Mathematics   Higher

Practice Papers
for SQA Exams

You are allowed 1 hour, 30 minutes to complete this paper.

You must not use a calculator.

Full marks will only be awarded where your answer includes relevant working.

You will not receive any marks for answers derived from scale drawings.

FORMULAE LIST

Trigonometric formulae

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Circle

The equation $x^2 + y^2 + 2nx + 2py + c = 0$ represents a circle centre $(-n, -p)$ and radius $\sqrt{n^2 + p^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\dfrac{1}{a}\cos ax + C$
$\cos ax$	$\dfrac{1}{a}\sin ax + C$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Scalar Product $\boldsymbol{a.b} = |\boldsymbol{a}||\boldsymbol{b}| \cos \theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

1. What is the limit of the sequence generated by the recurrence relation
$u_{n+1} = 0 \cdot 9\, u_n - 1$?

A   −10

B $-\dfrac{10}{9}$

C $\dfrac{20}{9}$

D   10

2. The line through points A $(2k, 3)$ and B $(k, 5)$ has gradient 4. What is the value of k?

A   −2

B $-\dfrac{1}{2}$

C $\dfrac{2}{3}$

D   8

3. The following two statements are true for the polynomial $f(x)$:

    (1) $x^2 - 4$ is a factor of $f(x)$

    (2) $f(-1) = 0$

Which of the following is a possible expression for $f(x)$?

A $(x - 1)^2(x - 2)$

B $(x - 2)(x + 2)$

C $(x - 1)(x - 2)^2$

D $(x + 1)(x - 2)(x + 2)$

4. A sequence is defined by the recurrence relation $u_{n+1} = -\dfrac{1}{2}\,u_n + 1$, $u_0 = 4$; what is the value of u_2?

A $-\dfrac{1}{2}$

B $\dfrac{3}{4}$

C $\dfrac{3}{2}$

D $\dfrac{5}{2}$

5. Solve the equation $2\cos x - \sqrt{2} = 0$ for $0 \le x \le \frac{\pi}{2}$

A $\quad \dfrac{\pi}{6}$

B $\quad \dfrac{\pi}{4}$

C $\quad \dfrac{3\pi}{4}$

D $\quad \dfrac{5\pi}{4}$

6. What is the gradient of the line perpendicular to the line with equation $4y = -3x + 2$?

A $\quad -4$

B $\quad -2$

C $\quad \dfrac{4}{3}$

D $\quad 7$

7. The points E(1, –1, –1), F(–1, –1, 0) and G(–7, –1, 3) are collinear. In what ratio does F divide EG?

A $\quad$ 2:1

B $\quad$ –1:3

C $\quad$ 1:3

D $\quad$ 3:1

8. What is the remainder when $2x^4 - 3x^3 - 3x + 1$ is divided by $x - 2$?
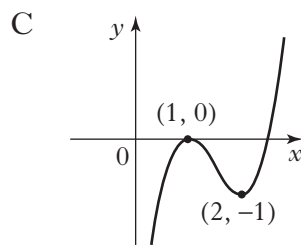
A $\quad -21$

B $\quad -9$

C $\quad -3$

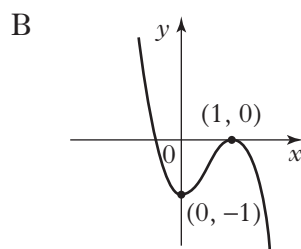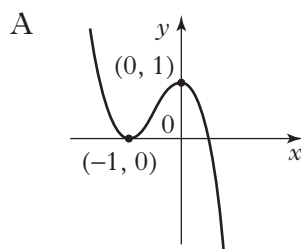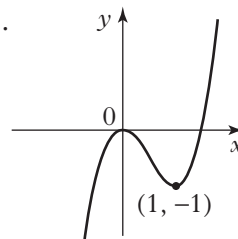D $\quad 3$

9. $(x + 4)(x - 2)$ can be written in the form $(x + a)^2 + b$. What is the value of b?

    A -12

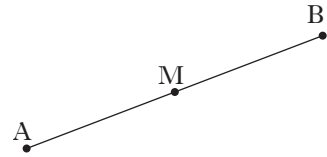   B -9

    C -8

    D 1

10. A function is defined by $f(x) = (1 - x^3)^{\frac{1}{3}}$. Find $f'(x)$

    A $-x^2(1 - x^3)^{-\frac{2}{3}}$

    B $(1 - x^3)^{-\frac{2}{3}}$

    C $(1 - 3x^2)^{\frac{1}{3}}$

    D $-4(1 - x^3)^{\frac{4}{3}}$

11. The diagram shows the graph with equation $y = f(x)$. Which of the following shows the graph with equation $y = -(f(x) + 1)$?

A

B

C

D
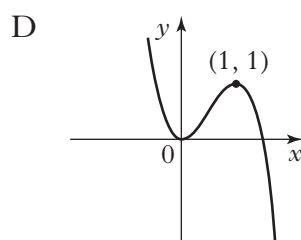
12. M is the midpoint of the line AB where A and B have coordinates (–1, 2, 0) and (–2, 3, 1) respectively. What is the position vector of M?

