1.

$$x^2 - x - 2 = 0$$

Compare $ax^2 + bx + c = 0$

$a = 1,\ b = -1,\ c = -2$

Discriminant $= b^2 - 4ac$

$= (-1)^2 - 4 \times 1 \times (-2)$

$= 1 + 8 = 9 > 0$

So there are two distinct Real roots.

Also $9 = 3^2$, a perfect square

So the roots are rational

Choice D

2 marks

- The 1st result you are using is concerning the value of the discriminant $b^2 - 4ac$ To solve $ax^2 + bx + c = 0$ the roots are $x = \frac{-b+\sqrt{b^2-4ac}}{2a}$ and $x = \frac{-b-\sqrt{b^2-4ac}}{2a}$ If $b^2 - 4ac > 0$ as is the case in this question there will be two distinct Real values for these roots. If $b^2 - 4ac = 0$ the two values above are equal (one root). If $b^2 - 4ac < 0$ then $\sqrt{b^2-4ac}$ is not Real (no roots)
- $\sqrt{9} = 3$ and so no surd remains in the above expressions. The roots are rational.

HMRN: p 27

2.

u_{n+1} 0·8u_n 3, u_0 5

So u_1 0·8 and u_2 = 0·8

Choice B

2 marks

- The process is as follows:.

$u_0 \longrightarrow u_1 \longrightarrow u_2$

multiply by 0·8 then add 3 | multiply by 0·8 then add 3

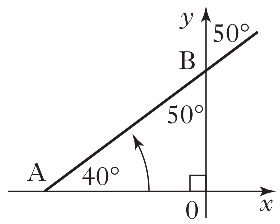
giving:

$5 \longrightarrow 7 \longrightarrow 8{\cdot}6$

HMRN: p 23

3.

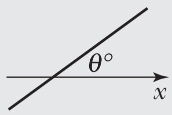
m_{AB} tan 40°



Choice D

2 marks

- The result used here is gradient m = tan θ



- The angle used is the angle the line makes with the positive direction of the x-axis. The 50° given in the question is the angle with the y-axis - that's the wrong axis!

HMRN: p 4

4.

Point of contact is (0,5).
Centre is (0,3)
So radius = 2 units
Centre (0,3) radius = 2
So equation of circle is :
$(x - 0)^2 + (y - 3)^2 = 2^2$
$\Rightarrow x^2 + (y - 3)^2 = 4$

Choice A

2 marks

- To find the equation of a circle you need to know two facts:

  The centre: (a, b) The radius: r

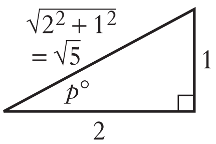
  The equation is then:

  $(x - a)^2 + (y - b)^2 = r^2$

- In this case the centre is known, (0,3), so what is the radius? The realisation is that the radius from the centre to the point of contact with the tangent lies on the y-axis and so the difference in height i.e $5 - 3 = 2$ is the length of the radius

  HMRN: p 39

5.



$\tan p^0 = \dfrac{1}{2}$

so $\cos(p - q)^\circ$

$= \cos p^\circ \cos q^\circ + \sin p^\circ \sin q^\circ$

$= \dfrac{2}{\sqrt{5}} \times \cos q^\circ + \dfrac{1}{\sqrt{5}} \times \sin q^\circ$

Choice D

2 marks

- When you are given sin, cos or tan equal to a number that is not an 'exact value' like $\frac{1}{2}, \frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}$ etc then you should draw a right-angled triangle and use Pythagoras' Theorem.
- The addition formula used here is given to you in the exam:

  $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

  – notice in the cosine addition formula the signs change: + becomes – and – becomes +.

  HMRN: p 35

6.

p and q are perpendicular
$\Rightarrow p.q = 0$

$$\Rightarrow \begin{pmatrix} a \\ -1 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ a \\ 3 \end{pmatrix} = 0$$

$\Rightarrow a \times (-1) + (-1) \times a + 2 \times 3 = 0$
$\Rightarrow -a + (-a) + 6 = 0$
$\Rightarrow -2a + 6 = 0 \Rightarrow 2a = 6 \Rightarrow a = 3$

Choice D

2 marks

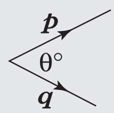
- The result used here is:

  $\boldsymbol{p}$ and $\boldsymbol{q}$ are perpendicular vectors $\Leftrightarrow \boldsymbol{p} \cdot \boldsymbol{q} = 0$

  (p and q are non-zero vectors)

- Remember that

  $\cos\theta^\circ = \dfrac{\boldsymbol{p} \cdot \boldsymbol{q}}{|\boldsymbol{p}||\boldsymbol{q}|}$

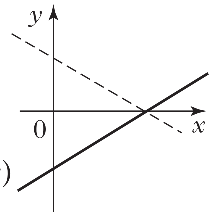


  $\boldsymbol{p}$ and $\boldsymbol{q}$ perpendicular means $\theta° = 90°$ so $\cos\theta° = \cos 90° = 0$. If the fraction on the right is zero then the numerator $\boldsymbol{p} \cdot \boldsymbol{q}$ is zero.

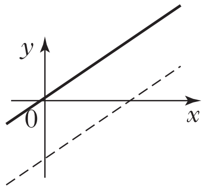
  HMRN: p 46

7.

$y = f(x) \rightarrow y = -f(x)$
This is reflection in the x-axis



$y = -f(x) \rightarrow y = 3 - f(x)$
this is a translation of 3 units parallel to y-axis



Choice C

2 marks

- One way of checking your choice of answer is to work with the point (0,3). The equation is $y = f(x)$ so if $x = 0$ gives $y = 3$ This means $3 = f(0)$.

  So what happens in the new equation when $x = 0$? $y = 3 - f(x)$ gives $y = 3 - f(0) = 3 - 3 = 0$.

  So the origin lies on the new line. Only choice C and D show this. Knowing $-f(x)$ is a 'flip' in the x-axis means a downward slope will change to an upward slope - choice C

  HMRN: p 9

8.

Let the limit be L

$\Rightarrow L = 0{\cdot}9L + 90$

$\Rightarrow L - 0{\cdot}9L = 90$

$\Rightarrow 0{\cdot}1L = 90 \Rightarrow L = \dfrac{90\,(\times 10)}{0{\cdot}1\,(\times 10)}$

$\Rightarrow L = \dfrac{900}{1} = 900$

Choice D

2 marks

- The multiplier here is 0·9 so you know a limit exists as this number lies between −1 and 1.
- The formula $L = \frac{b}{1-a}$ is not given to you in the exam and many mistakes arise from muddling 'a' and 'b'. It is recommended that you understand the algebraic method given in the solution:

  apply the recurrence relation to L and you will still get L as L is the limit.

