Mathematics Higher

Practice Papers
for SQA Exams

Exam L
Higher
Paper 2

You are allowed 1 hour, 10 minutes to complete this paper.

You may use a calculator.

Full marks will only be awarded where your answer includes relevant working.

You will not receive any marks for answers derived from scale drawings.

FORMULAE LIST

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Circle

The equation $x^2 + y^2 + 2nx + 2py + c = 0$ represents a circle centre $(-n, -p)$ and radius $\sqrt{n^2 + p^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Scalar Product $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Marks

1. Three circles have equations as follows:

Circle A : $x^2 + y^2 + 4x - 6y + 5 = 0$

Circle B : $(x - 2)^2 + (y + 1)^2 = 2$

Circle C : $(x - 2)^2 + (y + 1)^2 = 40$

(*a*) (i) State the centre of circle A **1**

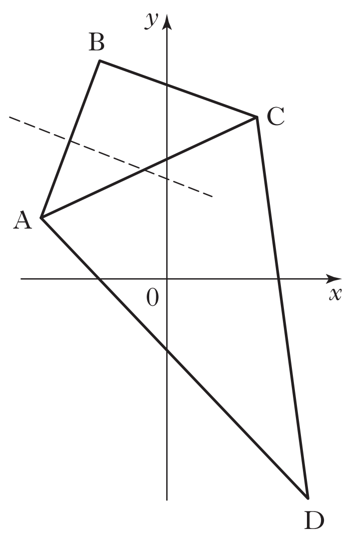
(ii) Show that the radius of circle A is $2\sqrt{2}$ **1**

(*b*) (i) Calculate the distance between the centres of circles A and B writing your answer as a surd in its simplest form **2**

(ii) Hence show that circles A and B do not intersect **2**

(*c*) Circles A and C intersect at points P and Q. Chord PQ has equation $y = x + 5$. Find the coordinates of points P and Q if P lies to the left of Q. **5**

2. The diagram shows kite ABCD with diagonal AC drawn. The vertices of the kite are A(−5, 2), B(−3, 8), C(3, 6) and D(5, −8).



The dotted line shows the perpendicular bisector of AB.

(*a*) Show that the perpendicular bisector of AB has equation $3y + x = 11$ **4**

(*b*) Find the equation of the median from C in triangle ACD **3**

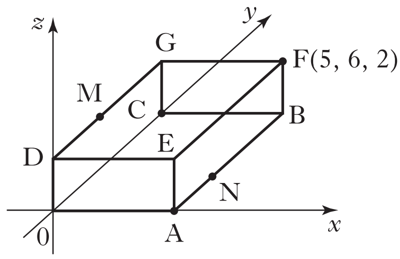
(*c*) The perpendicular bisector of AB and the median from C in triangle ACD meet at the point S. Find the coordinates of S. **3**

3. Solve the equation

$$3 \cos 2x° + 9 \cos x° = \cos^2 x° - 7 \text{ for } 0 \leq x < 360$$ **5**

Marks

4. OABC, DEFG is a cuboid.



The vertex F is the point (5, 6, 2).

M is the midpoint of DG.

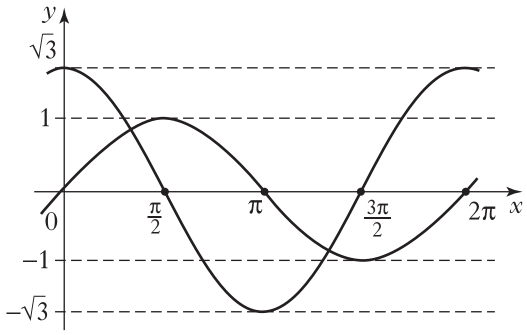
N divides AB in the ratio 1:2.

(*a*) Find the coordinates of M and N. **2**

(*b*) Write down the components of $\overrightarrow{MB}$ and $\overrightarrow{MN}$. **2**

(*c*) Find the size of angle BMN. **5**

5. The diagram below shows the graphs $y = f(x)$ and $y = g(x)$ where $f(x) = m \sin x$ and $g(x) = n \cos x$



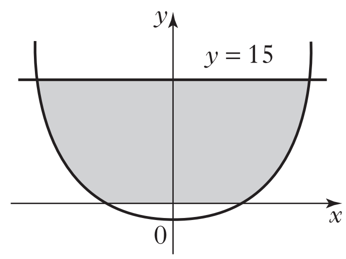
(*a*) Write down the values of m and n **2**

(*b*) Write $f(x) - g(x)$ in the form $k \sin(x - a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$ **4**

(*c*) Hence find, in the interval $0 \le x \le \pi$ the x-coordinate of the point on the curve $y = f(x) - g(x)$ where the gradient is 2. **2**

Marks

6. The diagram shows the graph with equation $y = x^4 - 1$

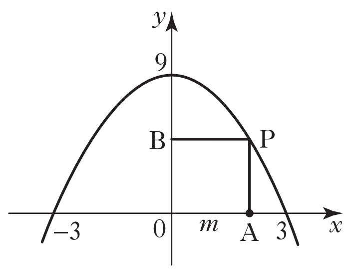


The graph has the y-axis as an axis of symmetry.

The shaded area lies between the curve, the x-axis and the line $y = 15$.

Calculate the exact value of the shaded area. **8**

7. The diagram shows a parabola with equation $y = f(x)$ passing through the points $(-3, 0)$, $(0, 9)$ and $(3, 0)$.



OAPB is a rectangle with A and B lying on the axes and P lying on the parabola as shown. OA = m, $0 \leq m \leq 3$

(*a*) If $f(x)$ is of the form $-x^2 + a$ where a is a constant, determine the value of a. **1**

(*b*) Show that AP = $9 - m^2$. **1**

(*c*) Find the value of m for which the area of the rectangle has a maximum. **6**

(*d*) Find the exact value of this maximum area. **1**

[End of question paper]