Mathematics
Higher
Paper 2
Practice Paper R

Time allowed
1 hour 10 minutes

NATIONAL QUALIFICATIONS

Read carefully

1 **Calculators may be used in this paper**.

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product : $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$, where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives :

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

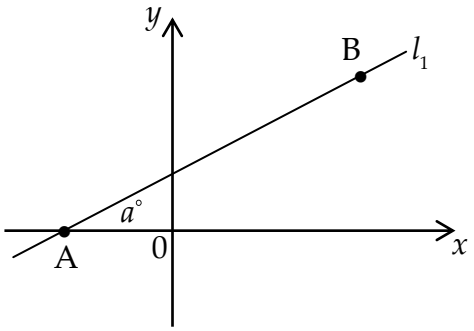
Table of standard integrals :

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted.

Marks

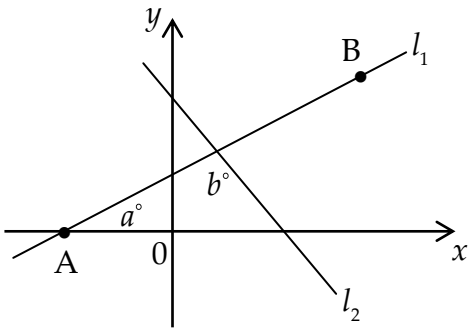
1. (*a*) A line, l_1, passes through the points $A(-3, 0)$ and $B(5, 4)$.



The line makes an angle of a° with the positive direction on the x-axis.

Find the value of a. **3**

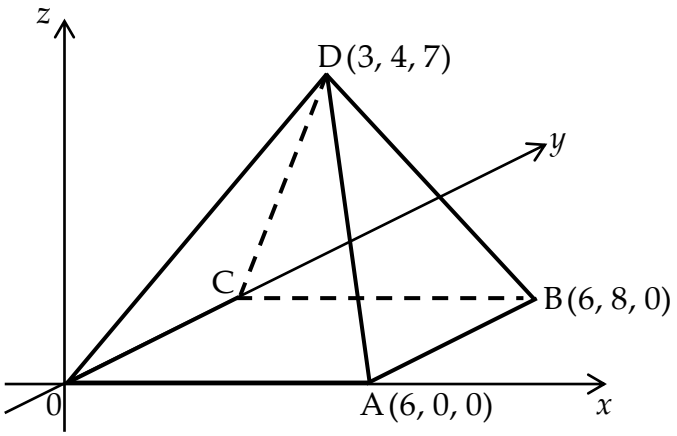
(*b*) A second line, l_2, with equation $4x + 3y = 12$, crosses the line in (*a*).



The angle between the two lines is b°, as shown.

Find the value of b. **4**

2. The rectangular based pyramid D,OABC has vertices $A(6, 0, 0)$, $B(6, 8, 0)$ and $D(3, 4, 7)$.



(*a*) (i) Write down the coordinates of C.

(ii) Express $\overrightarrow{AC}$ and $\overrightarrow{AD}$ in component form. **3**

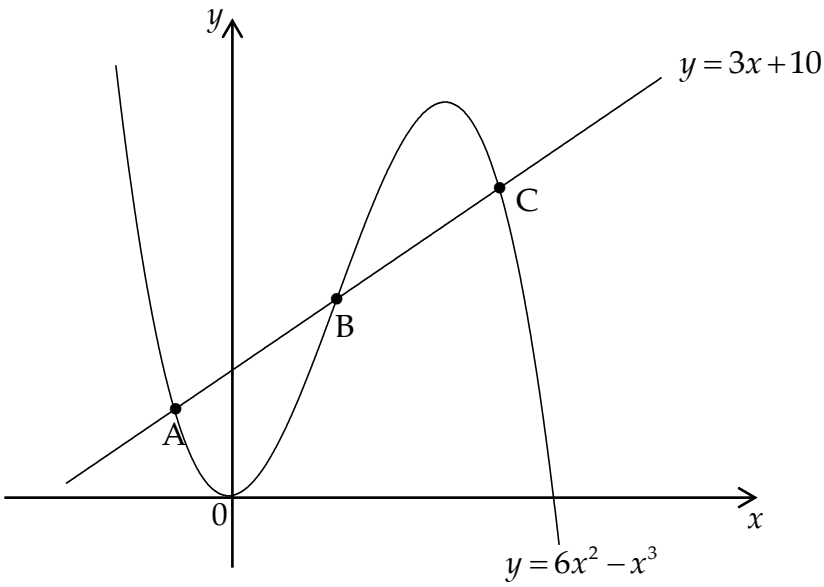
(*b*) Calculate the size of angle CAD. **5**

Marks

3. (*a*) (i) Show that $(x-2)$ is a factor of $x^3-6x^2+3x+10$.