  HMRN: p 24

9.

C(0,−3) and P(−2,0)

$m_{PC} = \dfrac{-3-0}{0-(-2)} = \dfrac{-3}{2} = -\dfrac{3}{2}$

$\Rightarrow m_1 = \dfrac{2}{3}$. So for the tangent:

A point is P(−2,0) and the gradient $= \dfrac{2}{3}$

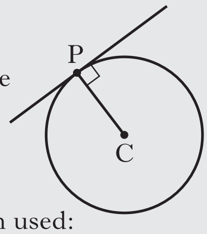
Equation is: $y - 0 = \dfrac{2}{3}(x - (-2))$

$\Rightarrow y = \dfrac{2}{3}(x+2)$

Choice A

2 marks

- The tangent is perpendicular to the radius from the centre C to the point of contact P.



- The perpendicular gradient result is then used:

  if $m = \dfrac{a}{b}$ then $m_{\perp} = -\dfrac{b}{a}$
  (invert and change the sign)

- Use $y - b = m(x - a)$ with $m = \frac{2}{3}$ and (a, b) being the point P(−2,0)

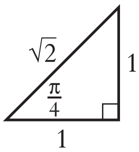
  HMRN: p 4

10.

$\sqrt{2}\cos\theta + 1 = 0 \Rightarrow \sqrt{2}\cos\theta = -1$

$\Rightarrow \cos\theta = -\dfrac{1}{\sqrt{2}}$. Required solution is in the 3^{rd} quadrant.

1^{st} quadrant angle = $\pi/4$



so $\alpha = \pi + \pi/4 = 4\pi/4 + \pi/4 = 5\pi/4$

Choice B

2 marks

- Lots of greek letters here!

  θ is the unknown angle

  α is the particular value of θ that you are trying to find i.e the root or solution of the equation

  $\pi \leq \alpha \leq 3\pi/2$ indicates that your solution is in the 3rd quadrant
- You should recognise $\frac{1}{\sqrt{2}}$ as an 'exact value'

HMRN: p 34

11.

$\int 6\cos 2x\,dx$

$= \dfrac{6\sin 2x}{2} + c$

$= 3\sin 2x + c$

Choice B

2 marks

- The result used is:

$$\int \cos ax\,dx = \frac{\sin ax}{a} + c$$

- In your exam you will be given this result in this format:

$f(x)$	$\int f(x)dx$
$\cos ax$	$\frac{1}{a}\sin ax + c$

HMRN: p 49

12.

$f(x) = \sqrt{x^2+1} = (x^2+1)^{1/2}$

$\Rightarrow f'(x) = \dfrac{1}{2}(x^2+1)^{-\frac{1}{2}} \times 2x$

$= x(x^2+1)^{-\frac{1}{2}}$

$= \dfrac{x}{(x^2+1)^{\frac{1}{2}}} = \dfrac{x}{\sqrt{x^2+1}}$

Choice B

2 marks

- This is the use of the 'chain rule'. The basic result is:

$$f(x) = x^{\frac{1}{2}} \Rightarrow f'(x) = \tfrac{1}{2}x^{-\frac{1}{2}}$$

  If x is replaced by a more complicated expression let's call it $g(x)$ (in this case $x^2 + 1$) you use:

$$f(x) = (g(x))^{\frac{1}{2}} \Rightarrow f'(x) = \tfrac{1}{2}(g(x))^{-\frac{1}{2}} \times g'(x).$$

  In this case the factor $g'(x)$ is $2x$
- The other results used are:

  $\sqrt{a} = a^{\frac{1}{2}}$ and $a^{-n} = \frac{1}{a^n}$

HMRN: p 48–49

13.

working through choices:

For $\boldsymbol{p}: \overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AD} + \overrightarrow{DC}$

$$\text{so } \boldsymbol{p} = \boldsymbol{s} - \boldsymbol{u} - \boldsymbol{q}$$
$$= -\boldsymbol{q} + \boldsymbol{s} - \boldsymbol{u}$$

so A is true.

For $q: \overrightarrow{CD} = \overrightarrow{CB} + \overrightarrow{AB} + \overrightarrow{AD}$

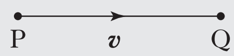
$$\text{so } \boldsymbol{q} = -\boldsymbol{p} + \boldsymbol{s} - \boldsymbol{u}$$

so B is false

Choice B

2 marks

- The direction of the arrows is very important. If you travel against the arrow it introduces a negative:



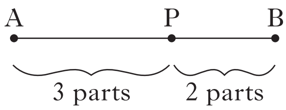
$\overrightarrow{PQ}$ represents $\boldsymbol{v}$

$\overrightarrow{QP}$ (against the arrow) represents $-\boldsymbol{v}$

- In a question where you have to work through the choices. you should double check by working through, in this case, Choice C and Choice D - both statements you will find are true.

HMRN: p 43

14.



y-coordinates:

$\underbrace{2 \quad -1}_{\text{down } 3} \; \underbrace{\quad -3}_{\text{down } 2}$

Choice A

2 marks

- The numbers are easy enough to reason out the -1 as the only suitable choice. Normally the method would be, for example,

$$\overrightarrow{AP} = \frac{3}{5}\overrightarrow{AB}$$
$$\Rightarrow 5\overrightarrow{AP} = 3\overrightarrow{AB} \Rightarrow 5(\boldsymbol{p} - \boldsymbol{a}) = 3(\boldsymbol{b} - \boldsymbol{a})$$
$$\Rightarrow 5\boldsymbol{p} - 5\boldsymbol{a} = 3\boldsymbol{b} - 3\boldsymbol{a} \Rightarrow 5\boldsymbol{p} = 3\boldsymbol{b} + 2\boldsymbol{a} = 3$$
$$\begin{pmatrix} 7 \\ -3 \\ 1 \end{pmatrix} + 2\begin{pmatrix} -3 \\ 2 \\ 6 \end{pmatrix}$$

so $5\boldsymbol{p} = \begin{pmatrix} 15 \\ -5 \\ 15 \end{pmatrix} \Rightarrow \boldsymbol{p} = \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}$ with -1 as the y-coordinate

You probably prefer working through the choices!

