1.
Let the limit be L
then $L = 0{\cdot}9L - 1$
$\Rightarrow L - 0{\cdot}9L = -1 \Rightarrow 0{\cdot}1L = -1$

$$\text{So } L = \frac{-1}{0.1}\frac{(\times 10)}{(\times 10)} = \frac{-10}{1} = -10$$

Choice A

2 marks

- Remember that limits only exist if the multiplier, in this case $0{\cdot}9$, lies between -1 and 1
- No calculater is available during Paper 1 so you have to be able to calculate $-\frac{1}{0.1}$. The easiest was is to multiply 'top' and 'bottom' by 10.

HMRN: p24

2.
$A(2k,3)$ and $B(k,5)$

$$\Rightarrow m_{AB} = \frac{3-5}{2k-k} = \frac{-2}{k} = -\frac{2}{k}$$

but $m_{AB} = 4$ So $-\frac{2}{k} = 4$

$$\Rightarrow -2 = 4k \Rightarrow k = -\frac{2}{4} = -\frac{1}{2}$$

Choice B

2 marks

- The gradient formula is:

  If $P(x_1, y_1), Q(x_2, y_2)$ then $m_{PQ} = \dfrac{y_2 - y_1}{x_2 - x_1}$.

  You are not given this formula during the exam.

- $-\frac{2}{k} = 4$ multiply both sides by k

  so $-\dfrac{2}{\cancel{k}_1} \times \cancel{k}^1 = 4 \times k \Rightarrow -2 = 4k$ etc....

- It is possible, to work through the choices, in each case finding points A and B and calculating the gradient. This will be more time consuming

HMRN: p3

3.
$x^2 - 4 = (x-2)(x+2)$ so both $(x-2)$ and $(x+2)$ are facters
Since $f(-1) = 0$, $(x+1)$ is a factor
so $f(x) = (x+1)(x-2)(x+2)$

Choice D

2 marks

- The 'difference of squares' pattern is:

  $$A^2 - B^2 = (A - B)(A + B)$$

  In this case you have $x^2 - 2^2$

- For a polynomial equation $f(x) = 0$ there is a correspondence between roots and factors:
  -1 is a root $\Leftrightarrow x + 1$ is a factor
  $f(-1) = 0$ tells you -1 is a root.

HMRN: p 26

4.

$$u_{n+1} = -\frac{1}{2}u_{n+1}, u_o = 4$$

$$\text{So } u_1 = -\frac{1}{2} \times 4 + 1 = -2 + 1 = -1$$

$$\text{and } u_2 = -\frac{1}{2} \times (-1) + 1$$

$$= \frac{1}{2} + 1 = \frac{3}{2}$$

Choice C

2 marks

- The process is:

$$u_0 \xrightarrow[\text{then add 1}]{\text{multiply by } -\frac{1}{2}} u_1 \xrightarrow[\text{then add 1}]{\text{multiply by } -\frac{1}{2}} u_2$$

In this case:

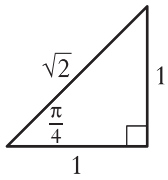
$$4 \xrightarrow{4\times(-\frac{1}{2}) + 1} -1 \xrightarrow{-1\times(-\frac{1}{2}) + 1} \frac{3}{2}$$

HMRN: p 23

5.

$$2\cos x - \sqrt{2} = 0 \Rightarrow 2\cos x = \sqrt{2}$$

$$\Rightarrow \cos x = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{1}{\sqrt{2}}$$



So $x = \dfrac{\pi}{4}$

(1st quadrant only)

Choice B

2 marks

- The initial strategy is to change the equation to the format: cos (angle) = number

  in this case $\cos x = \dfrac{\sqrt{2}}{2}$
- You have to recognise an exact value however $\dfrac{\sqrt{2}}{2}$ is not the normal form.
- $\dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{2}\times\sqrt{2}}{2\times\sqrt{2}} = \dfrac{\cancel{2}^{1}}{{}_{1}\cancel{2}\sqrt{2}} = \dfrac{1}{\sqrt{2}}$ is an alternative.

HMRN: p15, p34

6.

$$4y = -3x + 2$$

$$\Rightarrow y = -\frac{3}{4}x + \frac{2}{4}$$

$$\text{So } m = -\frac{3}{4} \Rightarrow m_1 = \frac{4}{3}$$

Choice C

2 marks

- The initial strategy here is to change the equation to the format: $y = mx + c$ in this case $m = -\dfrac{3}{4}$.
- In general if $m = \dfrac{a}{b}$ then $m_1 = -\dfrac{b}{a}$, the process is to 'invert' the fraction and change the sign. In this case negative changes to positive.

HMRN: p 4–5

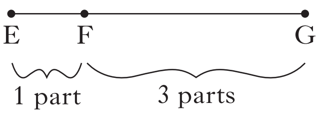
7.

E(1,–1,–1), F(–1,–1,0), G(–7,–1,3)

$$\overrightarrow{EF} = \boldsymbol{f} - \boldsymbol{e} = \begin{pmatrix}-1\\-1\\0\end{pmatrix} - \begin{pmatrix}-1\\-1\\-1\end{pmatrix} = \begin{pmatrix}-2\\0\\1\end{pmatrix}$$

$$\overrightarrow{FG} = \boldsymbol{g} - \boldsymbol{f} = \begin{pmatrix}-7\\-1\\3\end{pmatrix} - \begin{pmatrix}-1\\-1\\0\end{pmatrix} = \begin{pmatrix}-6\\0\\3\end{pmatrix}$$

$$\text{So } \overrightarrow{FG} = 3\overrightarrow{EF}$$



F divides EG in the ratio 1:3

Choice C

2 marks

- If you know three points are collinear then select two different 'journeys' between the points. In the solution $\overrightarrow{EF}$ and $\overrightarrow{FG}$ were selected. The 'position vector' result eg $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$ is used to calculate the components of these 'journeys'. Collinearity ensures that one 'journey' is a multiple of the other 'journey'. In this case $\overrightarrow{FG} = 3\overrightarrow{EF}$. Other selections, in this case, would give:

  $\overrightarrow{EF} = \frac{1}{4}\overrightarrow{EG}$, $\overrightarrow{FG} = \frac{3}{4}\overrightarrow{EG}$, $\overrightarrow{GE} = 4\overrightarrow{FE}$ etc.

  Then use this information to sketch the relationship between the points (see diagram in solution) and therefore determine the required ratio.

