Mathematics
Higher
Paper 1
Practice Paper R

Time allowed
1 hour 30 minutes

NATIONAL QUALIFICATIONS

Read carefully

Calculators may <u>NOT</u> be used in this paper.

Section A – Questions 1 – 20 (40 marks)

Section B (30 marks).

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product : $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$, where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives :

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals :

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

SECTION A

ALL questions should be attempted.

1. K and L have position vectors $\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$ respectively.

   What is the magnitude of $\overrightarrow{KL}$?

   A $\sqrt{5}$

   B $\sqrt{10}$

   C $\sqrt{22}$

   D $\sqrt{55}$

2. If $f(x) = x^3 - 4x + 7$, find $f'(-2)$.

   A 7

   B 8

   C 11

   D 12

3. Find $\int \left(x^{\frac{1}{4}} - x^{-2} \right) dx$

   A $\frac{1}{4}x^{-\frac{3}{4}} + 2x^{-3} + c$

   B $x^{\frac{3}{4}} + x^{-3} + c$

   C $x^{\frac{5}{4}} - x^{-1} + c$

   D $\frac{4}{5}x^{\frac{5}{4}} + x^{-1} + c$

4. A function f is defined on the set of real numbers by $f(x) = 4x + 5$.

Find an expression for $f(f(x))$.

A $f(f(x)) = 8x + 10$

B $f(f(x)) = 16x + 25$

C $f(f(x)) = 16x^2 + 20x + 5$

D $f(f(x)) = 16x^2 + 40x + 25$

5. Evaluate $4\sqrt{2}\sin\frac{\pi}{4}\cos\frac{2\pi}{3}$.

A -12

B -2

C $2\sqrt{3}$

D $2\sqrt{6}$

6. A circle with centre $(-3, 4)$ passes through the point $(-2, 2)$.

What is the equation of the circle?

A $(x-3)^2 + (y+4)^2 = 5$

B $(x-3)^2 + (y+4)^2 = 61$

C $(x+3)^2 + (y-4)^2 = 5$

D $(x+3)^2 + (y-4)^2 = 61$

7. $f(x) = 2x^3 - x^2 - 5x + 4$.

What is the remainder when $f(x)$ is divided by $(x+2)$?

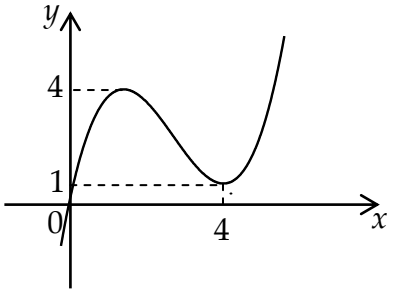
A -6

B -2

C 0

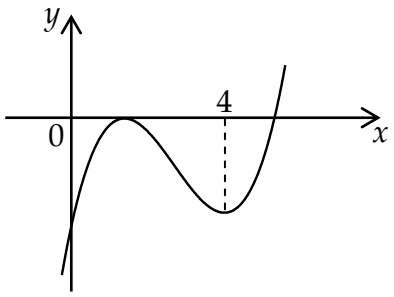
D 18

8. The diagram shows the part of the graph of the cubic $y = f(x)$.

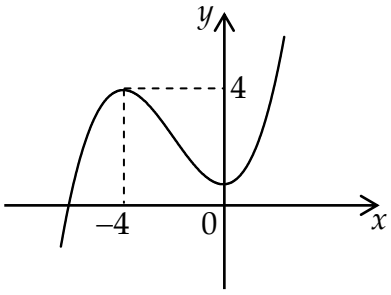


Which diagram below shows the graph of $y = 4 - f(x)$?