HMRN: p 44–45

15.

x-intercept $(-1, 0)$
corresponds to factor $(x + 1)$
x-intercept $(3, 0)$
corresponds to factor $(x - 3)$
so $y = k(x + 1)(x - 3)$
since (1,8) lies on the curve
$y = 8$ when $x = 1$
So $8 = k \times (1 + 1) \times (1 - 3)$
$\Rightarrow 8 = k \times 2 \times (-2) \Rightarrow 8 = -4k$

$$\Rightarrow k = \frac{8}{-4} = -2$$

So $y = -2(x + 1)(x - 3)$

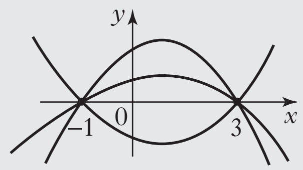
Choice A

2 marks

- Remember that to find x-axis intercepts you set $y = 0$ so for the graph with equation $y = k(x - a)(x - b)$ setting $y = 0$ gives:

  $k(x - a)(x - b) = 0 \Rightarrow x - a = 0$ or $x - b = 0$

  $\Rightarrow x = a$ or $x = b$ and the x-intercepts are the points $(a, 0)$ and $(b, 0)$. In the example the intercepts are $(-1,0)$ and $(3, 0)$ leading to $a = -1$ and $b = 3$ with the factors being $x - a = x - (-1) = x + 1$ and $x - b = x - 3$
- k is the 'scaling factor' in the y axis direction. There are many curves with equation $y = k(x + 1)(x - 3)$:



  only one passes through (1,8) namely $y = -2(x + 1)(x - 3)$ with $k = -2$.

  HMRN: p 29

16.

$y = k \times 2^{-x}$
(3,1) lies on the curve so
$y = 1$ when $x = 3$

$$\Rightarrow 1 = k \times 2^{-3} \Rightarrow 1 = k \times \frac{1}{2^3}$$

$\Rightarrow k = 2^3 = 8$

Choice D

2 marks

- There is an important general result about the graph with equation $y = f(x)$:

  point (a, b) lies on the graph $\Leftrightarrow$ $b = f(a)$

  In other words the values of the coordinates of a point on the graph will satisfy the equation of the graph.
- The result $a^{-n} = \frac{1}{a^n}$ is also used in this question

  HMRN: p 50

17.

$$3x^2 - 6x + 11$$
$$= 3\left[x^2 - 2x + \frac{11}{3}\right]$$
$$= 3\left[(x-1)(x-1) - 1 + \frac{11}{3}\right]$$
$$= 3\left[(x-1)^2 + \frac{8}{3}\right]$$
$$= 3(x-1)^2 + 8$$

compare $3(x + a)^2 + b$
$\Rightarrow b = 8$

Choice C

2 marks

- Each term has been divided by 3 but the factor 3 outside the square brackets will eventually multiply each term by 3 so that everything remains the same!
- In these 'completing the square' questions it is the coefficient of x^2 (in this case 3) that causes complications. You should always take this coefficient outside brackets and then divide each term by it. This is what has been done in this example.
- Alternative method in this case is :

  $3[x^2 - 2x] + 11$
  $= 3[(x - 1)(x - 1) - 1] + 11$
  $= 3(x - 1)^2 - 3 + 11$

  avoiding fractions!

  HMRN: p 13

18.

$v \cdot (v - w)$

$= v \cdot v - v \cdot w$

$= |v||v|\cos 0° - |v||w|\cos 60°$

$= 3 \times 3 \times 1 - 3 \times 3 \times \frac{1}{2}$

$= 9 - \frac{9}{2} = \frac{18}{2} - \frac{9}{2} = \frac{9}{2}$

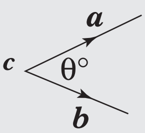
Choice B

2 marks

- The following properties of the dot product need to be known for this question:

  $a.(b \pm c) = a.b \pm a.c$

  and $a.b = |a||b|\cos\theta°$



  (this last is given on your formulae sheet)
- You also need to know:

  $\cos 60° = \frac{1}{2}$ and $\cos 0° = 1$
- $|v|$ and $|w|$ are the magnitudes of the vectors and are represented by the lengths of the lines in the diagram - all 3 units.

HMRN: p 47

19.

Division by zero is not allowed

Consider: $2(x^2 - 3x + 2) = 0$

$\Rightarrow 2(x - 1)(x - 2) = 0$

$\Rightarrow x = 1$ or $x = 2$

So all Real numbers are allowed apart from $x = 1$ and $x = 2$

Choice C

2 marks

- The 'Domain' of a function f is the set of numbers you are allowed to use in the formula for the function. The most common restrictions arise from two sources:

  1. Square roots of negative quantities

  2. Division by zero
- In this case the formula is

  $f(x) = \frac{5}{2(x^2 - 3x + 2)}$.

  There are no square roots so the only potential problem is a division by zero. So the question to ask is: can $2(x^2\ 3x\ 2)$ ever take the value zero. To answer this involves solving $2(x^2\ 3x\ 2) = 0$ as is shown in the solution to the question.

HMRN: p 9

20.

For x-intercept set $y = 0$

so $0 = 2\log_5(x + 3)$

$\Rightarrow \log_5(x + 3) = 0$

$\Rightarrow x + 3 = 5°$

$\Rightarrow x + 3 = 1 \Rightarrow x = -2$

intercept is $(-2,0)$

Choice D

2 marks

- There are several general results used in this question:

  1. For x-intercepts set $y = 0$

  2. $\underset{\text{(log statement)}}{\log_b a = c} \quad \leftrightarrow \quad \underset{\text{(exponential statement)}}{a = b^c}$

  3. $a° = 1$
- The factor 2 'vanishes' since you can divide both sides of the equation by 2.
- An alternative strategy would be to work through the choices substituting the x and y values into the equation to see if it makes sense.

HMRN: p 50–51

21.

$$y = \frac{1}{16}x^4 - \frac{1}{8}x^2 + x \qquad ✓$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{4}x^3 - \frac{1}{4}x + 1 \qquad ✓$$

The tangent $y = x + c$ has gradient 1

So set ✓

$$\frac{dy}{dx} = 1$$

$$\Rightarrow \tfrac{1}{4}x^3 - \tfrac{1}{4}x + 1 = 1$$

$$\Rightarrow \tfrac{1}{4}x^3 - \tfrac{1}{4}x = 0 \Rightarrow x^3 - x = 0 \qquad ✓$$

$$x\,(x^2 - 1) = 0 \Rightarrow x\,(x-1)\,(x+1) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 1 \text{ or } x = -1$$