HMRN: p 44

8.

$$\begin{array}{r|rrrrr} 2 & 2 & -3 & 0 & -3 & 1 \\ & & 4 & 2 & 4 & 2 \\ \hline & 2 & 1 & 2 & 1 & 3 \end{array}$$

The remainder is 3

Choice D

2 marks

- The 'synthetic division scheme' is used.
- Take care to look for 'missing terms'. In this case there is no 'x^2 term' and that is recorded as a 0 in the top row.
- $\begin{array}{r|l} a & \text{Polynomial coefficients} \\ \hline & \qquad\qquad\qquad\qquad b \end{array}$ This means if you divide the polynomial by $x-a$ the remainder is b.

HMRN: p 25–26

9.

$$\begin{aligned} &(x + 4)(x - 2) \\ &= x^2 + 2x - 8 \\ &= (x + 1)(x + 1) - 1 - 8 \\ &= (x + 1)^2 - 9 \end{aligned}$$

Compare: $(x + a)^2 + b$

so $b = -9$

Choice B

2 marks

- The question is trying to confuse you by not giving the quadratic in 'standard form' i.e. with no brackets: $x^2 + 2x - 8$
- Remember to take your answer, expand it and check it works. In this case:

  $(x + 1)^2 - 9 = x^2 + 2x + 1 - 9 = x^2 + 2x - 8$
- b is not 9, which is a common mistake. Fortunately that is not one of the choices.

HMRN: p13

10.

$$f(x) = (1 - x^3)^{1/3}$$

$$\Rightarrow f'(x) = \frac{1}{3}(1 - x^3)^{-2/3} \times (-3x^2)$$

$$= -x^2(1 - x^3)^{-2/3}$$

Choice A

2 marks

- The function given is of the format $f(x) = (g(x))^{1/3}$ where $g(x) = 1 - x^3$.

  The 'chain rule' is used in this case, so:

  $$f'(x) = \frac{1}{3}(g(x))^{1/3} \times g'(x)$$

  where $g'(x) = -3x^2$
- Note that

  $$\frac{1}{3} \times (-3x^2) = -\frac{1}{3} \times 3 \times x^2 = -1 \times x^2$$

HMRN: p 48–49

11.

$y = -(f(x) + 1) = -f(x) - 1$

$y = f(x) \rightarrow y = -f(x)$

This is reflection in the x-axis

$y = -f(x) \rightarrow y = -f(x) - 1$

followed by a translation of 1 unit in the negative direction parallel to the y-axis

Choice B

2 marks

- An alternative method would be to first consider $y = f(x) + 1$. This is the graph $y = f(x)$ shifted up 1 unit parallel to the y-axis. Then $y = -(f(x) + 1)$ is the new graph 'flipped' in the x-axis.
- Remember to follow your steps using each of the points (0,0) and (1,–1) to check your final choice works for the known points on the graph.

HMRN: p 9–10

12.

$$\boldsymbol{m} = \frac{1}{2}(\boldsymbol{a}+\boldsymbol{b}) = \frac{1}{2}\left[\begin{pmatrix}-1\\2\\0\end{pmatrix}+\begin{pmatrix}-2\\3\\1\end{pmatrix}\right]$$

$$= \frac{1}{2}\begin{pmatrix}-3\\5\\1\end{pmatrix} = \begin{pmatrix}-3/2\\5/2\\1/2\end{pmatrix}$$

Choice B

2 marks

- The connection between the coordinates of a point P and its position vector $\boldsymbol{p}$ is as follows:

  P(a,b,c) and $\boldsymbol{q} = \begin{pmatrix}a\\b\\c\end{pmatrix}$

  The mid point result for points:

  P(x_1,y_1,z_1), Q(x_2,y_2,z_2)

  Mid point$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

  works for position vectors

  HMRN: p 44–45

13.
For stationary points set $f'(x) = 0$
$\Rightarrow x^2 + 1 = 0 \Rightarrow x^2 = -1$
and since $x^2 \geq 0$ this equation has no Real solutions. f has no stationary points

Choice A

2 marks

- In this case notice $f'(x)$ is a quadratic. So $f'(x) = 0$ is therefore a quadratic equation. Any solution (root) of this equation gives a stationary point
- Alternative method: find discriminant of $x^2+1 = 0$ In this case compare $ax^2 + bx + c = 0$ giving $a = 1$, b = 0, $c = 1$ Discriminant $= 0^2 - 4 \times 1 \times 1 = -4$ A negative discriminant $\Rightarrow$ no Real roots $\Rightarrow$ no stat. pts

  HMRN: p 20, p 27

14.

$$m_{OA} = \frac{3}{2} \text{ so } \tan a° = \frac{3}{2}$$

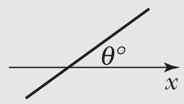
$$\Rightarrow a° = \tan^{-1}\left(\frac{3}{2}\right)$$

Choice D

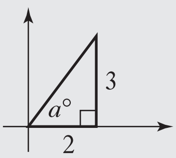
2 marks

- The result used is:

  gradient = tan θ



- Alternatively from the given diagram you get $\tan a° = \frac{3}{2}$



- Always use $\tan^{-1}$ to find the angle.

  HMRN: p 4

15.
$\log_4(x^2-4) - 2\log_4(x-2)$
$= \log_4(x^2-4) - \log_4(x-2)^2$

$$= \log_4 \frac{x^2-4}{(x-2)^2} = \log_4 \frac{(x-2)(x+2)}{(x-2)(x-2)}$$

$$= \log_4 \frac{x+2}{x-2}$$

Choice C

2 marks

- The 'laws of logs' used are:

  $m \log_b a = \log_b a^m$

  and $\log_b m - \log_b n = \log_b \frac{m}{n}$
- In fractions with algebraic expressions you should always factorise 'top & bottom' expression to see if cancellation can take place. In this case $x - 2$ can be cancelled.

  HMRN: p 50

16.

$$\int (2-3x)^{1/3}\,dx$$

$$= \frac{(2-3x)^{4/3}}{4/3 \times (-3)} + c = \frac{(2-3x)^{4/3}}{-4} + c$$

$$= -\frac{1}{4}(2-3x)^{4/3} + c$$

Choice A

2 marks

- The integration result used here is called a 'special integral'. The formula used in this case is:

$$\int (ax+b)^n dx = \tfrac{(ax+b)^{n+1}}{a(n+1)} + c$$

where $a = -3$, $b = 2$ and $n = \frac{1}{3}$. Notice the division by 'a' the coefficient of x, -3 in this case.

