Mathematics
Higher
Paper 1
Practice Paper P

Time allowed
1 hour 30 minutes

NATIONAL QUALIFICATIONS

Read carefully

Calculators may <u>NOT</u> be used in this paper.

Section A – Questions 1 – 20 (40 marks)

Section B (30 marks).

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product : $\boldsymbol{a} \,.\, \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or $\boldsymbol{a} \,.\, \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$, where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives :

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals :

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

SECTION A

ALL questions should be attempted.

1. A sequence is defined by the recurrence relation

$$u_{n+1} = 2u_n - 5, \quad u_0 = 6$$

What is the value of u_2 ?

A 9

B 6

C –1

D –5

2. Here are two statements about the line with equation $3x + 4y - 8 = 0$.

(1) This line is parallel to a line with gradient $-\frac{3}{4}$.

(2) This line cuts the y-axis at the point $(0, 8)$.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

3. Functions f and g are defined on suitable domains by

$$f(x) = 3x + 5 \text{ and } g(x) = 2 - x.$$

Find an expression for $f(g(x))$.

A $f(g(x)) = 11 - 3x$

B $f(g(x)) = 2x + 7$

C $f(g(x)) = 5 + 6x - 3x^2$

D $f(g(x)) = 10 + x - 3x^2$

4. A curve has equation $y = x^3 - 2x + 5$.

What is the gradient of the tangent at the point where $x = 2$?

A 1

B 2

C 9

D 10

5. A circle with centre $(-2, 1)$ passes through the point $(5, -2)$.

What is the equation of the circle?

A $(x+2)^2 + (y-1)^2 = 58$

B $(x+2)^2 + (y-1)^2 = 10$

C $(x-2)^2 + (y+1)^2 = 58$

D $(x-2)^2 + (y+1)^2 = 10$

6. Find $\int \frac{2}{\sqrt[3]{x}}\, dx$.

A $\frac{3}{2}x^{-\frac{4}{3}} + c$

B $3x^{\frac{2}{3}} + c$

C $\frac{2}{3}x^{\frac{4}{3}} + c$

D $\frac{2}{3}x^{\frac{5}{3}} + c$

7. $g(x) = x^3 - 2x^2 + x + 7.$

What is the remainder when $g(x)$ is divided by $(x+1)$?

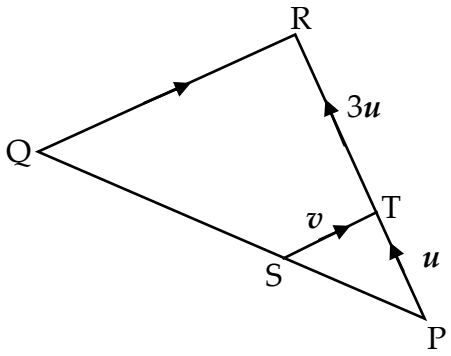
A -1

B 0

C 3

D 7

8. Vectors $\boldsymbol{u}$ and v are shown in the diagram below.



$|\mathrm{QR}| = 3|\mathrm{ST}|$

Find $\overrightarrow{\mathrm{PQ}}$ in terms of **u** and **v**.

A $3\boldsymbol{u} + \boldsymbol{v}$

B $3\boldsymbol{u} - 4\boldsymbol{v}$

C $4\boldsymbol{u} + 4\boldsymbol{v}$

D $4\boldsymbol{u} - 3\boldsymbol{v}$

9. P and Q are the points with coordinates $(-1, 0, 5)$ and $(2, 3, 3)$ respectively.

If $\overrightarrow{\mathrm{PR}} = 2\overrightarrow{\mathrm{PQ}}$, find the coordinates of R.

A $(1, 6, 21)$

B $(2, 6, 16)$

C $(4, 3, -7)$

D $(5, 6, 1)$

10. What is the exact value of $\sin\frac{5\pi}{4} + \cos\frac{\pi}{4}$?

A -2

B 0

C 1

D 2

11. Find $\int 5\cos(2x-1)\,dx$.

A $\frac{5}{2}\sin(2x-1)+c$

B $-10\sin(2x-1)+c$

C $\frac{5}{2}\cos(2x-1)+c$

D $-10\cos(2x-1)+c$

12. Given that $\log_2 y = 3\log_2 x + \log_2 8$, express y in terms of x.

A $y = 3x+3$

B $y = 3x+8$

C $y = 8x^3$

D $y = x^3+8$

13. Given that $y = \sin^4 x$, find $\frac{dy}{dx}$.

A $\sin^3 x$

B $4\cos^3 x$

C $4\sin^3 x\cos x$

D $4\cos^3 x\sin x$

14. If $5-6x-x^2$ is written in the form $p-(x+q)^2$, what is the value of p?

A -4

B -1

C 5

D 14

15. Solve $\tan^2 x = \frac{1}{3}$ for $\frac{\pi}{2} < x < \pi$.

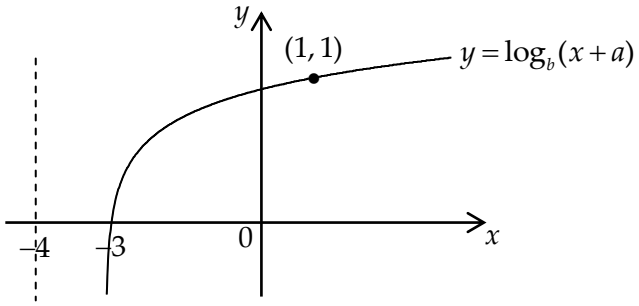
A $\frac{3\pi}{4}$

B $\frac{4\pi}{5}$

C $\frac{5\pi}{6}$

D $\frac{6\pi}{7}$

16. The diagram shows the graph with equation $y = \log_b(x + a)$.



What are the values of a and b?