For $x = 0$:

$$y = \frac{1}{16} \times 0^4 - \frac{1}{8} \times 0^2 + 0 = 0 \qquad ✓$$

so $y = x + c$ gives $0 = 0 + c \Rightarrow c = 0$

The tangent is $y = x$ with contact point $(0, 0)$

For $x = -1$:

$$y = \frac{1}{16} \times (-1)^4 - \frac{1}{8} \times (-1)^2 = (-1)$$

$$= -\frac{17}{16}$$

so $y = x + c$ gives

$$-\frac{17}{16} = -1 + c \Rightarrow c = -\frac{1}{16}$$

tangent is $y = x - \dfrac{1}{16}$, ✓

contact point is $\left(-1, -\dfrac{17}{16}\right)$

For $x = 1$:

$$y = \frac{1}{16} \times 1^4 - \frac{1}{8} \times 1^2 + 1 = \frac{15}{16}$$

so $y = x + c$ gives

$$\frac{15}{16} = 1 + c \Rightarrow c = -\frac{1}{16}$$

tangent is $y = x - \dfrac{1}{16}$ (same as for $x = -1$)

contact point is $\left(1, \dfrac{15}{16}\right)$. ✓

7 marks

Strategy
- Evidence that you know to differentiate will gain you this mark

Differentiation
- Be careful with the fractions here:

  $4 \times \frac{1}{16} = \frac{4}{16} = \frac{1}{4}$ and $2 \times \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$

Gradient
- Compare $y = mx + c$ and $y = x + c$. This leads to $m = 1$ for the gradient of the tangent line

Strategy
- This second strategy mark is given for knowing to set the gradient formula equal to 1.

Solving
- Remove fractions first by multiplying both sides of the equation by 4. Although this is a cubic, terms are missing and so is easily factorised once you realise to remove the common factor x

Calculation
- This processing mark involves a fair amount of calculation and is gained for clearly stating the possible values of c. These are:

  $c = 0$ and $c = -\dfrac{1}{16}$

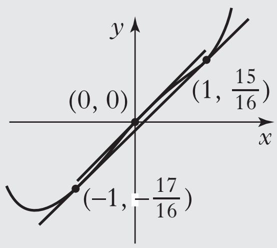
Interpretation
- With there being three points of contact:

  $(0,0)$, $\left(-1, -\dfrac{17}{16}\right)$ and $\left(1, \dfrac{15}{16}\right)$

  but only two equations for the tangent:

  $y = x$ and $y = x - \dfrac{1}{16}$

  a close examination of the graph shows $y = x - \frac{1}{16}$ is a tangent at two separate points on the curve as is shown in this diagram



HMRN: p 19–20

22. (*a*)

$f(x) = x^3 + 3x^2 - 4$ ✓

$\Rightarrow f'(x) = 3x^2 + 6x$

For stationary points set

$f'(x) = 0$ ✓

$\Rightarrow 3x^2 + 6x = 0 \Rightarrow 3x(x+2) = 0$ ✓

$\Rightarrow x = 0$ or $x = -2$ ✓

x :		-2		0	
$f'(x) = 3x(x+2)$:	$+$		$-$		$+$
Shape of graph :	/	—	\	—	/
nature:		max		min	

$f(-2) = (-2)^3 + 3x\,(-2)^2 - 4 = 0$

So $(-2, 0)$ is a maximum stationary point

$f(0) = 0^3 + 3 \times 0^2 - 4 = -4$ ✓

So $(0,-4)$ is a minimum stationary point ✓

6 marks

differentiate
- correct differentiation will gain this mark

Strategy
- to find the stationary points set $f'(x) = 0$

Solutions
- The common factor is $3x$ with two roots: 0 and -2

Justify
- A 'nature table' is needed to determine maximum.

***y*-coordinates**
- Take care with negatives. In paper 1 questions you have no access to a calculator. In general remember:

  squaring produces positive or zero answers, cubing a negative quantity gives a negative answer. In this case $(-2)^3 = -8$.

Statements
- Clear statements concerning the nature of the points i.e maximum or minimum etc are expected for this final mark.

  HMRN: p 20–21

22. (*b*) (i)

✓

$$\begin{array}{r|rrrr} -2 & 1 & 3 & 0 & -4 \\ & & -2 & -2 & 4 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

0 remainder so $f(-2) = 0$ so $x + 2$ is a factor ✓

2 marks

Strategy
- You should know to use $x = -2$ to show that $x + 2$ is a factor.

Calculation and conclusion
- Producing a zero in the 'synthetic division scheme' means that dividing by $x + 2$ gives a zero remainder and so shows that $x + 2$ is a factor.

  HMRN: p 25–26

22. (*b*) (ii)

$x^3 + 3x^2 - 4$

$= (x+2)(x^2 + x - 2)$ ✓

$= (x+2)(x+2)(x-1)$ ✓

$= (x+2)^2(x-1)$ ✓

3 marks

Interpretation
- If $x + 2$ is a factor then the factorisation will give: $(x + 2)$ (other factor).

Quadratic factor
- The row: 1 1 −2 in the 'synthetic division scheme' gives the coefficients of the 'other factor' i.e $x^2 + x - 2$

Complete factorisation
- $(x+2)(x^2 + x - 2)$ is not a *complete* factorisation

  HMRN: p 25–26

22. (*c*)

For x-intercepts set $y = 0$

So $x^3 + 3x^2 - 4 = 0$

$\Rightarrow (x - 1)(x + 2)^2 = 0$

$\Rightarrow x = 1$ or $x = -2$

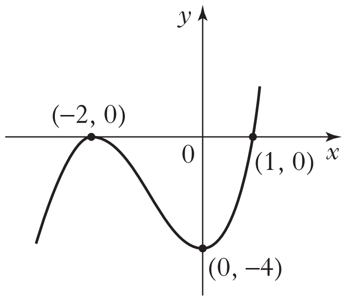
Intercepts are $(-2, 0)$ and $(1, 0)$ ✓

For y-intercept set $x = 0$

So $y = 0^3 + 3 \times 0^2 - 4 = -4$

Intercept is $(0, -4)$ ✓

Sketch:



✓

✓

4 marks

***x*-intercepts**

- You should always be aware that previous parts of questions are likely to be used in subsequent parts. In this case (b)(ii) covered the factorisation of $x^3 + 3x^2 - 4$ which is vital for calculating the two x-intercepts (−2,0) and (1,0)

***y*-intercept**

- 1 mark is allocated for this result.