- Note: $\frac{1}{3}+1=\frac{1}{3}+\frac{3}{3}=\frac{4}{3}$

and $\frac{4}{3}\times(-3) = -\frac{4\times \cancel{3}^1}{\cancel{3}_1} = -4$

- You could, of course, differentiate each choice using the 'chain rule' to determine the correct choice. The 'inverse' of integration is differentiation.

HMRN: p 49

17.

P(− 1,2,5) and Q(− 3, − 1, 4)

$\overrightarrow{QR} = -2\overrightarrow{PQ} \Rightarrow \boldsymbol{r} - \boldsymbol{q} = -2(\boldsymbol{q} - \boldsymbol{p})$

$\Rightarrow \boldsymbol{r} - \boldsymbol{q} = -2\boldsymbol{q} + 2\boldsymbol{p} \Rightarrow \boldsymbol{r} = -\boldsymbol{q} + 2\boldsymbol{p}$

So

$$\boldsymbol{r} = -\begin{pmatrix}-3\\-1\\4\end{pmatrix} + 2\begin{pmatrix}-1\\2\\5\end{pmatrix} = \begin{pmatrix}3+2\times(-1)\\1+2\times 2\\-4+2\times 5\end{pmatrix}$$

$$\Rightarrow \boldsymbol{r} = \begin{pmatrix}1\\5\\6\end{pmatrix} \text{ So R(1,5,6)}$$

Choice A

2 marks

- Rewriting a 'journey' like $\overrightarrow{AB}$ in terms of the position vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ of the points A and B allows you to use your algebra skills to solve a question like this. The basic result is:

$$\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$$

So, in this case, $\overrightarrow{QR} = -2\overrightarrow{PQ}$ is translated into $\boldsymbol{r} - \boldsymbol{q} = -2(\boldsymbol{q} - \boldsymbol{p})$. The aim is then to solve this equation for $\boldsymbol{r}$ since both p and q are known: $\boldsymbol{r} = -\boldsymbol{q} + 2\boldsymbol{p}$

- If A (x_1,y_1,z_1) then $a = \begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix}$

HMRN: p 44–45

18.

$f(x) = -2\sin 3x$

$f'(x) = -2\cos 3x \times 3 = -6\cos 3x$

Choice C

2 marks

- This question involves use of the 'chain rule'. In this case the result used is:

$f(x) = b\sin(g(x)) \Rightarrow f'(x) = b\cos(g(x)xg'(x)$ with $b = -2$ and $g'(x) = 3x$ so $g'(x) = 3$.

- Your formulae sheet gives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$

HMRN: p 48–49

19.

$y = k(x - 1)(x + 2)$

$\left(-\tfrac{1}{2}, 9\right)$ lies on the curve

So $y = 9$ when $x = -\tfrac{1}{2}$

$$\Rightarrow 9 = k\left(-\tfrac{1}{2} - 1\right)\left(-\tfrac{1}{2} + 2\right)$$

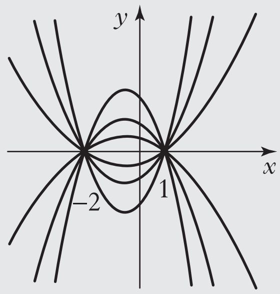
$$\Rightarrow 9 = k \times \left(\frac{-3}{2}\right) \times \left(\frac{3}{2}\right)$$

$$\Rightarrow 9 = -\frac{9}{4}k \Rightarrow 36 = -9k \Rightarrow k = -4$$

Choice B

2 marks

- To understand the role k plays in this question here is a diagram showing the graph $y = k(x - 1)(x + 2)$ for different values of k:



All these curves pass through (0, − 2) and (0, 1), These intercepts corresponding to factors $(x + 2)$ and $(x - 1)$.

Only one curve passes through $\left(-\tfrac{1}{2}, 9\right)$ and there will be one value of k that gives this curve. Substituting $x = -\tfrac{1}{2}$ and $y = 9$ in the equation $y = k(x-1)(x + 2)$ will determine what that value is.

HMRN: p 29

20.

$$\int_0^2 \frac{1}{(4x+1)^{3/2}}\,dx = \left[\frac{1}{2(4x+1)^{1/2}}\right]_0^2$$

$$= -\frac{1}{2\sqrt{4\times 2+1}} - \left(-\frac{1}{2\sqrt{4\times 0+1}}\right)$$

$$= -\frac{1}{2\sqrt{9}} + \frac{1}{2\sqrt{1}} = -\frac{1}{6} + \frac{1}{2}$$

$$= -\frac{1}{6} + \frac{3}{6} = \frac{2}{6} = \frac{1}{3}$$

Choice D

2 marks

- When limits appear on the integral sign the 'constant of integration' c is not required
- You are using:

$$\int_a^b f(x)dx = \left[F(x)\right]_a^b = F(b) - F(a)$$

where $F(x)$ is the result of integrating $f(x)$.

- You should know: $a^{1/2} = \sqrt{a}$

HMRN: p 49

21.

$\sin 2x - \sqrt{3}\sin x = 0$ ✓

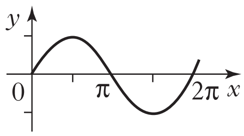
$\Rightarrow 2\sin x\cos x - \sqrt{3}\sin x = 0$

$\Rightarrow \sin x\,(2\cos x - \sqrt{3}) = 0$ ✓

$\Rightarrow \sin x = 0$ or $\cos x = \dfrac{\sqrt{3}}{2}$ ✓

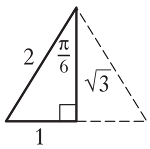
For $\sin x = 0$

$x = 0, \pi, 2\pi$



For $\cos x = \dfrac{\sqrt{3}}{2}$. x is in 1st or 4th quadrants

1st quadrant angle is $\dfrac{\pi}{6}$.



So

$$x = \frac{\pi}{6} \text{ or } 2\pi - \frac{\pi}{6} = \frac{12\pi}{6} - \frac{\pi}{6} = \frac{11\pi}{6}$$ ✓

Solutions are:

$$x = 0, \frac{\pi}{6}, \pi, \frac{11\pi}{6}, 2\pi\,.$$

5 marks

Strategy

- Using the 'Double Angle Formula' allows you to factorise the expression. The formula $\sin 2A = 2\sin a\cos a$ is given to you in the exam.

