St Peter the Apostle High

Mathematics Dept.

Higher Prelim Revision 3

Paper I - Non~calculator

Time allowed - 1 hour 10 minutes

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

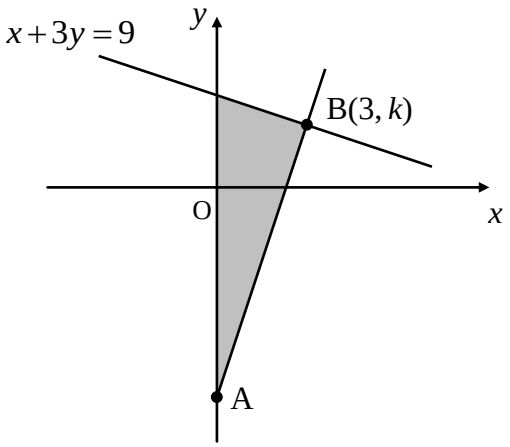
The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

All questions should be attempted

1. Part of the line with equation $x+3y=9$ is shown in the diagram. B lies on this line and has coordinates $(3, k)$.



(a) Find the value of k. **1**

(b) Given that the line AB is perpendicular to the line $x+3y=9$, find the equation of the line AB. **3**

(c) Hence write down the coordinates of A. **1**

(d) Calculate the area of the shaded triangle. **4**

2. (a) A function f has as its derivative $f'(x)=x^3-ax^2-4ax$.

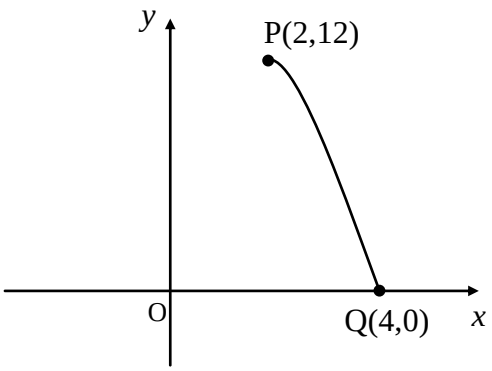
Find a if the function has a stationary point at $x=4$. **4**

(b) Hence find the rate of change of this function at $x=-2$ and comment on your result. **2**

3. A quadratic function, defined on a suitable domain, is given as $f(x)=12x-3x^2$.

The diagram shows part of the graph of this quadratic function, $y=f(x)$.

The graph passes through the points P(2,12) and Q(4,0) as shown.



(a) Sketch the graph of $y=-f(x)+6$ marking clearly the image points of P and Q and stating their coordinates. **3**

(b) Given that $g(x)=-f(x)+6$, write down a formula for $g(x)$. **2**

4. Find a given that $\int_a^2 (4+2x)\,dx = 0$, where $a<2$. **5**

6. Two functions are defined on suitable domains as $f(x) = x + 1$ and $g(x) = x^2 + 6x + 13$.

Given that the function *h* is such that $h(x) = g(f(x))$, express *h* in the form

$$h(x) = (x + a)^2 + b,$$ where *a* and *b* are integers,

and hence write down the minimum value of *h* and the corresponding replacement for *x*. **6**

8. Find $f'(x)$ when $f(x) = \dfrac{x^2 - 2\sqrt{x}}{x}$, expressing your answer with positive indices, and hence calculate the value of the gradient of the tangent to the curve $y = f(x)$ at $x = \frac{1}{4}$. **6**

9. What can you say about p if the equation, in *x*, $\dfrac{x}{p} + \dfrac{9}{px} = 1$ has **no real** roots ? **6**

[END OF QUESTION PAPER]