Mathematics Higher

Practice Papers
for SQA Exams

Exam M
Higher
Paper 2

You are allowed 1 hour, 10 minutes to complete this paper.

You may use a calculator.

Full marks will only be awarded where your answer includes relevant working.

You will not receive any marks for answers derived from scale drawings.

FORMULAE LIST

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Circle

The equation $x^2 + y^2 + 2nx + 2py + c = 0$ represents a circle centre $(-n, -p)$ and radius $\sqrt{n^2 + p^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Scalar Product $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Marks

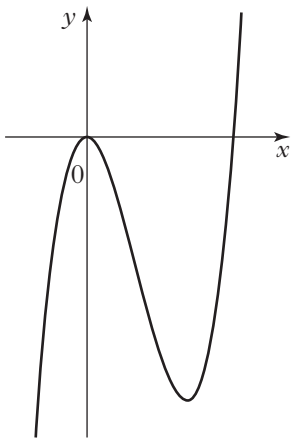
1. (*a*) Express $3\sin x° - \cos x°$ in the form $k\sin(x - a)°$ where $k > 0$ and $0 \leq a \leq 90$ **4**

(*b*) Hence solve the equation $3\sin x° - \cos x° = 1$ for $0 \leq x \leq 90$ **3**

2. The vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$ have components $\begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ respectively. Calculate the size of angle ABC. **5**

3. Prove that for all values of c the equation $x^2 - 2x + c^2 + 2 = 0$ has no real roots. **4**

4. The diagram shows the graph with equation $y = \frac{1}{3}x^3 - 2x^2$



(*a*) A tangent to this curve has gradient -4. Find the x-coordinate of the point of contact **5**

(*b*) Hence find the equation of this tangent. **2**

5. A drug is given to a patient. The concentration, C_t milligrams per millilitre (mg/ml), of the drug in the patient's blood t hours after it is administered is given by the formula:

$$C_t = C_o e^{-\frac{t}{4}}$$

where C_o is the concentration in the blood immediately after the drug was administered.

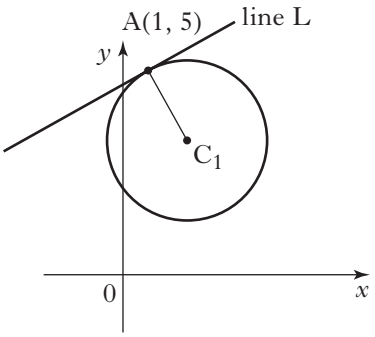
(*a*) If the concentration is 3·5 mg/ml after 3 hours, what was the concentration of the drug just after it was administered? **3**

(*b*) In general, after a dose of this drug has been administered, how long does it take for the initial concentration to halve? **4**

Marks

6. The line L is a tangent to the circle with centre C_1 and equation

$$x^2 + y^2 - 4x - 6y + 8 = 0.$$

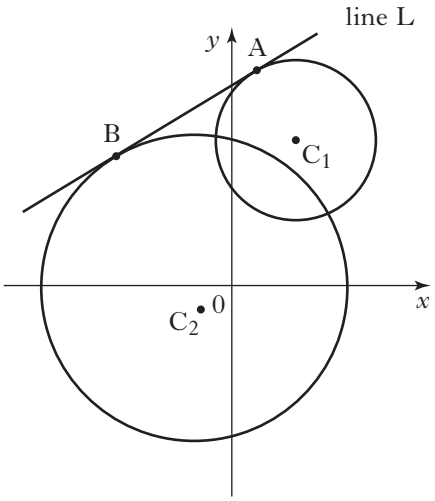


The point of contact A has coordinates (1,5).

(*a*) Show that the equation of line L is $2y - x = 9$. **4**

The circle with centre C_2 has equation

$$x^2 + y^2 + 2x + 2y - 18 = 0$$



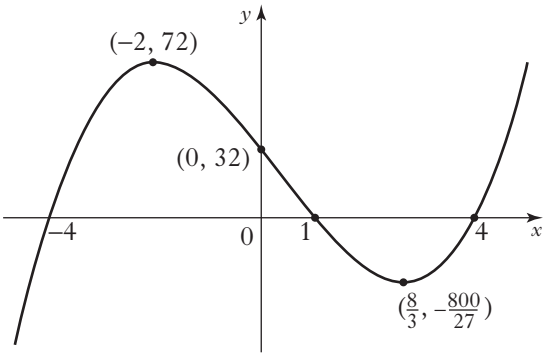
(*b*) Show that line L is also a tangent to this circle. **5**

(*c*) If B is the point of contact, find the exact length of AB. **2**

7. The graph shows a cubic function with equation $y = f(x)$.

The graph has stationary points at (–2,72) and $\left(\frac{8}{3}, -\frac{800}{27}\right)$.

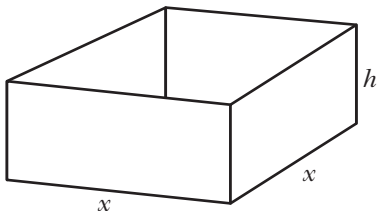
The graph intersects the axes at the points (–4,0), (1,0), (4,0) and (0,32).



Sketch the graph of $y = f'(x)$. **3**

Marks

8. An open box is in the shape of a cuboid and was made from a sheet of tin.



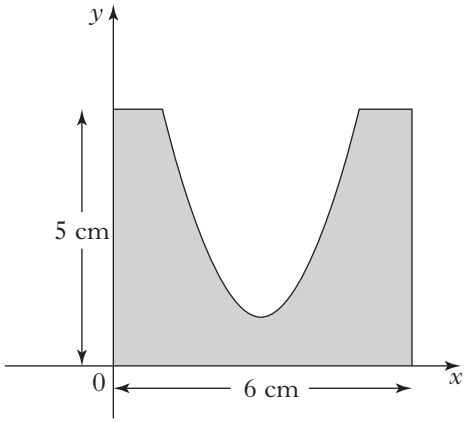
The box has a square base of side x cm and a height of h cm. The volume of the box is $62\frac{1}{2}\text{cm}^3$

(*a*) Show that the Area, A cm^2, of tin required to make the box is given by

$$\text{A}(x) = \frac{250}{x} + x^2$$ **3**

(*b*) Find the value of x for which this area is a minimum. **5**

9. The diagram shows a rectangular metal plate with dimensions 5cm × 6cm.



The plate has a parabolic section removed from it.

The equation of the parabola used to make this section is $y = x^2 - 6x + 10$

The scale of the diagram is
1 unit = 1 cm

Find the shaded area, in square centimetres, of the metal plate **8**

[End of question paper]