Factorise

- The common factor is $\sin x$

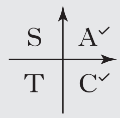
Solve

- Notice $2\cos x - \sqrt{3} = 0$

  $\Rightarrow 2\cos x = \sqrt{3} \Rightarrow 2\cos x = \frac{\sqrt{3}}{2}$

Angles

- For $\sin x$ or $\cos x$ equal to values -1, 0 or 1 you should use the $y = \sin x$ or $y = \cos x$ graph to determine the angles.
- Take care that the angles you give ≤ as solutions are allowed. In this case the interval in which the angles lie is $0 \le x \le 2\pi$.
- For quadrant use:
  ✓ shows cosine positive



HMRN: p15, p37

22. (*a*)

$y = x^3 - 3x^2 - 24x - 28$ ✓

$\Rightarrow \frac{dy}{dx} = 3x^2 - 6x - 24$ ✓

For stationary points set $\frac{dy}{dx} = 0$ ✓

$\Rightarrow 3x^2 - 6x - 24 = 0$

$\Rightarrow 3(x^2 - 2x - 8) = 0$

$\Rightarrow 3(x + 2)(x - 4) = 0$

$\Rightarrow x + 2 = 0$ or $x - 4 = 0$

$\Rightarrow x = -2$ or $x = 4$. ✓

x:		-2		4	
$\frac{dy}{dx} = 3(x+2)(x-4)$:	$+$		$-$		$+$
Shape of graph:	/	—	\	—	/
nature:		max		min	

✓

when $x = -2$

$y = (-2)^3 - 3 \times (-2)^2 - 24 \times (-2) - 28$

$= -8 - 12 + 48 - 28 = 0$

so $(-2, 0)$ is a maximum stationary point

when $x = 4$

$y = 4^3 - 3 \times 4^2 - 24 \times 4 - 28$

$= 64 - 48 - 96 - 28 = -108$ ✓

so $(4, -108)$ is a minimum stationary point ✓

7 marks

Strategy
- Evidence that you knew to differentiate will gain you this strategy work.

Differentiate
- The rule used here is:

$$f(x) = ax^n \Rightarrow f'(x) = nax^{n-1}$$

where a is a constant

Strategy
- The 2nd strategy mark is for setting $\frac{dy}{dx}$ equal to zero. The places on a curve where the gradient is zero are the stationary points

***x*-values**
- Remember when solving quadratic equations to check for common factors, in this case 3. This reduces the size of the coefficients and makes the subsequent factorisation easier.

Justify
- Evidence has to be given about the 'nature' of the stationary points. There are three types:

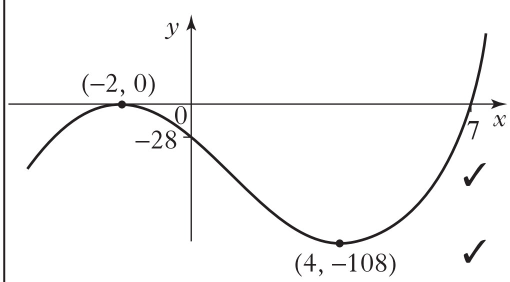
- The 'nature table' gives the justification and will gain you this mark.

***y*-values.**
- There is a mark allocated for the correct calculation of the two y-values for the stationary points
- Remember to use the 'original' formula $y =$ … and not the $\frac{dy}{dx}$ … gradient formula for this calculation

Communication
- Statements must be made stating 'maximum' or 'minimum'.

HMRN: p 20–21

22. (*b*)



✓ ✓

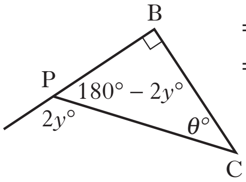
2 marks

Sketch
- 1 mark here will be for showing the correctly shaped graph with the maximum and minimum.
- 1 mark is for the 'annotation' i.e. labelling the points with $(-2,0)$ and $(4,-108)$
- Set $x = 0$ in $y = x^3 - 3x^2 - 24x - 28$ for y-intercept

HMRN: p 21

23. (*a*)

From the diagram



$\theta + 180 - 2y = 90$

$\Rightarrow \theta = 90 - 180 + 2y$

$\Rightarrow \theta = 2y - 90$ ✓

1 mark

Angle

- "Find θ in terms of y." means you are aiming for : θ = (an expression involving y only) So θ cannot appear on the 'right-hand side'.
- You are using: the angle sum in a triangle is 180°

23. (*b*)

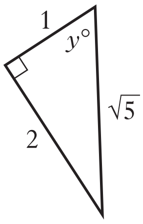
$\sin\theta° = \sin(2y-90)°$

$= \sin 2y°\cos 90° - \cos 2y° \sin 90°$ ✓

$= \sin 2y° \times 0 - \cos 2y° \times 1$ ✓

$= -\cos 2y° = -(2\cos^2 y° - 1)$ ✓

$= -2\cos^2 y° + 1 = 1 - 2\cos^2 y°$



now $\cos y° = \dfrac{1}{\sqrt{5}}$ ✓

So $\sin\theta° = 1 - 2\times\left(\dfrac{1}{\sqrt{5}}\right)^2$ ✓

$= 1 - 2\times\dfrac{1}{5} = 1 - \dfrac{2}{5} = \dfrac{3}{5}$ ✓

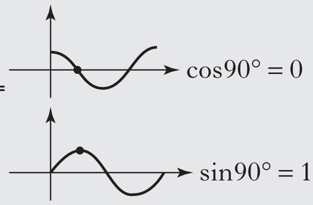
6 marks

Expansion

- The formula: $\sin(A \pm B) = \sin a \cos B \pm \cos a \sin B$ is given to you in your exam.

Simplify

- The values $\cos 90° = 0$ and $\sin 90° = 1$ are known from the graphs.



– remember no calculators in Paper 1:

Strategy

- Use of the 'Double angle' formula

Calculation

- The exact value of $\cos y°$ is obtained using Pythagoras' Theorem in Δ APD and then using SOHCAHTOA.