Sketch

- 1 mark is allocated for your sketch showing the cubic shape correctly with the two stationary points clearly shown and labelled.
- The 2nd mark is given for your sketch clearly showing the intercepts i.e. (0,–4) and (1,0) etc

HMRN: p 21

23. (*a*)

$f(x) = 2x - 1$, $g(x) = \log_{12}x$

$f(g(x)) + g(f(x)) = 0$ ✓

$\Rightarrow f(\log_{12}x) + g(2x - 1) = 0$ ✓

$\Rightarrow 2\log_{12}x - 1 + \log_{12}(2x - 1) = 0$ ✓

$\Rightarrow \log_{12}x^2 + \log_{12}(2x - 1) = 1$

$\Rightarrow \log_{12}x^2(2x - 1) = 1$ ✓

$\Rightarrow x^2(2x - 1) = 12^1$ ✓

$\Rightarrow 2x^3 - x^2 - 12 = 0$

2	2	−1	0	−12
		4	6	12
	2	3	6	0

✓

So $x = 2$ satisfies the equation and is therefore a solution

6 marks

Composition

- Combining f and g to get the formula $f(g(x))$ is called the 'composition' of f and g. Showing either $f(\log_{12}x)$ or $g(2x - 1)$ gains you this first mark

Composition

- reaching either $2\log_{12}x - 1$ or $\log_{12}(2x - 1)$ will gain you this 2nd mark

Composition

- Correctly finding the 2nd of either $2\log_{12}x - 1$ or $\log_{12}(2x - 1)$ will give you this 3rd mark

Log Law

- The rule used here is:

$$\log_b m + \log_b n = \log_b mn$$

Exponential form

- The result used is:

$$\underset{\text{(logarithmic form)}}{\log_b a = c} \quad \leftrightarrow \quad \underset{\text{(exponential form)}}{a = b^c}$$

Calculation

- At this stage you must clearly show that $x = 2$ is a solution of $2x^3 - x^2 - 12 = 0$. The table shows that $f(2) = 0$ where $f(x) = 2x^3 - x^2 - 12$.

HMRN: p 10, p 25, p 50

23. (*b*)
From part(a) equation becomes:
$(x - 2)(2x^2 + 3x + 6) = 0$
Consider $2x^2 + 3x + 6 = 0$ ✓
Discriminant $= 3^2 - 4 \times 2 \times 6$
$= 9 - 48 = -39$
Since Discriminant < 0 there are no Real solutions. ✓
The only Real solution is $x = 2$.

2 marks

Strategy

- One solution, $x = 2$, comes from the factor $x - 2$ equating to zero. Any other solutions will therefore come from equating the other factor to zero i.e $2x^2 + 3x + 6 = 0$. If your working shows you knew this you will gain this strategy mark.

Communication

- There must be a clear reason for 'no Real solutions'. In this case the discriminant of the quadratic equation is negative. Your working should state this fact quite clearly. The result used is:

  Discriminant $< 0 \Rightarrow$ no Real roots.

  HMRN: p 26–27

1. (*a*) (i)

$x^2 \ y^2 \ 4x \ 6y \ 5 \ 0$

Centre is (2, 3) ✓

1 mark

Centre

- The result used is:

$x^2 \ y^2 \ 2gx \ 2fy \ c \ 0$

Centre: (g, f)

This result is on your formulae sheet in the exam.

HMRN: p 39

1. (*a*) (ii)

Radius $= \sqrt{(-2)^2 + 3^2 - 5}$

$= \sqrt{4+9-5} = \sqrt{8} = 2\sqrt{2}$ ✓

1 mark

Radius

- Notice that

$\sqrt{8} = \sqrt{4\times 2} = \sqrt{4}\times\sqrt{2} = 2\times\sqrt{2} = 2\sqrt{2}$

- The radius formula: $\sqrt{g^2 + f^2 - c}$ for the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is given to you on your formulae sheet in the exam.

HMRN: p 39

1. (*b*) (i)

Circle B:

$(x-2)^2 + (y+1)^2 = 2$

has centre (2, −1) ✓

Distance between (−2,3) and (2,−1) is:

$\sqrt{(-2-2)^2 + (3-(-1))^2}$

$= \sqrt{(-4)^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$ ✓

2 marks

Centre

- The result used is:

For circle $(x \, a)^2 \ (y \, b)^2 \ r^2$ the centre is (a, b) a result also given to you in your exam.

Calculation

- The distance formula is used here. If A (x_1,y_1) ad B(x_2,y_2) then

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

HMRN: p 7, p 39

1. (*b*) (ii)

Circle A: radius $2\sqrt{2}$

Circle B: radius $= \sqrt{2}$

Distance between centres

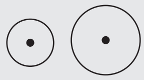
Sum of radii $2\sqrt{2} + \sqrt{2} = 3\sqrt{2}$ ✓

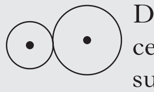
less than $4\sqrt{2}$, the distance between centres

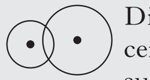
circles do not intersect ✓

2 marks

Strategy

- I  Distance between centres is greater than the sum of the radii

II  Distance between centres is equal to the sum of the two radii

III  Distance between centres is less than the sum of the two radii.

Communication

- There must be a comparison given. In this case a statement that the sum $\left(3\sqrt{2}\right)$ is less than the centre distance $\left(4\sqrt{2}\right)$ so there is no intersection.

  This is situation I shown above.

  HMRN: p 41

1. (*c*)

Solve $\left.\begin{aligned} y = x + 5 \\ x^2 + y^2 + 4x - 6y + 5 = 0 \end{aligned}\right\}$

Substitute $y = x + 5$ in circle equation: ✓

$x^2 + (x+5)^2 + 4x - 6(x+5) + 5 = 0$ ✓

$\Rightarrow x^2 + x^2 + 10x + 25 + 4x - 6x - 30 + 5 = 0$

$\Rightarrow 2x^2 + 8x = 0 \Rightarrow 2x(x + 4) = 0$ ✓

$\Rightarrow x = 0$ or $x = -4$ ✓

when $x = -4y = -4 + 5 = 1$

when $x = 0y = 0 + 5 = 5$

So P(−4, 1) and Q(0, 5) ✓

5 marks

Strategy

- In general to find where graphs $y = f(x)$ and $y = g(x)$ intersect then you equate the formulae i.e. $f(x) = g(x)$ and solve the resulting equation. However in this case the equation of the circle cannot be written as "$y = \ldots$" so substitution is used.

Substitution

- replace all occurrences of y by x 5 in the circle equation

'Standard form'

- reducing the equation to $2x^2 + 8x = 0$ gains you this mark

Solve for *x*

- The common factor is $2x$ with roots 4 and 0.