A



B



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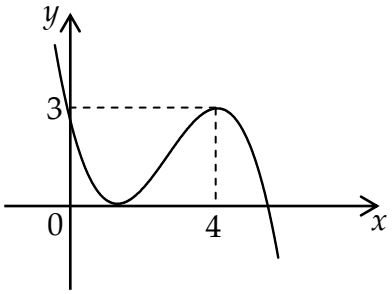

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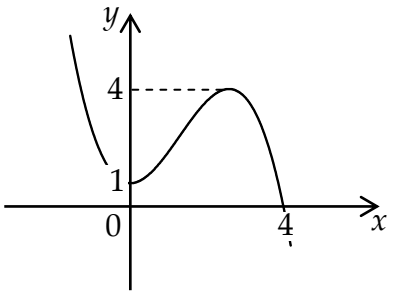
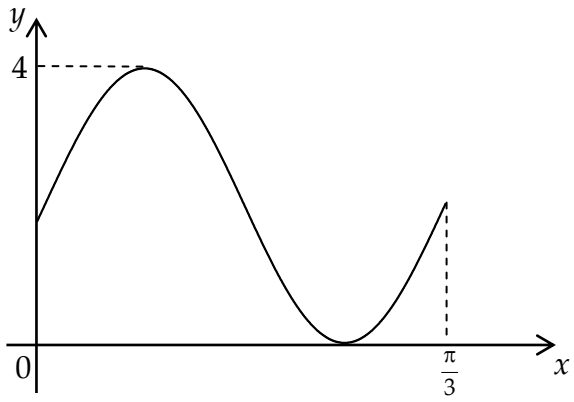


9. The graph shown in the diagram has equation $y = p + 2\sin(qx)$.



What are the values of p and q?

	p	q
A	2	3
B	4	3
C	2	6
D	4	6

10. A sequence is generated by the recurrence relation $u_{n+1} = 7 - 2u_n$.

If $u_2 = 5$, what is the value of u_0 ?

A 3

B 5

C 7

D 13

11. For what value of k does the equation $kx^2 - 6x + 1 = 0$ have equal roots?

A 0

B 1

C 9

D 12

12. Find $\int (2x+7)^4 \, dx$.

A $\frac{1}{10}(2x+7)^5 + c$

B $\frac{1}{5}(x^2+7x)^5 + c$

C $2(2x+7)^3 + c$

D $4(x^2+7x)^3 + c$

13. Given that $f'(x) = 6x^2$ and $f(1) = 5$, find a formula for $f(x)$ in terms of x.

A $f(x) = 12x - 7$

B $f(x) = 6x^2 - 1$

C $f(x) = 2x^3$

D $f(x) = 2x^3 + 3$

14. What are the coordinates of the centre of the circle with equation

$$3x^2 + 3y^2 - 6x + 18y - 5 = 0\,?$$

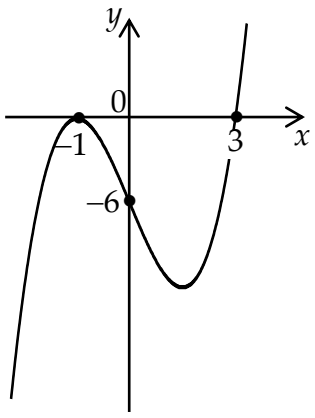
A $(1, -3)$

B $(3, -9)$

C $(-2, 6)$

D $(-6, 18)$

15. The diagram shows part of the graph of a cubic function.



What is the equation of this graph?

A $y = -2(x-1)^2(x+3)$

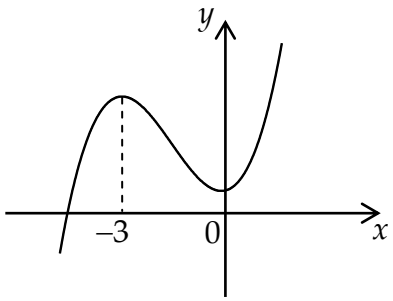
B $y = -(x+1)^2(x-3)$

C $y = (x-1)^2(x+3)$

D $y = 2(x+1)^2(x-3)$

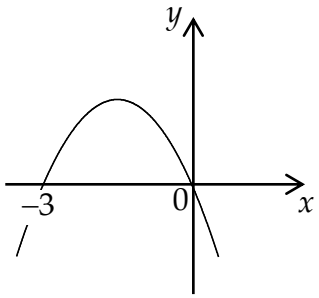
16. The diagram shows part of the graph of the cubic $y = f(x)$.

There are stationary points at $x = 0$ and $x = -3$.