Value

- Substitution of $\frac{1}{\sqrt{5}}$ for $\cos y°$ in the expression

Simplify

- Notice : $\left(\dfrac{1}{\sqrt{5}}\right)^2 = \dfrac{1}{\sqrt{5}} \times \dfrac{1}{\sqrt{5}} = \dfrac{1\times 1}{\sqrt{5}\times\sqrt{5}} = \dfrac{1}{5}$

HMRN: p 15, p 35–36

24. $\log_{\sqrt{2}} x - \log_{\sqrt{2}} 2 = 2$

$\Rightarrow \log_{\sqrt{2}} \dfrac{x}{2} = 2$ ✓ ✓

$\Rightarrow \dfrac{x}{2} = \left(\sqrt{2}\right)^2 = 2$ ✓

$\Rightarrow x = 4.$ ✓

4 marks

Log Law

- The 'law' used is $\log_b m - \log_b n = \log_b \frac{m}{n}$

Exponential form

- Rewrite $\log_b a = c$ as $a = b^c$

Start Solution

- $\left(\sqrt{2}\right)^2 = \sqrt{2}\times\sqrt{2} = 2$

Finish Solutions

- $\dfrac{x}{2} = 2 \Rightarrow \dfrac{x}{2}\times 2 = 2\times 2 \Rightarrow x = 4$

HMRN: p 50

25.

$f'(x) = x^2(x-1) = x^3 - x^2$ ✓

$\Rightarrow f(x) = \int x^3 - x^2 \; dx$ ✓

$= \dfrac{x^4}{4} - \dfrac{x^3}{3} + c$ ✓

now $f(2) = \dfrac{1}{3}$ ✓

$\Rightarrow \dfrac{2^4}{4} - \dfrac{2^3}{3} + c = \dfrac{1}{3}$

$\Rightarrow 4 - \dfrac{8}{3} + c = \dfrac{1}{3}$ ✓

$\Rightarrow c = \dfrac{1}{3} + \dfrac{8}{3} - 4 = 3 - 4 = -1$

$so\ f(x) = \dfrac{1}{4}x^4 - \dfrac{1}{3}x^3 - 1$

5 marks

Strategy

- In questions where you are given $f'(x)$ (or $\dfrac{dy}{dx}$) and asked to find $f(x)$ (or y) then you need to use integration:

$$f(x) \xrightarrow{\text{differentiation}} f'(x)$$
$$f(x) \xleftarrow{\text{integration}} f'(x)$$

Preparation

- Multiplying out brackets is essential before you integrate

Integrate

- You are using $\int x^n dx = \dfrac{x^{n+1}}{n+1} + c$

Substitution

- The 'constant of integration' c is crucial in this question. $f(2) = \frac{1}{3}$ means when $x = 2$ the formula gives $\frac{1}{3}$. c can therefore be found.

Calculation

- No calculator — so practice your fraction work!

HMRN: p31

1. (*a*)

$3\sin x° - \cos x° = k\sin(x-a)°$

$\Rightarrow 3\sin x° - \cos x°$

$= k\sin x°\cos a° - k\cos x°\sin a°$ ✓

now equate the coefficients of $\sin x°$ and $\cos x°$

$\left.\begin{array}{l} k\cos a° = 3 \\ k\sin a° = 1 \end{array}\right\}$ since both $\sin a°$ and $\cos a°$ are positive, $a°$ is in the 1st quadrant. ✓

Divide:

$$\frac{k\sin a°}{k\cos a°} = \frac{1}{3} \Rightarrow \tan a° = \frac{1}{3}$$

so $a° = \tan^{-1}\left(\dfrac{1}{3}\right) \doteq 18{\cdot}4°$ ✓

(to 1 decimal place)

Square and add:

$(k\cos a°)^2 + (k\sin a°)^2 = 3^2+1^2$

$\Rightarrow k^2\cos^2 a° + k^2\sin^2 a° = 9 + 1$

$\Rightarrow k^2(\cos^2 a° + \sin^2 a°) = 10$

$\Rightarrow k^2 \times 1 = 10 \Rightarrow k^2 = 10$

$\Rightarrow k = \sqrt{10}\ (k > 0)$ ✓

So $3\sin x° - \cos x°$

$= \sqrt{10}\sin(x - 18{\cdot}4)°$

(correct to 1 decimal place)

4 marks

Strategy

- Your first step is to expand $k\sin(x-a)°$
- On your formulae sheet is:

  $\sin(A \pm B) = \sin a\cos B \pm \cos a\sin B$

  This is the expansion you should use
- Notice that there is an implied step in the working:

  $k\sin(x-a)° = k(\sin x°\cos a° - \cos x°\sin a°)$

  $= k\sin x°\cos a° - k\cos x°\sin a°$
- If this expansion does not appear in your working you will not be awarded this mark.

Coefficients

- The method is:

  $3\sin x° - 1\cos x°$

  $k\sin x°\cos a° - k\cos x°\sin a°$

  so $k\cos a° = 3$ and $k\sin a° = 1$

Angle

- A common mistake is to divide in the wrong order. The result you are using is $\frac{\sin a°}{\cos a°} = \tan a°$

  In this case since is on the 'top' of the fraction but comes from the 'bottom' equation so the fraction is $\frac{1}{3}$ not $\frac{3}{1}$.

Amplitude

- You should try to understand the method for calculating k. This will help you deal with more unusual questions that sometimes arise - these require understanding, not the blind application of a formula.

HMRN: p53–54

1. (*b*)

$3\sin x^\circ - \cos x^\circ = 1$

$\Rightarrow \sqrt{10}\ \sin(x - 18{\cdot}4\ldots)^\circ = 1$ ✓

$\Rightarrow \sin(x - 18{\cdot}4\ldots)^\circ = \dfrac{1}{\sqrt{10}}$

$\Rightarrow x^\circ - 18{\cdot}4\ldots^\circ = \sin^{-1}\left(\dfrac{1}{\sqrt{10}}\right)$

$= 18{\cdot}4\ldots^\circ$ ✓

so $x = 18{\cdot}43\ldots + 18{\cdot}43\ldots$

$= 36{\cdot}86\ldots$

$\Rightarrow x \doteqdot 36{\cdot}9$ (to decimal place)

$(0 \le x \le 90)$ ✓

3 marks

Strategy
- Using your result from part(a) you can rewrite the given equation in a form that is easier to solve

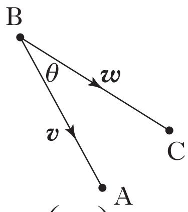
Solve for $(x–a)^\circ$
- Your aim is to move to an equation of the form sin (angle) = number
- Since the interval of allowable values of x is $0 \le x \le 90$ only the 1st quadrant value is considered in this particular example

Solve for x
- $x – 18{\cdot}4 = 18{\cdot}4 \Rightarrow x = 18{\cdot}4 + 18{\cdot}4$

HMRN: p 54

2.