Coordinates

- Coordinates are asked for not just values of x and y

  HMRN: p 40

2. (*a*)

A(5, 2) and B(3, 8) ✓

$$\Rightarrow m_{AB} = \frac{8-2}{-3-(-5)} = \frac{6}{2} = 3$$

$$\Rightarrow m_1 = -\frac{1}{3}$$ ✓

Midpoint of AB is

$$\left(\frac{-5+(-3)}{2}, \frac{2+8}{2}\right) = (-4,5)$$ ✓

For the perpendicular bisector:
A point on the line is (4, 5)

and the gradient $-\frac{1}{3}$
So equation is

$3y\ 15\ \ (x\ 4)$
$3y\ 15\ \ x\ 4$ ✓
$3y\ \ x\ \ 11$

4 marks

Strategy
- Here you are using the gradient formula:

  P(x_1,y_1), Q (x_2,y_2) gives $m_{PQ} = \frac{y_2-y_1}{x_2-x_1}$

Perpendicular Gradient
- If $m = \frac{a}{b}$ then $m_{\perp} = -\frac{b}{a}$. For m 3 you think of 3 as $\frac{3}{1}$. Inverting and changing sign then gives $-\frac{1}{3}$ as shown in the solution

Strategy
- You have to know that 'bisector' means find the mid point of AB
- The mid point formula is:

  P(x_1,y_1),Q(x_2,y_2). Midpoint is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Equation
- Using $y - b = m(x - a)$ with $m = -\frac{1}{3}$ and the point (a,b) is (−4,5)

  HMRN: p 3–4, p 7

2. (*b*)

A(5,2) and D(5,8)
Midpoint of AD is

$$\left(\frac{-5+5}{2}, \frac{2+(-8)}{2}\right) = M(0,-3)$$ ✓

So usin g C(3,6) and M(0,3)

$$m_{CM} = \frac{6-(-3)}{3-0} = \frac{9}{3} = 3$$ ✓

For the median:
A point on the line is (0, 3) and the gradient is 3
So equation is y (3) 3(x 0)
$y\ \ 3\ \ 3x$
$y\ 3x\ \ 3$ ✓

3 marks

Strategy
- The median is the line from C to the midpoint of the opposite side AD. You will therefore need to find the coordinates of the midpoint.

Gradient
- Medians do not normally involve 'perpendicular' and so when m_{CM} is calculated you use this value, 3, to find the equation of the median.

Equation
- Use $y - b = m(x - a)$ with $m = 3$ and (a,b) being the point (0,−3). Alternatively spot that (0,−3) is the y-intercept of the line and use $y = mx + c$ with $m = -3$ and $c = -3$ to give $y = 3x - 3$

  HMRN: p 7

2. (c)

To find the intersection point S: ✓

Solve:

$$\left.\begin{aligned} 3y+x&=11 \\ y-3x&=-3 \end{aligned}\right\} \begin{aligned} &\rightarrow 3y+x=11 \\ _{(\times 3)}&\rightarrow \underline{3y-9x=-9} \end{aligned}$$

subtract: $10x = 20$

$\Rightarrow x = 2$ ✓

now substitute $x = 2$ in

$y - 3x = -3$

$\Rightarrow y - 3 \times 2 = -3 \Rightarrow y - 6 = -3$

$\Rightarrow y = 3$ ✓

so S(2,3)

3 marks

Strategy

- To find the point of intersection of two lines you solve the two equations of the lines simultaneously

Find one variable

- An alternative is to multiply the 1st equation by 3 then add to give $10y = 30 \Rightarrow y = 3$

Second variable

- Use the 'easier' equation when doing the substitution. If you found y 3 first then the 1st equation is 'easier'.

HMRN: p 6

3.

$3\cos 2x° \;\; 9\cos x° \;\; \cos^2 x° \;\; 7$

$3(2\cos^2 x° 1)\, 9\cos x°\, \cos^2 x°\, 7$ ✓

$6\cos^2 x°\, 3\; 9\cos x° \;\; \cos^2 x°\, 7$

$5\cos^2 x° \;\; 9\cos x° \;\; 4 \;\; 0$ ✓

$(5\cos x° \;\; 4)(\cos x° \;\; 1) \;\; 0$ ✓

$5\cos x° \;\; 4 \;\; 0$ or $\cos x° \;\; 1 \;\; 0$

or $\cos x° \;\; 1$ ✓

For $\cos x° = -\dfrac{4}{5}$:

$x°$ is in 2nd or 3rd quadrants

1st quadrant angle is 36·9°

so $x \;\; 180 \;\; 36{\cdot}9$ or

$x \;\; 180 \;\; 36{\cdot}9$

$x \;\; 143{\cdot}1$ or $x \;\; 216{\cdot}9$

(continued next page)

Strategy

- There are three expansions of $\cos 2x°$·
$\cos 2x° \;\; 2\cos^2 x \; 1$ or $\cos^2 x \; \sin^2 x$ or $1 \; 2\sin^2 x$. Which of these you use is dictated by the surrounding 'landscape' in the equation: There is a '$\cos x$ term' and a $\cos^2 x$ term but no '$\sin x$ term' or '$\sin^2 x$ term'. So $\cos 2x° \;\; 2\cos^2 x \; 1$ is used as the other two forms involve $\sin^2 x$.

'Standard form'.

- You recognize the equation as a quadratic equation in $\cos x$ and arrange it into the standard form $5\cos^2 x° + 9\cos x° + 4 = 0$

Factorisation

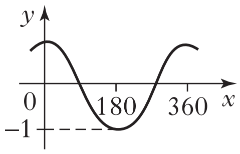
- Compare $5c^2 \;\; 9c \;\; 4 \;\; (5c \;\; 4)(c \;\; 1)$
- remember that you should always check your answer in a factorisation by multiplying out i.e. working backwards

Solving for cos *x*

- From $(5c + 4)(c + 1) = 0$ to $c = -\frac{4}{5}$ or $c = -1$ is no different to what you do at this stage except that the single variable c is replaced by the expression $\cos x°$

3. Continued.

For $\cos x° = -1$



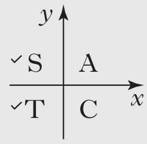
So $x = 180$

Solutions are:
143·1, 180, 216·9 ✓
(to 1 decimal place).

5 marks

Solutions

- A lot of knowledge and work needed for this final processing mark!
- The quadrant diagram is used:



for $\cos x°$ negative.

HMRN: p37

4. (*a*)

M(0,3,2) ✓
N(5,2,0) ✓

2 marks

Point M

- M is the midpoint. Half-way along DG which is 6 units long (y-coordinate) is 3 units

Point N

- N is $\frac{1}{3}$ of the way along AB so $\frac{1}{3}$ of 6 2 units is the y-coordinate.

