St Peter the Apostle High

Mathematics Dept.

Higher Prelim Revision 2

Paper I - Non~calculator

Time allowed - 1 hour 10 minutes

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre (-g, -f) and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:	$\sin(A\pm B) = \sin A\cos B \pm \cos A\sin B$
	$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
	$\sin 2A = 2\sin A\cos A$
	$\cos 2A = \cos^2 A - \sin^2 A$
	$= 2\cos^2 A - 1$
	$= 1 - 2\sin^2 A$

All questions should be attempted

1. The diagram shows triangle OAB with M being the mid-point of AB. The coordinates of A and B are (-2,6) and (20,0) respectively.



- (a) Establish the coordinates of M.
- (b) Hence find the equation of the median OM.
- (c) A line through B, perpendicular to OM meets OM produced at C.



- (i) Find the equation of the line BC and hence establish the coordinates of C. 4
- (ii) What can you say about triangles OAM and BMC? Explain your answer.

2. A curve has as its equation
$$y = \frac{x^2 - 4x}{\sqrt{x}}$$
, where $x \in R$ and $x > 0$.

Find the gradient of the tangent to this curve at the point where x = 4.

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The function has stationary points at P(0,8), Q(5,0) and R(10,-8) as shown. Sketch a possible graph for y = f'(x), where f'(x) is the derivative of f(x).

4. Two functions, defined on suitable domains, are given as

 $g(x) = x^2 - 3x$ and h(x) = 2x + 1.

Show that the composite function g(h(x)) can be written in the form a(ax+b)(x-b), where a and b are constants, and state the value(s) of a and b. 4

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6. The diagram below, which is not to scale, shows part of the graph of the line with equation y = 6x - 2. Also shown are ordinates at x = 1 and at x = 1 + a.



Find *a* given that the shaded part of the diagram has an area of 4 square units.

7. Two sequences are defined by the following recurrence relationships

 $U_{n+1} = 0.6U_n + 20$ and $U_{n+1} = 0.9U_n + b$, where b is a constant.

- (a) Explain why both sequences have a limit as $n \to \infty$. 1
- (b) Find the value of b if both these sequences have the same limit.

9. A curve has as its equation $y = (p+1)x^3 - 3px^2 + 4x + 1$, where p is a positive integer.

(a) Find
$$\frac{dy}{dx}$$
.

(b) Hence establish the value of p given that this curve has only **one stationary point**. **5**

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