Mathematics
Higher
Paper 1
Practice Paper N

Time allowed
1 hour 30 minutes

NATIONAL QUALIFICATIONS

Read carefully

Calculators may <u>NOT</u> be used in this paper.

Section A – Questions 1 – 20 (40 marks)

Section B (30 marks).

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product : $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$, where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives :

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals :

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

SECTION A

ALL questions should be attempted.

1. A sequence is defined by the recurrence relation $u_{n+1} = 2u_n + 1,\ u_0 = 3.$

   What is the value of u_2 ?

   A 1

   B 5

   C 11

   D 15

2. The line with equation $kx - 2y + 9 = 0$ is parallel to the line with gradient 7.

   What is the value of *k*?

   A –7

   B $-\frac{1}{7}$

   C 7

   D 14

3. A circle has equation $x^2 + y^2 - 8x + 2y - 1 = 0.$

   What is the radius of this circle?

   A $\sqrt{6}$ units

   B $\sqrt{7}$ units

   C $\sqrt{18}$ units

   D $\sqrt{69}$ units

4. What is the derivative of $\frac{x^3-2}{3x}$ with respect to x?

A $\frac{2}{3}x+\frac{2}{3}x^{-2}$

B $6x+6x^{-2}$

C $2x^2$

D $\frac{2}{3}x^3+\frac{2}{3}$

5. Find $\int \frac{1}{2x^4}\,dx$.

A $-\frac{1}{8x^3}+c$

B $-\frac{1}{6x^3}+c$

C $\frac{1}{8x^3}+c$

D $\frac{5}{2x^5}+c$

6. If $x^2-12x+37$ is written in the form $(x-p)^2+q$, find the value of q.

A 1

B 25

C 31

D 37

7. A sequence is generated by the recurrence relation $u_{n+1}=0\cdot 8u_n+16$.

What is the limit of this sequence as $n\to\infty$?

A 2

B $13\frac{1}{4}$

C 16

D 80

8. A circle with centre $(-1, 5)$ passes through the point $(2, 7)$.

What is the equation of the circle?

A $(x-1)^2+(y+5)^2=145$

B $(x-1)^2+(y+5)^2=13$

C $(x+1)^2+(y-5)^2=145$

D $(x+1)^2+(y-5)^2=13$

9. The vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ with components $\boldsymbol{p}=\begin{pmatrix}1\\k\\2\end{pmatrix}$ and $\boldsymbol{q}=\begin{pmatrix}k\\-3\\-2\end{pmatrix}$ are perpendicular.

What is the value of k?

A -2

B -1

C 0

D 1

10. Here are two statements about the equation $2x^2-8x+3=0$

(1) The roots are real;

(2) The roots are irrational.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

11. What is the value of $\cos\frac{5\pi}{3}-\tan\frac{7\pi}{4}$?

A -1

B $-\frac{1}{2}$

C $\frac{\sqrt{3}}{2}$

D $\frac{3}{2}$

12. Given that $\log_2 \frac{1}{8} = p$, find the value of p.

A -3

B -2

C $\frac{1}{64}$

D 3

13. Find $\int (3x-11)^5 \, dx$

A $\frac{1}{15}(3x-11)^4 + c$

B $\frac{1}{2}(3x-11)^4 + c$

C $\frac{1}{18}(3x-11)^6 + c$

D $15(3x-11)^6 + c$

14. K and L are the points with coordinates $(0, -1, 4)$ and $(3, -2, 5)$ respectively.

If $\overrightarrow{KM} = 3\overrightarrow{KL}$, find the coordinates of M.

A $(1, -1, 3)$

B $(1, 0, -1)$

C $(9, -4, 7)$

D $(9, -9, 27)$

15. $h(x) = \dfrac{4}{x^2 - 2x - 8}$.

For what values of x is $h(x)$ undefined?

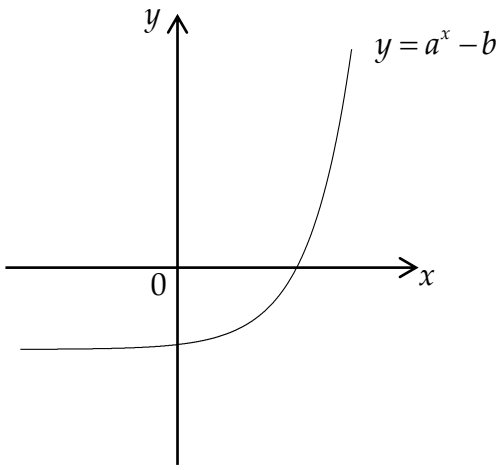
A -2 and 4

B -1 and 2

C 0 and 4

D 2 and -4

16. Here are two statements about the graph with equation $y = a^x - b$, shown opposite.



(1) $0 < a < 1$;

(2) y is always increasing

Which of the following is true?