A $\begin{pmatrix} 3/2 \\ -5/2 \\ -1/2 \end{pmatrix}$

B $\begin{pmatrix} -3/2 \\ 5/2 \\ 1/2 \end{pmatrix}$

C $\begin{pmatrix} 1/2 \\ -1/2 \\ -1/2 \end{pmatrix}$

D $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

13. $f'(x) = x^2 + 1$ for a function f. Which statement is true?

A f has no stationary points

B f has exactly one stationary point

C f has exactly two stationary points

D f has more than two stationary points

14. The diagram shows the line OA where A is the point with coordinates (2,3). $a°$ is the angle between OA and the positive direction of the x-axis. Which of the following gives the value of a?
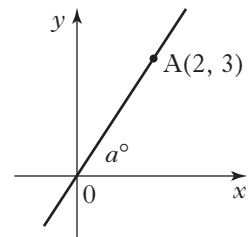
A $-\tan^{-1}\dfrac{2}{3}$

B $\tan^{-1}\dfrac{2}{3}$

C $-\tan^{-1}\dfrac{3}{2}$

D $\tan^{-1}\dfrac{3}{2}$

15. Which of the following expressions gives a simplification of
$\log_4 (x^2 - 4) - 2\log_4 (x - 2)$?

A   0

B $\log_4 x(x - 2)$

C $\log_4 \dfrac{x + 2}{x - 2}$

D $\log_4 (x - 2)^2(x + 2)$

16. Find $\int (2 - 3x)^{\frac{1}{3}} dx$

A $-\dfrac{1}{4}(2 - 3x)^{\frac{4}{3}} + c$

B $\dfrac{1}{3}(2 - 3x)^{-\frac{2}{3}} + c$

C $9(2 - 3x)^{-\frac{1}{3}} + c$

D $(2 - x) + c$

17. Points P and Q have coordinates (−1, 2, 5) and (−3, −1, 4) respectively. If $\overrightarrow{QR} = -2\overrightarrow{PQ}$ what are the coordinates of R ?

A   (1, 5, 6)
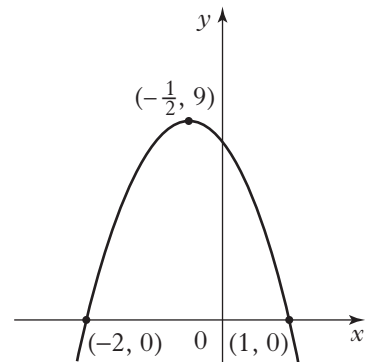
B   (5, −3, −14)

C   (−7, −7, 2)

D   (−8, −2, −10)

18. A function f is defined by $f(x) = -2 \sin 3x$. Find $f'(x)$.

A $6 \sin 3x$

B $-\dfrac{2}{3} \cos 3x$

C $-6 \cos 3x$

D $-2 \sin 3$

19. The graph shows a parabola with equation of the form $y = k(x-1)(x+2)$. What is the value of k ?

A $-\dfrac{36}{5}$

B -4

C $-\dfrac{1}{176}$

D 9

20. Here is an integration formula:

$$\int \frac{1}{(4x+1)^{3/2}}\, dx = -\frac{1}{2(4x+1)^{1/2}} + c$$

Use this formula to calculate the exact value of $\displaystyle\int_0^2 \frac{1}{(4x+1)^{3/2}}\, dx$.

A -8

B $-\dfrac{26}{27}$

C $\dfrac{1}{6}$

D $\dfrac{1}{3}$

[End of section A]

21. Solve the equation $\sin 2x - \sqrt{3}\sin x = 0$ for $0 \le x \le 2\pi$ **5**

22. (*a*) Find the stationary points on the curve with equation $y = x^3 - 3x^2 - 24x - 28$ and justify their nature   **7**

    (*b*) The curve intersects the *x*-axis at (7,0). Sketch the curve   **2**

23. ABCD is a rectangle with point P lying on side AB, 1 unit from A. AD = 2 units

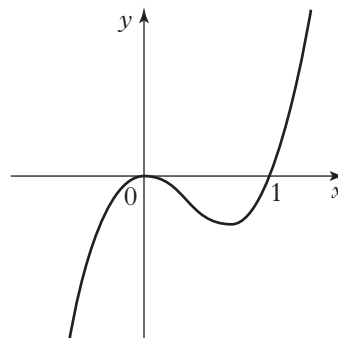The dotted line PD on the diagram shows the bisector of angle APC with angle APD = angle DPC = $y°$
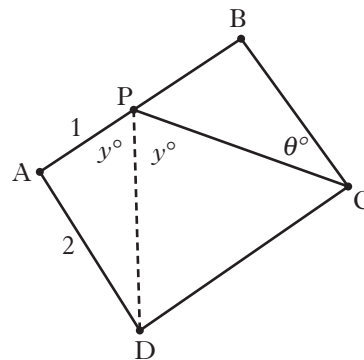
(*a*) Find $\theta°$ in terms of $y°$ **1**

(*b*) Hence find the exact value of $\sin \theta°$ **6**

24. Solve the equation
$\log_{\sqrt{2}} x - \log_{\sqrt{2}} 2 = 2$ **4**

25. The diagram shows the cubic graph with equation $y = x^2(x - 1)$. A function *f* is such that $y = f'(x)$ is the same as the graph shown in the diagram.

If $f(2) = \dfrac{1}{3}$, find the formula for $f(x)$.   **5**

[End of section B]

[End of question paper]