St Peter the Apostle High

Mathematics Dept.

Higher Prelim Revision 2

Paper 2 - Calculator

Time allowed   -   1 hour 30 minutes

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.
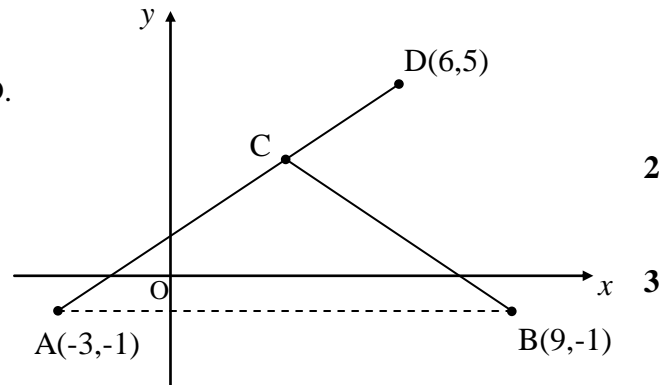
The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2 \cos^2 A - 1$$
$$= 1 - 2 \sin^2 A$$

1. The diagram shows a line joining the
points A(-3,-1) and D(6,5).
B has coordinates (9,-1) and C is a point on AD.

(a) Find the equation of the line AD. **2**

(b) Hence establish the coordinates of C
given that triangle ABC is isosceles. **3**

(c) Use gradient theory to calculate
the size of angle BCD, giving your
answer correct to the nearest degree. **3**

2. A lead shot is discharged from a gun at a clay pigeon.

The height, h feet, of the shot after t seconds is given by
the function
$$h(t) = 288t - 48t^2 .$$

(a) What is the maximum height the shot can reach ? **4**

(b) For the shot to actually break the clay pigeon it must strike the pigeon at a
speed greater than <u>or</u> equal to 48 feet per second.

The speed, s, of the shot after t seconds can be found
from $s = h'(t)$, where $0 < t \le 3$.

Will the shot break the clay pigeon after a flight of $2\cdot7$ seconds ? Explain. **2**

(c) Calculate the maximum **height** the shot can reach **and** still break the clay pigeon. **3**

**5.** A sequence is defined by the recurrence relation $U_{n+1} = aU_n + b$, where a and b are constants.

(*a*) Given that $U_0 = a - 2$ and $b = 1$, show clearly that $U_1 = a^2 - 2a + 1$. **2**

(*b*) Hence find an expression for U_2 in terms of a. **2**

(c) Given now that $U_2 = 37$, form an equation and solve it to find a.

Explain why there is only one possible answer for a. **4**

6. A titanium rod from a nuclear reactor is a solid prism which slots into an elliptical chamber along with three other identical rods. It has a cross-sectional shape made up of two straight lines and a curved edge.
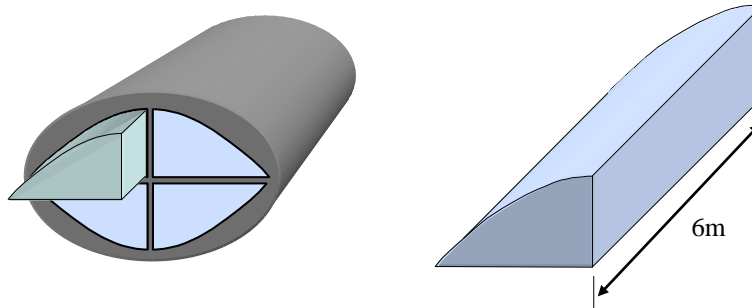
Each rod has a depth of 6 metres.

The cross section of a rod is shown geometrically in the coordinate diagram below where the **units are in metres**. The diagram is not drawn to scale.

The curved section is part of the graph of the curve with equation $y = 5 + 2x - \frac{1}{4}x^2$.
PQ is horizontal and QR is vertical.
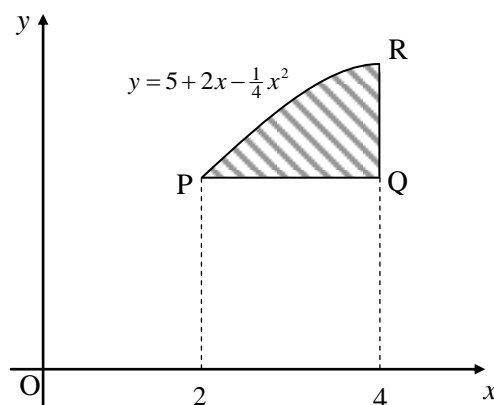
(a) Calculate the shaded area in square metres. **7**

(b) Hence calculate the **total volume** of titanium contained in **all four rods**. **2**

8. Three functions are defined on suitable domains as

$$f(x) = x - 1, \quad g(x) = 3x^2 - 3 \quad \text{and} \quad h(x) = x^3 - 6x.$$

(a) Given that $y = g(f(x)) - h(x)$, find a formula for y in its simplest form. **3**

(b) Hence find the coordinates of the maximum turning point of the
graph of $y = g(f(x)) - h(x)$, **justifying your answer**. **4**

9. An equation is given as $ax(x-1) = c(x-1)$, where $a \neq 0$, $c \neq 0$, and a and c are constants.

(a) Show clearly that this equation can be written in the form

$$ax^2 - (a+c)x + c = 0.$$ **2**

(b) What condition needs to be met for this quadratic equation to have equal roots? **4**

[END OF QUESTION PAPER]