Use $\cos\theta = \dfrac{\boldsymbol{v}\cdot\boldsymbol{w}}{(\boldsymbol{v})(\boldsymbol{w})}$



when $\boldsymbol{v} = \begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ ✓

$\boldsymbol{v}\cdot\boldsymbol{w} = \begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$

$= -2\times1 + 3\times(-1) + 5\times3 = 10$ ✓

$|\boldsymbol{v}| = \sqrt{(-2)^2 + 3^2 + 5^2} = \sqrt{4+9+25}$

$= \sqrt{38}$ ✓

$|\boldsymbol{w}| = \sqrt{1^2 + (-1)^2 + 3^2} = \sqrt{1+1+9}$

$= \sqrt{11}$ ✓

so $\cos\theta = \dfrac{10}{\sqrt{38}\ \sqrt{11}}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{10}{\sqrt{38}\ \sqrt{11}}\right)$ ✓

$\Rightarrow \theta = 60{\cdot}71\ \ldots \doteqdot 60{\cdot}7$ (to 1 dec. place)

5 marks

Strategy
- The formula given to you in the exam is: " $\boldsymbol{a}\cdot\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$" so you must know to rearrange this into the form used in this question: $\cos\theta = \dfrac{\boldsymbol{a}\cdot\boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|}$

As is the case for all formulae: LEARN THEM!

Dot product
- There is a check you can do with the 'dot product'

If $\boldsymbol{a}\cdot\boldsymbol{b} > 0$ the an is acute $(0^\circ < \theta^\circ < 90^\circ)$

If $\boldsymbol{a}\cdot\boldsymbol{b} < 0$ then the angle is obtuse $(90^\circ < \theta^\circ < 180^\circ)$

Magnitudes
- You will be awarded 1 mark for each correct answer.

Angle
- Many mistakes are made in this calculation. Use:

$\boxed{\text{INV}}\boxed{\cos}\boxed{(}\boxed{1}\boxed{0}\boxed{\div}\boxed{\sqrt{}}\boxed{3}\boxed{8}\boxed{\times}\boxed{\sqrt{}}\boxed{1}\boxed{1}\boxed{)}\boxed{)}$

as the keying sequence followed by $\boxed{\text{EXE}}$ (true for most calculators)

HMRN: p 46

3.

$x^2 - 2x + c^2 + 2 = 0$ ✓

Discriminant

$= (-2)^2 - 4 \times 1 \times (c^2 + 2)$ ✓

$= 4 - 4(c^2 + 2) = 4 - 4c^2 - 8$

$= -4c^2 - 4 = -4(c^2 + 1)$ ✓

Since $c^2 + 1 > 0$ then $-4(c^2 + 1) < 0$

So for all values of c the Discriminant < 0 and the equation has no Real roots ✓

4 marks

Strategy
- The positive/zero/negative nature of the discriminant of this quadratic equation has to be determined.

Substitution
- Comparing $ax^2 + bx + c' = 0$

  with $1x^2 - 2x + (c^2 + 2) = 0$

  gives $a = 1$, $b = -2$ and $c' = c^2 + 2$ (different c's!)

  So $b^2 - 4ac' = (-2)^2 - 4 \times 1 \times (c^2 + 2)$

Simplify
- Be careful with negatives eg $-4\,(c^2 + 2) = -4\,c^2 - 8$

Proof
- $-4 \times (c^2+1)$ is negative × positive = negative $c^2 + 1$ is always positive since c^2 is always positive or zero.

  HMRN: p 27

4. (*a*)

$$y = \frac{1}{3}x^3 - 2x^2$$ ✓

$$\Rightarrow \frac{dy}{dx} = x^2 - 4x$$ ✓

The tangent has gradient -4

so $\dfrac{dy}{dx} = -4$ ✓

$\Rightarrow x^2 - 4x = -4$

$\Rightarrow x^2 - 4x + 4 = 0$ ✓

$\Rightarrow (x - 2)(x - 2) = 0$

$\Rightarrow x = 2$ which is the required x-coordinate ✓

5 marks

Strategy
- Knowing to differentiate will earn you this mark.

Differentiate
- Notice the coefficients: $3 \times \frac{1}{3} = 1$ and $2 \times (-2) = -4$

Strategy
- This 2nd strategy mark is for setting the gradient equal to -4

Solve
- 2 marks are available: for starting the process and then completing it. You should recognise a quadratic equation and write it in 'standard form' i.e. $x^2 - 4x + 4 = 0$.

  HMRN: p 20

4. (*b*)

When $x = 2$

$$y = \frac{1}{3}\times 2^3 - 2\times 2^2 = \frac{8}{3} - 8 = -\frac{16}{3} \quad ✓$$

A point on the tangent is

$$\left(2, -\frac{16}{3}\right)$$

and the gradient = −4

So the equation is:

$$y - \left(-\frac{16}{3}\right) = -4(x-2)$$

$$\Rightarrow\ y + \frac{16}{3} = -4x + 8$$

$$\Rightarrow 3y + 16 = -12x + 24$$

$$\Rightarrow 3y + 12x = 8 \quad ✓$$

2 marks

Calculation

- To find the equation of the tangent you will need to know the coordinates of a point on that tangent. All you know so far is the *x*-coordinate ($x = 2$) and so calculation of the *y*-coordinate is essential.

Equation

- Here you are using $y - b = m(x - a)$ where $m = -4$ and the point (a, b) is $\left(2, -\frac{16}{3}\right)$
- Do not use decimal approximations in coordinate work - you will lose marks if you do eg - 5·3 should not be used for $-\frac{16}{3}$.

HMRN: p 20

5. (*a*)

$C_t = C_o e^{-t/4}$

In this case $C_t = 3{\cdot}5$ when $t = 3$

$$\Rightarrow 3{\cdot}5 = C_o e^{-3/4} \Rightarrow 3{\cdot}5 \frac{C_o}{e^{3/4}} \quad ✓$$

$$\Rightarrow C_o = 3{\cdot}5\,e^{3/4} = 7{\cdot}409\ldots \quad ✓$$

The concentration just after administration was 7·4 mg/ml (to 1 decimal place) ✓

3 marks

Strategy

- In a question like this you should know that you will have to pick out values for the letters in the formula and do a substitution. Sometimes it helps to label the formula:

$$\underbrace{C_t}_{\text{concentration after } t \text{ hours}} = \underbrace{C_o}_{\text{concentration at the start}}\, e^{-\frac{\overbrace{t}^{\text{the time elapsed } (t \text{ hours})}}{4}}$$

Change of Subject.