HMRN: p 42

4. (*b*)

M(0,3,2) and B(5,6,0)

$$\overrightarrow{MB} = \boldsymbol{b} - \boldsymbol{m}$$

$$= \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}$$ ✓

also M (0,3,2) and N (5,2,0)

$$\overrightarrow{MN} = \boldsymbol{n} - \boldsymbol{m}$$

$$= \begin{pmatrix} 5 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \\ -2 \end{pmatrix}$$ ✓

2 marks

Components

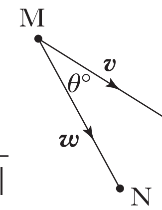
- The basic result used is:

P(x_1, y_1, z_1) and Q (x_2, y_2, z_2)

so $\overrightarrow{PQ} = \boldsymbol{q} - \boldsymbol{p} = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} - \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{pmatrix}$

HMRN: p 43–44

4. (*c*)



use $\cos\theta \dfrac{\boldsymbol{v}.\boldsymbol{w}}{|\boldsymbol{v}|\ |\boldsymbol{w}|}$ ✓

where $\boldsymbol{v} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 5 \\ -1 \\ -2 \end{pmatrix}$

$$\boldsymbol{v}.\boldsymbol{w} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix} . \begin{pmatrix} 5 \\ -1 \\ -2 \end{pmatrix}$$

$= 5 \times 5 + 3 \times (-1) + (-2) \times (-2)$

$= 25 - 3 + 4 = 26$ ✓

$|\boldsymbol{v}| = \sqrt{5^2 + 3^2 + (-2)^2}$

$= \sqrt{25 + 9 + 4} = \sqrt{38}$ ✓

$|\boldsymbol{w}| = \sqrt{5^2 + (-1)^2 + (-2)^2}$ ✓

$= \sqrt{25 + 1 + 4} = \sqrt{30}$

So $\cos\theta = \dfrac{26}{\sqrt{38}\sqrt{30}}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{26}{\sqrt{38}\sqrt{30}}\right)$

So $\theta = 39{\cdot}64\ldots \doteqdot 39{\cdot}6$ (to 1 dec pl.) ✓

5 marks

Strategy
- This strategy is for the use of the 'scalar' or 'dot' product formula.

Calculation
- The 'dot product' formula is:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} . \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = x_1x_2 + y_1y_2 + z_1z_2$$

Magnitudes
- The magnitude formula is

$$\left|\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}\right| = \sqrt{x_1^2 + y_1^2 + z_1^2}$$

Angle
- Be careful with your calculator calculation

$\boxed{\text{INV}}\boxed{\cos}\boxed{(}\boxed{2}\boxed{6}\boxed{\div}\boxed{(}\boxed{\sqrt{}}\boxed{3}\boxed{8}\boxed{\times}\boxed{\sqrt{}}\boxed{3}\boxed{0}\boxed{)}\boxed{)}\boxed{\text{EXE}}$

The brackets are vital: $\cos^{1}(\ldots)$ and $(\sqrt{38} \times \sqrt{30})$.

HMRN: p 46

5. (*a*)

$m = 1$ and $n = \sqrt{3}$ ✓ ✓

2 marks

interpret graphs
- The 'normal' sine graph has amplitude = 1 This gives m = 1. The amplitude of the Cosine graph shown is $\sqrt{3}$ so $n = \sqrt{3}$.

HMRN: p16–17

5. (*b*)

$f(x) = \sin x$ and $g(x) = \sqrt{3}\cos x$

so $f(x) - g(x) = \sin x - \sqrt{3}\cos x$

let $\sin x - \sqrt{3}\cos x$

$= k\sin(x - a),\ k > 0$

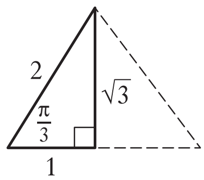
$\Rightarrow \sin x - \sqrt{3}\cos x$

$= k\sin x\cos a - k\cos x\sin a$ ✓

now equate coefficients of $\sin x$ and $\cos x$:

$\left.\begin{aligned} k\cos a &= 1 \\ k\sin a &= \sqrt{3} \end{aligned}\right\}$ since both $\sin a$ and $\cos a$ are positive a is in 1st quadrant ✓

Divide: $\dfrac{k\sin a}{k\cos a} = \dfrac{\sqrt{3}}{1}$



$\Rightarrow \tan a = \sqrt{3}$

$\Rightarrow a = \dfrac{\pi}{3}$ ✓

Square and add:

$(k\cos a)^2 + (k\sin a)^2 = 1^2 + \left(\sqrt{3}\right)^2$

$\Rightarrow k^2\cos^2 a + k^2\sin^2 a = 1 + 3$

$\Rightarrow k^2(\cos^2 a + \sin^2 a) = 4$

$\Rightarrow k^2 \times 1 = 4 \Rightarrow k = 2\ (k > 0)$ ✓

So $f(x) - g(x) = \sin x - \sqrt{3}\cos x$

$= 2\sin\left(x - \dfrac{\pi}{3}\right)$

4 marks

Strategy
- You must clearly show the expansion of $k\sin(x\ \ a)$. To do this you use the addition formulae:
  $\sin(A\ \ B)\ \sin A\cos B\ \ \cos A\sin B$
  which is given on your formulae sheet.

Compare coefficients

compare $1\sin x \quad -\sqrt{3}\cos x$

$k\sin x\cos a \quad - \quad k\cos x\sin a$

giving $k\cos a = 1$ and $k\sin a = \sqrt{3}$

Find *a*
- You should recognise $\sqrt{3}$ as an exact value leading to the angle $\pi/3$

Find *k*
- For finding a you used $\frac{\sin a}{\cos a} = \tan a$ and for finding k you use $\sin^2 a\ \cos^2 a\ 1$. If you try to apply a learnt 'formula' for finding k sometimes mistakes creep in. It is better to understand that squaring and adding both sides of the two equations leads to the value of k and just 'do the mathematics' at the time, $k^2\ 4$. $k\ 2$ is not an allowable value since $k\ \ 0$.

  HMRN: p 53–54

5. (*c*)

$$y = 2\sin\left(x - \frac{\pi}{3}\right)$$

$$\Rightarrow \frac{dy}{dx} = 2\cos\left(x - \frac{\pi}{3}\right) \quad ✓$$

For a gradient of 2 set $\frac{dy}{dx} = 2$

But $0 \leq x \leq \pi$ so $x = \frac{\pi}{3}$ is the only solution ✓

2 marks

Strategy

- You are told the gradient so you have to differentiate and find the x-value that satisfies $\frac{dy}{dx} = 2$. The word 'hence' is very important. It is telling you to use the previous result. You will lose marks if you do not do this.

