Mathematics Higher Paper 1 Practice Paper O

Time allowed 1 hour 30 minutes NATIONAL QUALIFICATIONS

Read carefully

Calculators may <u>NOT</u> be used in this paper.

Section A – Questions 1 – 20 (40 marks)

Section B (30 marks).

- 1. Full credit will be given only where the solution contains appropriate working.
- 2. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre (-g, -f) and radius $\sqrt{g^2 + f^2 - c}$. The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product : $a \cdot b = |a| |b| \cos \theta$, where θ is the angle between *a* and *b*.

or
$$\boldsymbol{a} \cdot \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$
, where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:
$$sin(A \pm B) = sin A cos B \pm cos A sin B$$

 $cos(A \pm B) = cos A cos B \mp sin A sin B$
 $sin 2A = 2 sin A cos A$
 $cos 2A = cos^2 A - sin^2 A$
 $= 2 cos^2 A - 1$
 $= 1 - 2 sin^2 A$

Table of standard derivatives :

f(x)	f'(x)
sin ax	a cos ax
cos ax	$-a\sin ax$

Table of standard integrals :

f(x)	$\int f(x)dx$
sin ax	$-\frac{1}{a}\cos ax + C$
cos ax	$\frac{1}{a}\sin ax + C$

SECTION A

ALL questions should be attempted.

- 1. Given that $f(x) = 2x^4 5x$, find f'(2).
 - A 6
 - B 22
 - C 42
 - D 59
- 2. Find $\int (x-3)(3x+1) dx$. A 6x+cB x^3-4x^2-3x+c C $6x^3-3x+c$ D $3x^4-8x^2-3x+c$
- 3. P and Q have coordinates (2, -3, 2) and (1, 0, 5).

What is the distance between P and Q?

- A $\sqrt{12}$ units
- B $\sqrt{19}$ units
- C $\sqrt{37}$ units
- D $\sqrt{67}$ units
- 4. If $x^2 8x + 3$ is expressed in the form $(x p)^2 + q$, what is the value of q?
 - A –13
 - В —1
 - C 3
 - D 19

- 5. Here are two statements about the equation $3x^2 5x + 1 = 0$.
 - (1) The roots are equal.
 - (2) The roots are rational.

Which of the following is true?

- A Neither statement is correct.
- B Only statement (1) is correct.
- C Only statement (2) is correct.
- D Both statements are correct.
- 6. Find all the values of *x* in the interval $0 \le x < 2\pi$ for which $\cos x = -\frac{\sqrt{3}}{2}$.
 - A $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ B $\frac{5\pi}{6}$ and $\frac{5\pi}{3}$ C $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$ D $\frac{\pi}{3}$ and $\frac{7\pi}{6}$
- 7. S is the point with coordinates (2, -1, 1), T(4, 1, 5) and U(5, 2, 7).Find the ratio in which T divides SU.
 - A 1:2
 - B 1:3
 - C 2:1
 - D 3:1
- 8. Given that $y = 4\sin(3x-2)$, find $\frac{dy}{dx}$.
 - A $-\sin(3x-2)$
 - B $-\cos(3x-2)$
 - C $12\sin(3x-2)$
 - D $12\cos(3x-2)$

9. The angle between the line shown in the diagram and the *x*-axis is $\frac{\pi}{3}$. What is the gradient of the line? A $\sqrt{3}$ B $\frac{\sqrt{3}}{2}$ C $\frac{1}{\sqrt{3}}$ D $\frac{1}{2}$

10. Given that $\log_a 9 = \frac{2}{3}$, what is the value of *a*?

- A 6
- B 13.5
- C 27
- D 81

11. What is the maximum value of $9-4\sin\left(x-\frac{\pi}{5}\right)$?

- A 1
- B 5
- C 9
- D 13

12. Find $\int (3x-11)^4 dx$. A $\frac{4}{3}(3x-11)^3 + c$ B $4(3x-11)^3 + c$ C $\frac{1}{15}(3x-11)^5 + c$ D $4(3x-11)^5 + c$



14. Given that $h(x) = \frac{1}{x^2 - 16}$, what is the largest possible domain for *h*?

- $A \quad \square \$, the set of real numbers.
- B $\square \{0\}$
- C □ -{-4, 4}
- D $\Box \{-16, 16\}$

15. Vector *t* has components $\begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$. *u* is a unit vector such that u = kt, where k > 0.

Find the value of *k*.

 $\begin{array}{c} A & \frac{1}{25} \\ B & \frac{1}{5} \\ C & 1 \\ D & 5 \end{array}$

16. The diagram shows the graph of y = f(x).



Which diagram below shows the graph of y = 2 - f(x)?





17. The equation of the parabola shown is of the form y = kx(x-4).

What is the value of *k*?

- A -4
- В —1
- C 1
- D 2

18. Simplify $2\log_3 x - \log_3(x+1)$.

- A $\log_3\left(\frac{x^2}{x+1}\right)$ B $\log_3\left(\frac{x+1}{2x}\right)$ C $\log_3(x-1)$ D $\log_3(2x^2+2x)$
- 19. What is the solution to $x^2 + 4x 5 < 0$?
 - A -5 < x < 1
 - B x < -1 or x > 5
 - C x < -4 or x > -1
 - D 1 < x < 4
- 20. If $v = 10t^2$ and the rate of change of v with respect to t at t = k, k > 0 is 160, find the value of k.
 - A 0
 - B 4
 - C 8
 - D 16



SECTION B

ALL questions should be attempted.

Marks

2

4

21. A circle with equation $x^2 + y^2 + 6x + 2y + 9 = 0$ has centre C₁.



(*a*) Write down the coordinates of the centre C_1 and find the length of the radius of this circle.

A second circle with equation $(x-3)^2 + (y-7)^2 = 36$ has centre C₂.



- (b) (i) Find the distance between the centres C_1 and C_2 .
 - (ii) Hence find the minimum distance between the circumferences of the two circles.

3

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2

5

- **22.** A is the point with coordinates (1, -1, 2), B(3, 0, 3) and C(-2, 3, 4).
 - (a) Express \overrightarrow{AB} and \overrightarrow{AC} in component form. 2
 - (*b*) Find the size of angle BAC.
- **23.** Solve $2\sin 2x = 5\cos x$ for $0 \le x < 2\pi$.
- **24.** The diagram shows part of the quartic with equation y = g(x). There are stationary points at x = -2, x = 0 and x = a.



On separate diagrams sketch the graph of

(a)
$$y = g'(x)$$
. 3

- (b) y = g'(x-3).
- **25.** Find the values of *x* for which the function $f(x) = 5 + 24x + 3x^2 x^3$ is decreasing.
- 26. P is the point with coordinates (-1, -6) and Q is (3, 10).
 Find the locus of points which are equidistant from both P and Q.
 4

End of question paper