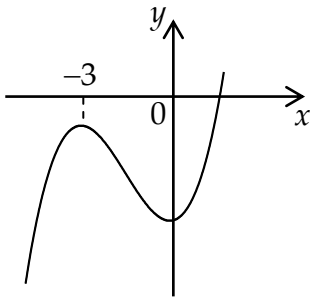


Which diagram below shows the graph of $y = f'(x)$?

A



B



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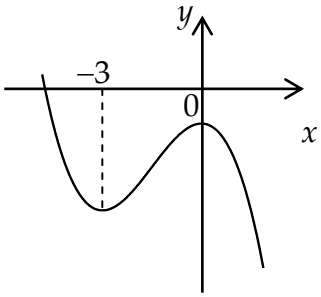

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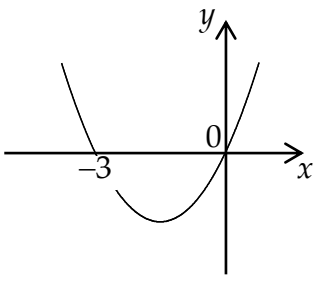


17. If $4x^2 + 8x - 1$ is expressed in the form $4(x+p)^2 + q$, what is the value of q?

A -5

B -2

C -1

D 0

18. If $3\log_2 t - \log_2 5 = 3$, find the value of t.

A $\frac{8}{3}$

B $\sqrt[3]{40}$

C $\frac{13}{3}$

D 5

19. If $p = 4x^{-3}$ find the rate of change of p with respect to x when $x = 2$.

A -12

B -3

C $-\frac{3}{4}$

D $\frac{1}{2}$

20. What is the solution of $8 - 2x - x^2 < 0$?

A $-2 < x < 4$

B $x < -2$ or $x > 4$

C $x < -4$ or $x > 2$

D $-4 < x < 2$

End of Section A

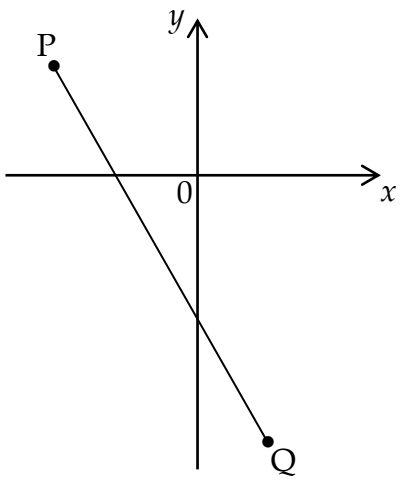
SECTION B

ALL questions should be attempted.

Marks

21. A line joins the points $P(-4, 3)$ and $Q(2, -7)$.

Find the equation of the perpendicular bisector of PQ.



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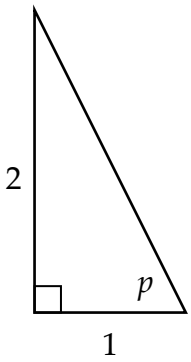
22. Show that the line with equation $y = 2x + 10$ is a tangent to the circle with equation $x^2 + y^2 - 2x - 4y - 15 = 0$ and find the coordinates of the point of contact of the tangent and circle. **6**

23. The diagram shows a right-angled triangle with height 2 units, base 1 unit and an angle of p.



(*a*) Find the exact values of:

(i) $\cos p$;

(ii) $\cos 2p$. **3**

(*b*) By writing $3p = 2p + p$, find the exact value of $\cos 3p$. **4**

24. A function f is defined by $f(x) = x^3 - 2x^2 - 4x + 1$, where $0 \le x \le 3$.

Find the maximum and minimum values of f. **5**

Marks

25. (*a*) Express $2\sqrt{2}\cos x^\circ - 2\sqrt{2}\sin x^\circ$ in the form $k\cos(x-a)^\circ$, where $k > 0$ and $0 \le a < 360$. **4**

(*b*) Find:

(i) the maximum value of $3 + 2\sqrt{2}\sin x^\circ - 2\sqrt{2}\cos x^\circ$;

(ii) a value of x where this maximum value occurs in the interval $0 \le x < 360$. **4**

End of question paper