  Differentiation involves the chain rule:

  $y = 2\sin(g(x)) \Rightarrow \frac{dy}{dx} = 2\cos(g(x)) \times g'(x)$

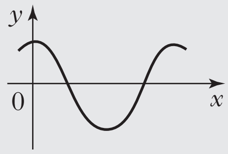
  In this case $g(x) = x - \frac{\pi}{3}$ so $g(x)$ 1 so

  $\frac{dy}{dx} = 2\cos\left(x - \frac{\pi}{3}\right) \times 1 = 2\cos\left(x - \frac{\pi}{3}\right)$

Equation and solution

- For $\cos\theta = 1$

  $\theta = \ldots -2\pi, 0, 2\pi \ldots$



- In this question the angle is $x - \frac{\pi}{3}$ so to finally find x you must add $\frac{\pi}{3}$.

  HMRN: p 48

6.

Intersection with $y = 15$ solve:

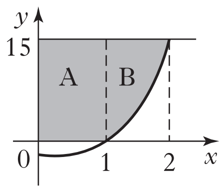
$$\left.\begin{array}{l} y = 15 \\ y = x^4 - 1 \end{array}\right\} \begin{array}{l} \text{so } x^4 - 1 = 15 \\ \Rightarrow x^4 = 16 \Rightarrow x = 2 \text{ or } -2 \end{array}$$

For x-intercept set $y = 0$ ✓

so $x^4 - 1 = 0 \Rightarrow x^4 = 1$

$\Rightarrow x = 1$ or -1 ✓

Here is the diagram:



Area A is a rectangle

Area A = 1 × 15 = 15 unit^2

(continued to next page)

Limits

- It is essential to find the x values of the intersection of the lines $y = 15$ and x-axis with the curve. These values will be used later in the integral for finding the area between two graphs.

***x*-values**

- The relevant values are x 1 and x 2. Notice that all the work can be done in the 1[st] quadrant and then doubled at the end since the y-axis is an axis of symmetry for the diagram.

6. Continued

Area B =

$$= \int_1^2 15-(x^4-1)\ dx \quad ✓$$

$$= \int_1^2 16-x^4dx = \left[16x-\frac{x^5}{5}\right]_1^2 \quad ✓$$

$$= \left(16\times 2-\frac{2^5}{5}\right)-\left(16\times 1-\frac{1^5}{5}\right) \quad ✓$$

$$= 32-\frac{32}{5}-16+\frac{1}{5} = 16-\frac{31}{5} = \frac{49}{5}\text{ unit}^2$$

✓

Area A + Area B

$$= 15+\frac{49}{5} = \frac{75}{5}+\frac{49}{5} = \frac{124}{5}\text{ unit}^2 \quad ✓$$

By symmetry the required area

$$= 2\times\frac{124}{5} = \frac{248}{5}\text{ units}^2 \quad ✓$$

8 marks

Strategy
- You should know to use integration to find the area between y 15 and the curve (area B) and add the area of Rectangle A. There are other methods possible.

Integration
- Notice

$$\int a\ dx = ax+c \text{ and } \int x^n = \frac{x^{n+1}}{n+1}+c$$
(a is a constant)

- The constant c is not needed when there are limits

Limits
- A mark is allocated for correct use of limits 1 and 2

Evaluate
- Using $\int_a^b f(x)dx = [F(x)]_a^b = F(b)-F(a)$

Where F(k) is the result of integrating $f(x)$

Strategy
- Knowing what to add together!

Calculation
- Final answer is gained by doubling as you have only found the area in the 1st quadrant.

HMRN: p 32–33

7. (a)

(0,9) lies on the curve

so y 9 when x 0

y x^2 a gives 9 0^2 a

a 9. ✓

1 mark

Calculation
- If (p,q) lies on a graph with equation $y = f(x)$ then $q = f(p)$. In otherwords the coordinates of the point can be substituted into the equation. In this case this gives the value of a.

HMRN: p 9

7. (b)

$f(x)$ x^2 9 9 x^2

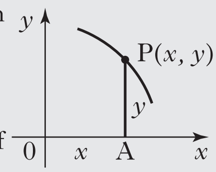
to find coordinates of P set x m

so $f(m)$ 9 m^2 P(m,9 m^2)

AP 9 m^2 ✓

1 mark

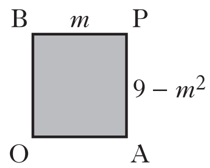
Calculation of *y*-coordinate
- As can be seen from this diagram the length AP is the y-coordinate of P. The x-coordinate of P you know is m i.e $x = m$



HMRN: p 9

7. (*c*)

The area of the rectangle, A(m) is given by:



$A(m) = m\,(9 - m^2)$ ✓

$= 9m - m^3 \Rightarrow A'(m) = 9 - 3m^2$ ✓

For stationary value set $A'(m) = 0$ ✓

$\Rightarrow 9 - 3m^2 = 0 \Rightarrow 3m^2 = 9$

$\Rightarrow m^2 = 3$

So $m = \sqrt{3},\ m \neq -\sqrt{3}$ since $m > 0$ ✓

x:		$\sqrt{3}$	
$A'(m) = 9 - 3m^2$:	+		−
Shape of graph:	/	—	\
nature:		max	

✓

So $m = \sqrt{3}$ gives a maximum value for the area of rectangle OAPB. ✓

6 marks

Communication
- Area of a rectangle is given by length × breadh and in this case the dimensions are m units × $(9 - m^2)$ units

Differentiate
- The function $A'(m)$ gives the area of the rectangle whereas $A'(m)$ is the gradient function and determines the slope of the graph y A(m) showing the various areas as m changes.

Strategy
- Setting $A'(m)$ to zero determines which values of m give stationary points on the Area graph.

Solve
- Here there is a positive and a negative value of m. Only the positive value makes sense as you are told that $0 \leq m \leq 3$.

Justify
- The 'nature' table is required to show that $m = \sqrt{3}$ does give a maximum value for the Area.

Communication
- A clear statement summarising your findings is needed for this final mark.

HMRN: p 21–22

7. (*d*)

$$A\left(\sqrt{3}\right) = \sqrt{3}\left(9 - \left(\sqrt{3}\right)^2\right)$$

$$= \sqrt{3}(9 - 3)$$

$$= \sqrt{3} \times 6$$ ✓

So $6\sqrt{3}$ unit^2 is the maximum area.

1 mark

Calculation
- When you read the word 'exact' you know to steer clear of decimal approximations. Your answer will be an integer value, a fraction (rational number) or a surd (involving roots) or perhaps an expression involving or e ,... but never approximate decimals.

HMRN: p 20