- C_t is the subject of the given formula. You are asked to find C_o - the concentration initially. This means you aim for C_o = (a number)

Calculation

- On the calculator use.

$\boxed{3}\boxed{\bullet}\boxed{5}\boxed{\times}\boxed{e^x}\boxed{(}\boxed{3}\boxed{\div}\boxed{4}\boxed{)}\boxed{\text{EXE}}$

HMRN: p 50–51

5. (*b*)

Required to find t so that

$$C_t = \frac{1}{2}C_o$$

$$\Rightarrow \frac{1}{2}C_o = C_o e^{-t/4}$$ ✓

$$\Rightarrow \frac{1}{2} = e^{-t/4}$$ ✓

$$\Rightarrow \log_e \tfrac{1}{2} = -\frac{t}{4}$$ ✓

$$\Rightarrow t = -4\log_e \tfrac{1}{2} = 2{\cdot}772\ldots$$

This is 2 hours and $0{\cdot}772\ldots \times 60$ min.

It takes 2 hours 46 minutes (to the nearest minute) ✓

4 marks

Interpretation
- You are being asked to calculate t for C_t, the concentration after t hours, to equal half of the initial concentration i.e. $\frac{1}{2}C_o$

1st step to solving
- Both sides of the equation can be divided by C_o. This creates an exponential equation with only the variable t.

Log Statement
- You use this conversion: $c = b^a \Leftrightarrow \log_b c = a$

  In this case $a = -\frac{t}{4}$, $b = e$ and $c = \frac{1}{2}$

Calculation
- The $\boxed{\text{ln}}$ key calculates '$\log_e$'

  HMRN: p 50–51

6. (*a*)

$x^2 + y^2 - 4x - 6y + 8 = 0$

Centre: $C_1(2, 3)$ ✓

For $C_1(2,3)$and $A(1,5)$ ✓

$$m_{CA} = \frac{5-3}{1-2} = \frac{2}{-1} = -2$$ ✓

$$\Rightarrow m_1 = \frac{1}{2}$$

Point on the tangent is A(1,5)

and the gradient = $\frac{1}{2}$ so the equation is:

$$y - 5 = \frac{1}{2}(x-1)$$

$\Rightarrow 2y - 10 = x - 1$

$\Rightarrow 2y - x = 9$ ✓

4 marks

Centre
- This uses the result:

  circle: $x^2 + y^2 + 2gx + 2fy + c = 0$

  Centre: $(-g\ ,\ -f)$

  The process involves halving and changing signs of the coefficients of x and y to get the coordinates of the centre of the circle.

Gradient of radius
- The gradient formula is:

  $P(x_1, y_1), Q(x_2, y_2) \Rightarrow m_{PQ} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Strategy
- The tangent is perpendicular to the radius to the point of contact i.e. C_1P
- Perpendicular gradients are obtained by using $m = \frac{a}{b} \Rightarrow m_1 = -\frac{b}{a}$. In this case -2 is thought of as $-\frac{2}{1}$

Equation
- Use $y - b = m(x - a)$. In this case you use $m = \frac{1}{2}$ with (a, b) being the point A(1,5)

  HMRN: p 3–4, p39

6. (*b*)

For intersection of line and circle solve:

$$\left.\begin{aligned} 2y - x &= 9 \\ x^2 + y^2 + 2x + 2y - 18 &= 0 \end{aligned}\right\} \quad ✓$$

Substitute $x = 2y - 9$ in circle equation: ✓

$(2y-9)^2 + y^2 + 2(2y-9) + 2y - 18 = 0$

$\Rightarrow 4y^2 - 36y + 81 + y^2 + 4y - 18 + 2y - 18 = 0$

$\Rightarrow 5y^2 - 30y + 45 = 0$ ✓

$\Rightarrow 5(y^2 - 6y + 9) = 0$

$\Rightarrow 5(y - 3)(y - 3) = 0$

$\Rightarrow y = 3$ ✓

and since there is only one solution the line is a tangent to the circle ✓

5 marks

Rearrangment

- Change $2y - x = 9$ to $x = 2y - 9$ ready for substitution

Strategy

- The strategy mark here is awarded for substitution of the line equation into the circle equation.

'Standard form'

- You should recognise a quadratic equation and therefore write it in the standard order, namely:

  $5y^2 - 30y + 45 = 0$

Solve

- It is always easier to factorise quadratic expressions if any common factor is removed first. In this case 5 is the common factor

Justify

- How do you prove a line is tangent to a circle? You find the points of intersection. If the line is a tangent there will be only 1 point. You write a clear statement to this effect to gain this 'communication' work.

  HMRN: p 40

6. (*c*)

when $y = 3$

$x = 2 \times 3 - 9 = -3$ ✓

so the point of contact is B(−3,3)

For A(1,5) and B(−3,3)

$$AB = \sqrt{(1-(-3))^2 + (5-3)^2}$$

$$= \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5} \quad ✓$$

2 marks

Other coordinate

- Having determined the value of the y-coordinate in part(b) you now have to substitute this back (use the line equation) to find the x-coordinate

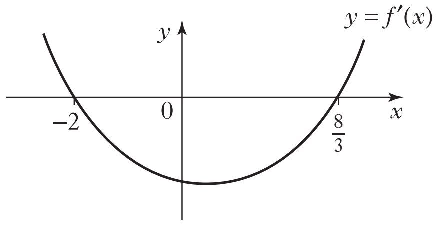
Distance

- You use the distance formula: $P(x_1, y_1)$, $Q(x_2, y_2)$

  $$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

  HMRN: p 7

7. ✓



✓ ✓

3 marks

Stationary Points

- Any stationary point on the graph $y = f(x)$ has a zero value for the gradient. So on $y = f'(x)$, the graph that shows the gradient values, an x-axis intercept (zero value) indicates a stationary point on the original graph. In this case your sketch should cross the x-axis at $(-2,0)$ and $(\frac{8}{3}, 0)$

Correct relationship

- Between -2 and $\frac{8}{3}$ the original graph goes downhill

  $\Rightarrow$ gradient is negative $\Rightarrow y = f'(x)$ graph is below the x-axis

- Less than -2 and greater than $8/3$ original graph goes uphill $\Rightarrow$ gradient positive $\Rightarrow y = f'(x)$ graph is above the x-axis

Shape

- Differentiating a cubic produces a quadratic $\Rightarrow$ your graph should show a parabola.