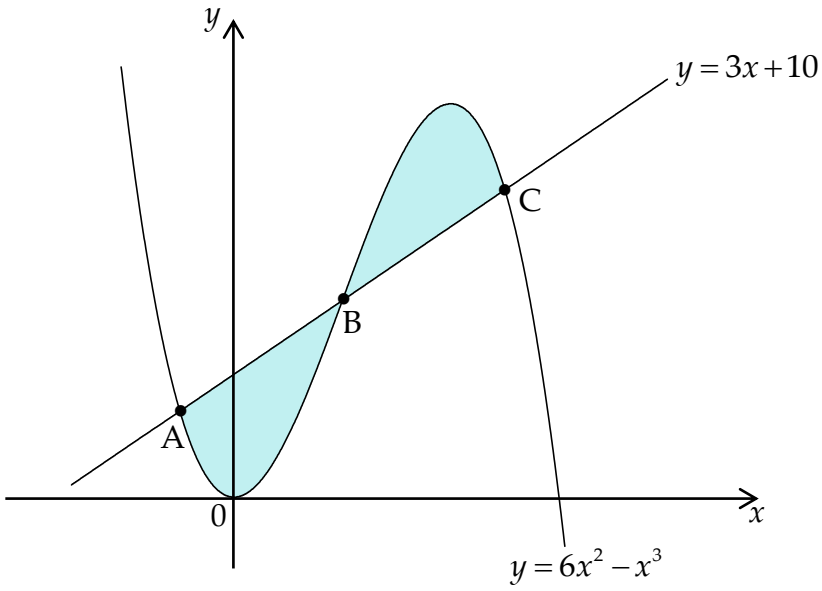
(ii) Hence factorise $x^3-6x^2+3x+10$ fully. **4**

The line with equation $y=3x+10$ intersects the curve with equation $y=6x^2-x^3$ at the points A, B and C.



(*b*) Find the x-coordinates of the points A and C. **3**

The area between the curve and the line from A to C is shaded in the diagram below.



(*c*) Calculate the total shaded area shown in the diagram. **7**

Marks

4. Solve $2\cos 2x - \sin x + 1 = 0$ for $0 \le x < 2\pi$. **6**

5. A new '24 hour anti-biotic' is being tested on a patient in hospital.

It is know, that over a 24 hour period, the amount of anti-biotic remaining in the bloodstream is reduced by 80%.

On the first day of the trial, an initial 250 mg dose is given to a patient at 7 a.m.

(*a*) After 24 hours and just prior to the second dose being given, how much anti-biotic remains in the patient's bloodstream? **1**

The patient is then given a further 250 mg dose at 7 a.m. and at this time each subsequent morning thereafter.

(*b*) A recurrence relation of the form $u_{n+1} = au_n + b$ can be used to model this course of treatment.

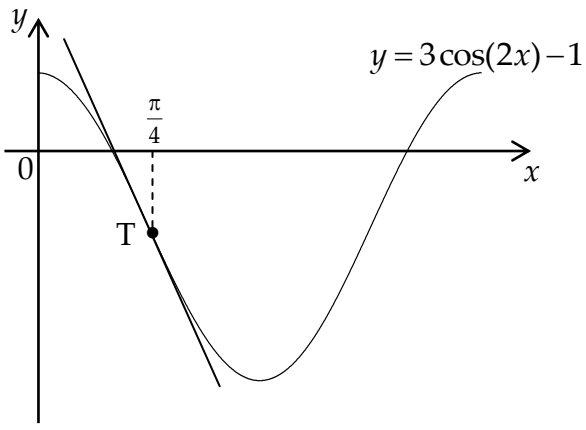
Write down the values of *a* and *b*. **2**

It is also known that more than 350 mg of the drug in the bloodstream results in unpleasant side effects.

(*c*) Is it safe to administer this anti-biotic over an extended period of time? **4**

6. The diagram shows part of the graph of $y = 3\cos(2x) - 1$.

Find the equation of the tangent at the point T, where $x = \frac{\pi}{4}$.



7

7. Solve $\log_x(x+2) + \log_x(2x-3) = 2,\ x > \frac{3}{2}$. **5**

Marks

8. A circle has the following properties:

- The x-axis and the line $y = 20$ are tangents to the circle.
- The circle passes through the points $(0, 2)$ and $(0, 18)$.
- The centre lies in the first quadrant.

Find the equation of this circle. **6**

End of Question Paper