	a	b
A	3	4
B	4	5
C	3	5
D	4	4

17. What is the nature of the roots of the quadratic equation $x^2 + 10x = 25$?

A Two real equal roots

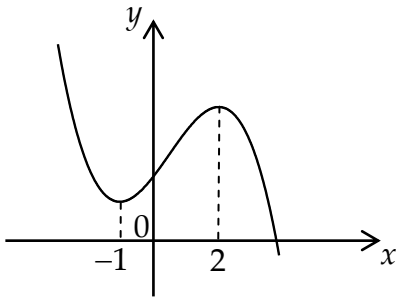
B Two real distinct roots

C No real roots

D Three real distinct roots

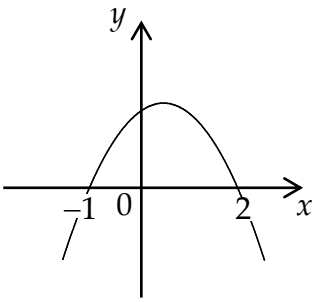
18. The diagram shows part of the graph of cubic with equation $y = g(x)$.

The graph has turning points at $x = -1$ and $x = 2$.

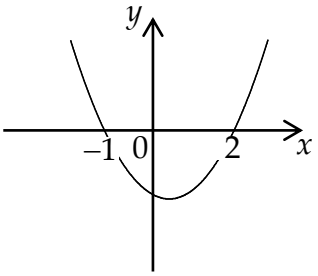


Which diagram below shows the graph of $y = g'(x)$?

A



B



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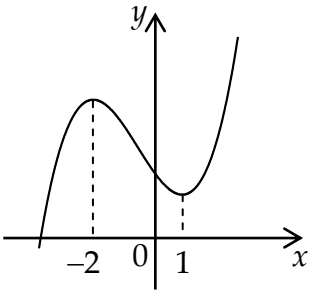

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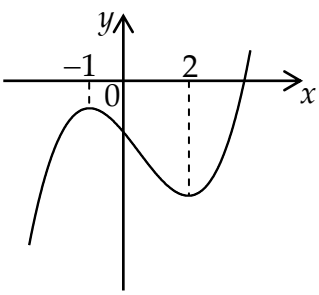


19. Solve $x^2 - 8x + 15 \geq 0$.

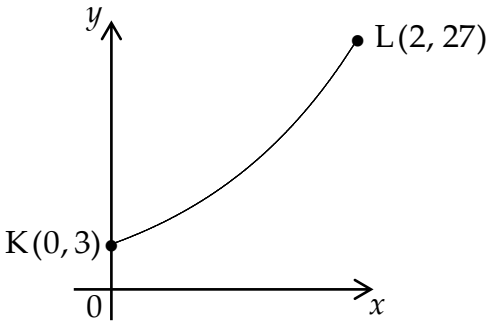
A $-5 \leq x \leq -3$

B $x \leq -5$ or $x \geq -3$

C $3 \leq x \leq 5$

D $x \leq 3$ or $x \geq 5$

20. The diagram shows part of the curve $y = f(x)$.



The curve passes through the points $K(0, 3)$ and $L(2, 27)$.

Which of the following represents the equation of the curve?

A $y = x^2 + 3$

B $y = 3^{x+1}$

C $y = e^{x+3}$

D $y = 3^x + 24$

End of Section A

SECTION B

ALL questions should be attempted.

Marks

21. A function f is defined by $f(x) = 2x^3 - 3x^2$, where x is a real number.

(a) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x and y-axes. **3**

(b) Find the stationary points on the curve $y = f(x)$ and determine their nature. **6**

(c) (i) Sketch the curve $y = f(x)$.

(ii) Hence solve $2x^3 > 3x^2$. **3**

22. Two sequences are generated by the recurrence relations

$$u_{n+1} = 0 \cdot 4u_n + 8 \cdot 4$$
$$v_{n+1} = kv_n + 2$$

The two sequences approach the same limit as $n \to \infty$.

(a) Evaluate this limit. **2**

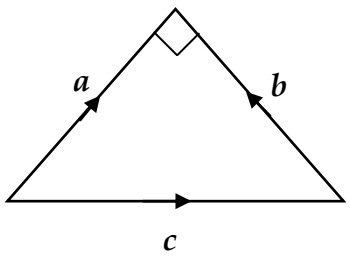
(b) Hence determine the value of k. **2**

23. Given that $\sin a = \frac{4}{5}$ and $\sin b = \frac{2}{\sqrt{5}}$, where $0 \le a < \frac{\pi}{2}$ and $0 \le b < \frac{\pi}{2}$, find the exact values of :

(a) $\sin(a+b)$; **4**

(b) $\tan(a+b)$. **4**

24. In the triangle opposite $|\boldsymbol{a}| = |\boldsymbol{b}| = 2$ units



Find $\boldsymbol{a} . (\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$ **6**

End of question paper