Mathematics Higher Paper 2 Practice Paper S

Time allowed 1 hour 10 minutes NATIONAL QUALIFICATIONS

Read carefully

- 1 Calculators may be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre (-g, -f) and radius $\sqrt{g^2 + f^2 - c}$. The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product : $a \cdot b = |a| |b| \cos \theta$, where θ is the angle between *a* and *b*.

or
$$\boldsymbol{a} \cdot \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$
, where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:
$$sin(A \pm B) = sin A cos B \pm cos A sin B$$

 $cos(A \pm B) = cos A cos B \mp sin A sin B$
 $sin 2A = 2 sin A cos A$
 $cos 2A = cos^2 A - sin^2 A$
 $= 2 cos^2 A - 1$
 $= 1 - 2 sin^2 A$

Table of standard derivatives :

f(x)	f'(x)
sin ax	a cos ax
cos ax	$-a\sin ax$

Table of standard integrals :

f(x)	$\int f(x)dx$
sin ax	$-\frac{1}{a}\cos ax + C$
cos ax	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted.

- 1. (a) Find the equation of the tangent to the curve $y = x^3 x 10$ at the point where x = 1.
 - (*b*) Show that this line is also a tangent to the circle with equation $x^2 + y^2 - 6x - 8y + 5 = 0$ and state the coordinates of the point of contact.

The diagram opposite shows a rhombus ABCD.
 AC and BD are diagonals of the

rhombus.

Diagonal AC has equation 2x + y + 3 = 0.

D is point with coordinates (-4, 0).

E is the point of intersection of the diagonals.



- (*a*) Find the equation of diagonal BD.
- (*b*) Hence find the coordinates of E.



4.	If co	$s_{2x} = \frac{7}{15}$, find the exact value of :	
	(<i>a</i>)	$\cos^2 x$.	2
	(<i>b</i>)	$\tan^2 x$.	3

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4 2

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- 5. A sequence is defined by the recurrence relation u_{n+1} = au_n + b, u₀ = 0.
 (a) If u₁ = 25, write down the value of b.
 (b) Given that u₃ = 31, find the possible values of a.
 4 This sequence tends to a limit as n→∞.
 (c) Find the limit of the sequence.
- 6. (a) Express $4\cos x^\circ 7\sin x^\circ$ in the form $k\sin(x-a)^\circ$, where k > 0 and $0 \le a < 360$. 4
 - (*b*) Hence solve $4\cos x^\circ = 7\sin x^\circ + 3$ for $0 \le x < 360$.
- 7. A new fish farm is being established consisting of a number of rectangular enclosures.

Each enclosure is made up from eight identical rectangular cages.

Each cage measures x metres by y metres.

The total length of edging around the top of all of the caging is 480 metres.

(*a*) Show that the total surface area, in square metres, of the top of the eight cages is given by $A(x) = 320x - \frac{20}{3}x^2$.

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- (b) (i) Find the value of x which maximises this surface area.
 - (ii) Hence find the dimensions of each enclosure.





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8. The diagram shows the graphs of $y = 2\cos(2x)^\circ + 3$ and $y = 7 + 5\cos(x)^\circ$ for $0 \le x \le 360$.

The two graphs intersect at T, which has coordinates (p, q).



- (*a*) Find the exact value of $\cos p$.
- (*b*) Determine the value of *p*.

