Mathematics	Time allowed	NATIONAL
Higher	1 hour 10 minutes	QUALIFICATIONS
Paper 2		
Practice Paper P		

Read carefully

1 **Calculators may be used in this paper**.

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product : $\quad \boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

$$\text{or} \quad \boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3, \text{ where } \boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.$$

Trigonometric formulae:
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives :

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals :

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\dfrac{1}{a}\cos ax + C$
$\cos ax$	$\dfrac{1}{a}\sin ax + C$

ALL questions should be attempted.

Marks

1. (*a*) Given that $(x+1)$ is a factor of $2x^3 + 3x^2 + kx - 6,$ find the value of k. **3**

(*b*) Hence, or otherwise, solve $2x^3 + 3x^2 + kx - 6 = 0$. **4**

2. OABC,DEFG is a rectangular prism as show.
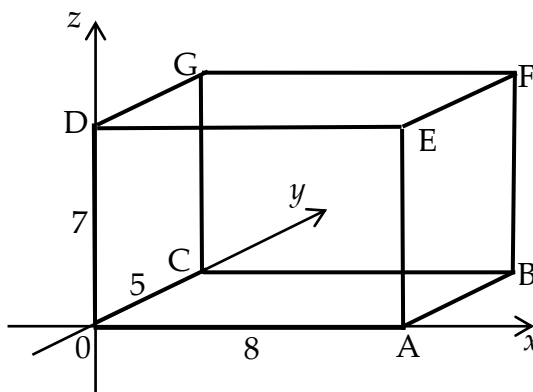
OA is 8 units long, OC is 5 units and OD is 7 units.

(*a*) Write down the coordinates of B and G. **2**

(*b*) Calculate the size of angle BEG. **6**

3. A circle, centre C, has equation $x^2 + y^2 - 4x - 2y - 20 = 0$.

(*a*) Find the centre C and radius of this circle. **2**

(*b*) (i) Show that the point $P(5, -3)$ lies on the circumference of the circle.

(ii) Find the equation of radius CP. **4**

(*c*) Find the equation of the chord which passes through $(7, 1)$ and is
perpendicular to radius CP. **3**

4. Solve $3\cos 2x = 11\cos x - 6$ for $0 \le x < 2\pi$. **6**

5. (*a*) Diagram 1 shows part of the graph with equation $y = x^3 - 5x^2 + 2x + 8$.
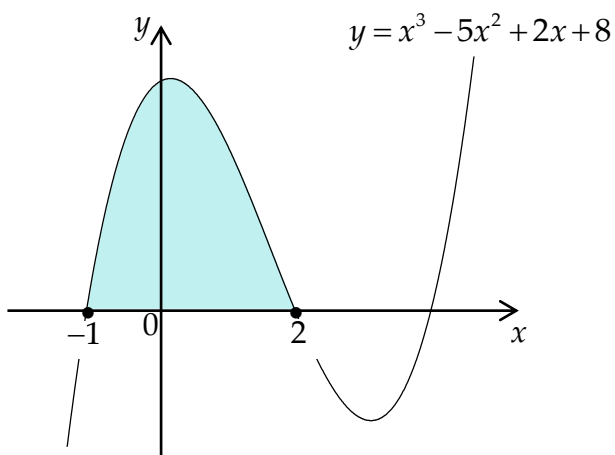
Calculate the shaded area.

$y = x^3 - 5x^2 + 2x + 8$

Diagram 1

5

(*b*) Given that

$$\int_{-1}^{p} (x^3 - 5x^2 + 2x + 8)\, dx = 12 \cdot 13$$

find the total shaded area in diagram 2.

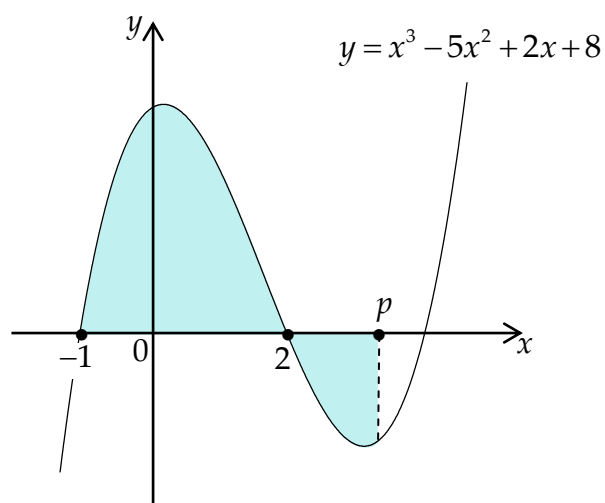$y = x^3 - 5x^2 + 2x + 8$

Diagram 2

2

6. Find the smallest integer value of c for which

$$g(x) = (x - 2)(x^2 - 2x + c)$$
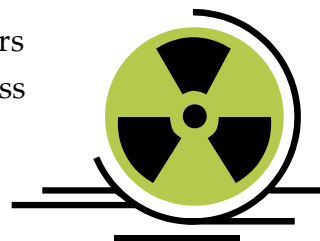
has only one real root.

5

7. (*a*) Write $2\sin x + \sqrt{5}\cos x$ in the form $k\sin(x + a)$, where $k > 0$ and $0 \le a < \frac{\pi}{2}$.

4

(*b*) Sketch the graph of $y = 4\sin x + 2\sqrt{5}\cos x$ for $0 \le x \le 2\pi$.

4

8. For a particular radioactive isotope, the mass of the
original isotope remaining, *m* grams, after time *t* years
is given by $m = m_0 e^{-0.18t}$ where m_0 is the original mass
of the isotope.

    (*a*) If the original mass is 1000g, find the mass of the original isotope
remaining after 20 years.   **2**

The half-life of the isotope is the time taken for half the original mass to decay.

    (*b*) Find the half life of this isotope.   **3**

9. Find $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\frac{\sin 4x}{\sin 2x}\right) dx.$ **5**

End of Question Paper