to differentiate •2 solve derivative to zero •3 differentiate •4 solve for h •5 justification	(a) •1 $r^2 = 144 - h^2$ (b) •1 $V = \frac{1}{3}\pi(144 - h^2)h$ •2 $V(h) = 48\pi h - \frac{1}{3}\pi h^3$ (c) •1 strategy to diff. (stated or implied) •2 solve $V'(h) = 0$ (stated or implied) •3 $V'(h) = 48\pi - \pi h^2$ •4 $48\pi - \pi h^2 = 0$ $h^2 = 48$ $h = \sqrt{48} = 4\sqrt{3}$ •5 $\begin{array}{c c c} \longrightarrow & 4\sqrt{3} & \longrightarrow \\ \hline + & 0 & - \\ \hline \swarrow & \text{max} & \searrow \end{array}$
		Total 70 marks