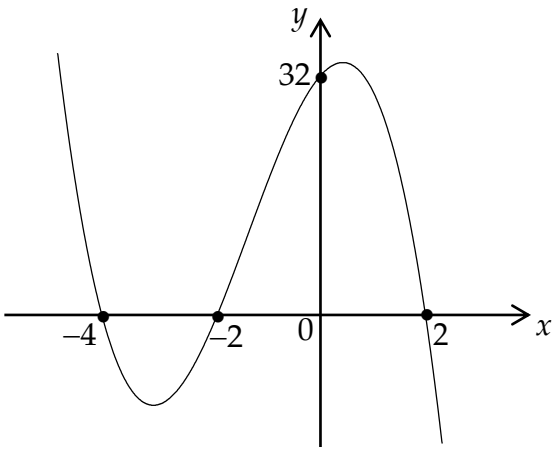
A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

17. The diagram shows part of the graph of a cubic.



What is the equation of this graph?

A $y = -32(x+2)(x-2)(x-4)$

B $y = (x+2)(x-2)(x-4)$

C $y = -2(x-2)(x+2)(x+4)$

D $y = 4(x-2)(x+2)(x+4)$

18. Given that $\log_4 y = 2 - \log_4 5x$, express y in terms of x.

A $y = \frac{2}{5x}$

B $y = \frac{16}{5x}$

C $y = \frac{1}{25x^2}$

D $y = 25x^2$

19. If $\boldsymbol{p} . (\boldsymbol{p} - \boldsymbol{q}) = 18$ and $|\boldsymbol{p}| = 3$, find the value of $\boldsymbol{p} . \boldsymbol{q}$

A –15

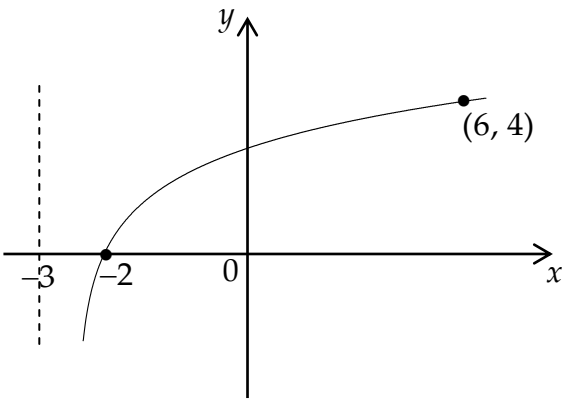
B –9

C 2

D 15

20. The diagram shows part of the curve with equation $y = p\log_3(x + k)$.

What is the value of p?



A –3

B –2

C 2

D 4

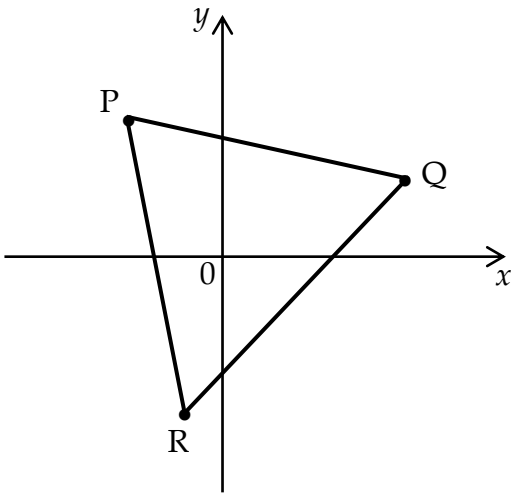
End of Section A

SECTION B

ALL questions should be attempted.

Marks

21. Triangle PQR has vertices P(–3, 5), Q(7, 3) and R(–1, –5), as shown.



(*a*) Find the equation of the median RM. **3**

(*b*) Find the equation of the altitude AP. **3**

(*c*) Find the coordinates of the point of intersection of RM and AP. **2**

22. Find the stationary points on the curve given by $y = x^3 - 9x^2 + 24x - 2$ and determine their nature. **7**

23. (*a*) Functions f and g are defined on suitable domains by

$$f(x) = 2x^2 + 5 \text{ and } g(x) = x - 1$$

Find $f(g(x))$. **2**

(*b*) Sketch the curve with equation $y = f(g(x))$. **3**

24. (*a*) Show that $2\sin\left(x + \frac{\pi}{6}\right) - 2\cos x = \sqrt{3}\sin x - \cos x.$ **2**

(*b*) Express $\sqrt{3}\sin x - \cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. **4**

(*c*) Hence, or otherwise, solve $2\sin\left(x + \frac{\pi}{6}\right) = 2\cos x + \sqrt{3}$, where $0 \le x \le 2\pi$. **4**

End of question paper