  HMRN: p 19

8. (*a*)

Volume = $x \times x \times h = x^2h$ ✓

so $x^2h = 62\tfrac{1}{2} \Rightarrow h = \dfrac{62\tfrac{1}{2}}{x^2} = \dfrac{125}{2x^2}$

Base area = x^2

Side area = xh

$= x \times \dfrac{125}{2x^2} = \dfrac{125}{2x}$ ✓

Total area = 4 × side area + Base area

$\Rightarrow A(x) = 4 \times \dfrac{125}{2x} + x^2 = \dfrac{250}{x} + x^2$ ✓

3 marks

Strategy

- Volume = length × breadth × height

Strategy

- Finding h in terms of $x\left(h = \frac{125}{2x^2}\right)$ and then substituting in the area expressions is the key to solving this question.

Proof

- Make sure all steps are shown. The answer is given so the examiner needs to see each step of your reasoning.

8. (*b*)

$A(x) = 250x^{-1} + x^2$ ✓

$\Rightarrow A'(x) = -250x^{-2} + 2x$

$$= -\frac{250}{x^2} + 2x$$ ✓

For stationary values set

$A'(x) = 0$ ✓

$$\Rightarrow \frac{-250}{x^2} + 2x = 0 \ (\times x^2)$$

$\Rightarrow -250 + 2x^3 = 0 \Rightarrow 2x^3 = 250$

$\Rightarrow x^3 = 125 \Rightarrow x = 5$ ✓

x:		5	
$A'(x) = \frac{250}{x^2} + 2x$:	$-$		$+$
Shape of graph:	$\backslash$	$\_$	$/$
nature:		min	

✓

So $x = 5$ gives a minimum value for the area.

5 marks

Preparation

- You should know that the term $\frac{250}{x}$ cannot be directly differentiated. Changing the expression to $250x^{-1}$ fits it to the following rule:

  $f(x) = ax^n \Rightarrow f'(x) = nax^{n-1}$

  in this case $a = 250$ and $n = -1$.

Differentiation

- It is useful for subsequent work to change negative indices to positive. In this case $-250x^{-2} = -\frac{250}{x^2}$

Strategy

- '$A'(x) = 0$' has to be stated to gain this mark.

Solve

- You should multiply both sides by x^2. The aim is to rid the equation of fractions
- Cubing a positive number gives a positive, but cubing a negative number gives a negative, so $x^3 = 125$ has only one positive solution. This is not like squaring: $x^2 = 25$ gives $x = 5$ or $x = -5$.

Justify

- 1 mark is allocated for the 'nature table'.

  HMRN: p 20–22

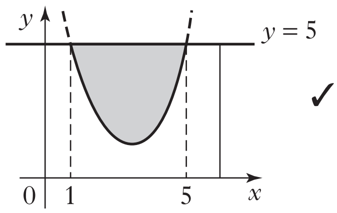
9.

First find the points of intersection of $y = 5$ with the curve:

Solve:

$$\left.\begin{array}{l} y = 5 \\ y = x^2 - 6x + 10 \end{array}\right\} \begin{array}{l} x^2 - 6x + 10 = 5 \\ \Rightarrow x^2 - 6x + 5 = 0 \\ \Rightarrow (x-1)(x-5) = 0 \\ \Rightarrow x = 1 \text{ or } x = 5 \end{array}$$ ✓ ✓

Diagram:



✓

shaded area in this diagram is given by: $\int_1^5 5 - (x^2 - 6x + 10)\,dx$ ✓

$$= \int_1^5 5 - x^2 + 6x - 10\,dx$$ ✓

$$= \int_1^5 -x^2 + 6x - 5dx$$ ✓

$$= \left[-\frac{x^3}{3} + 3x^2 - 5x\right]_1^5$$

$$= \left(-\frac{5^3}{3} + 3\times 5^2 - 5\times 5\right) - \left(-\frac{1^3}{3} + 3\times 1^2 - 5\times 1\right)$$

$$= -\frac{125}{3} + 75 - 25 + \frac{1}{3} - 3 + 5$$

$$= 52 - \frac{124}{3} = \frac{156}{3} - \frac{124}{3} = \frac{32}{3}\text{cm}^2$$ ✓

So Area of plate $= \underset{\text{(area of rectangle)}}{5\times 6} - \frac{32}{3}$

$$= 30 - \frac{32}{3} = \frac{90}{3} - \frac{32}{3} = \frac{58}{3}$$

$$= 19\frac{1}{3}\text{cm}^2$$ ✓

8 marks

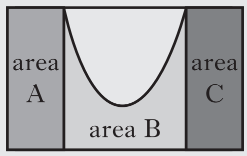
Strategy
- The strategy is to find the intersection points of line $y = 5$ and parabola $y = x^2 - 6x + 10$ as the x-values of these points will subsequently be crucial in setting up integrals to find areas.

Solve
- Always rearrange a quadratic equation into 'standard form', in this case $x^2 - 6x + 5 = 0$ so that the factorisation allows this process: (factor1)(factor2) = 0 ⇒ factor 1 = 0 or factor 2 = 0

Strategy
- This strategy mark is for how you will split areas up. In the solution given the approach is:

  (Rectangle Area) – (Parabolic Area).

  An alternative approach would have been to add together 3 areas to calculate the plate area as shown in this diagram.



area A + area B + area C

Strategy
- Did you know to integrate to find this area?

Limits
- Left to right on the diagram (1 and 5)

  Bottom to top on the integral (1 and 5)

Integration
- You should always 'simplify' first before you integrate. In this case don't integrate $5-(x^2 - 6x + 10)$, simplify first to $-x^2 + 6x - 5$ and then integrate.

Evaluate
- Even with a calculator available these calculations are very difficult to get correct. You should take great care and double-check your working always.

Area
- Finally make sure you answer the question. $\frac{32}{3}\text{cm}^2$ is not the required answer! Your strategy was to subtract the parabolic area from the rectangle.

  HMRN: p32–33