Mathematics Higher

Practice Papers
for SQA Exams

Exam L
Higher
Paper 1
Non-calculator

You are allowed 1 hour, 30 minutes to complete this paper.

You must not use a calculator.

Full marks will only be awarded where your answer includes relevant working.

You will not receive any marks for answers derived from scale drawings.

FORMULAE LIST

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Circle

The equation $x^2 + y^2 + 2nx + 2py + c = 0$ represents a circle centre $(-n, -p)$ and radius $\sqrt{n^2 + p^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Scalar Product $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

SECTION A

1. Here are two statements about the roots of equation $x^2 - x - 2 = 0$

(1) The roots are rational;

(2) The roots are real.

Which of the following is true?

A Neither statement is correct

B Only statement (1) is correct

C Only statement (2) is correct

D Both statements are correct.

2. A sequence is defined by the recurrence relation

$$u_{n+1} = 0{\cdot}8\, u_n + 3 \text{ with } u_0 = 5$$

What is the value of u_2?

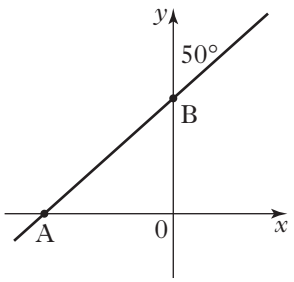
A 6·8

B 8·6

C 19·0

D 35·0

3. A line AB makes an angle of 50° with the positive direction of the y-axis as shown in the diagram.



What is the gradient of line AB?

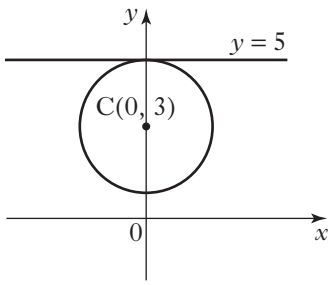
A $\tan 130°$

B $-\dfrac{1}{\tan 50°}$

C $\tan 50°$

D $\tan 40°$

4. The line $y = 5$ is a tangent to a circle with centre C(0, 3) as shown in the diagram. What is the equation of the circle?



A $x^2 + (y - 3)^2 = 4$

B $x^2 + (y - 3)^2 = 9$

C $x^2 + (y + 3)^2 = 4$

D $(x - 3)^2 + y^2 = 9$

5. Given that $\tan p° = \frac{1}{2}$ with $0 \leq p < 90$, which of the following is an expression for $\cos (p - q)°$?

A $\frac{2}{\sqrt{5}} - \cos q°$

B $\frac{\sqrt{3}}{2} \cos q° + \frac{1}{2} \sin q°$

C $\frac{2}{\sqrt{3}} \cos q° + \frac{1}{\sqrt{3}} \sin q°$

D $\frac{2}{\sqrt{5}} \cos q° + \frac{1}{\sqrt{5}} \sin q°$

6. The vectors $\boldsymbol{p} = \begin{pmatrix} a \\ -1 \\ 2 \end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix} -1 \\ a \\ 3 \end{pmatrix}$ are perpendicular. What is the value of a?

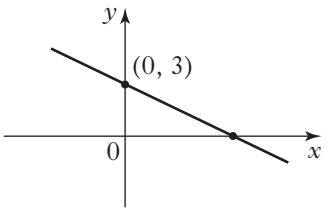
A $-\frac{3}{2}$

B 0

C $\frac{5}{2}$

D 3

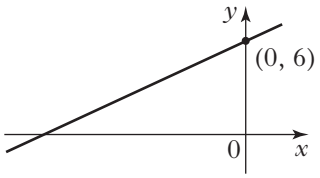
7. The diagram shows a straight line graph with equation $y = f(x)$.



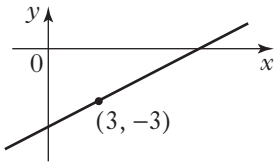
The line passes through the point (0, 3).

Which of the following diagrams could be the graph with equation $y = 3 - f(x)$?

A



B



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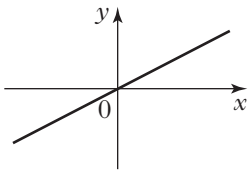

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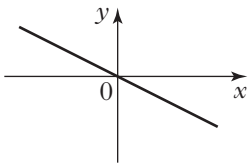


8. A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}9u_n + 90$
What is the limit of this sequence?

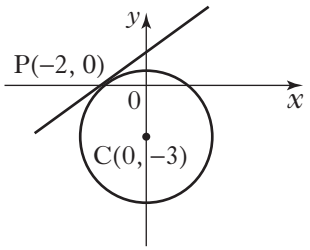
A −900

B 94·5

C 100

D 900

9. The diagram shows a circle, centre C(0,–3) with a tangent drawn at the point P(–2, 0).



What is the equation of this tangent?

A $y = \frac{2}{3}(x + 2)$

B $y + 2 = -\frac{2}{3}x$

C $y + 3 = -\frac{3}{2}x$

D $y = \frac{3}{2}(x + 2)$

10. The equation $\sqrt{2}\cos\theta + 1 = 0$ has solution $\theta = \alpha$ where $\pi \leq \alpha \leq \frac{3\pi}{2}$. What is the value of α?

A $\frac{3\pi}{4}$

B $\frac{5\pi}{4}$

C $\frac{4\pi}{3}$

D $\frac{3\pi}{2}$

11. Find $\int 6\cos 2x\,dx$

A $-12\sin 2x + c$

B $3\sin 2x + c$

C $-6\sin 2x + c$

D $6\sin(x^2) + c$

12. If $f(x) = \sqrt{x^2+1}$ what is $f'(x)$?

A 1

B $\dfrac{x}{\sqrt{x^2+1}}$

C $\dfrac{x}{\left(\sqrt{x^2+1}\right)^3}$

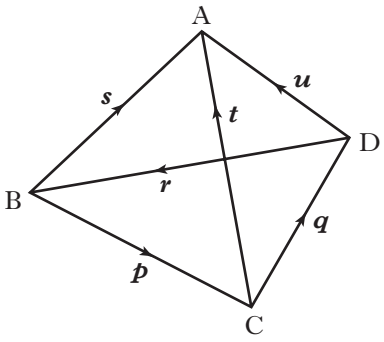
D $2x\sqrt{x^2+1}$

13. In the diagram ABCD represents a tetrahedron.

$\overrightarrow{BC}$ represents $\boldsymbol{p}$, $\overrightarrow{CD}$ represents $\boldsymbol{q}$,

$\overrightarrow{DB}$ represents $\boldsymbol{r}$, $\overrightarrow{BA}$ represents $\boldsymbol{s}$,

$\overrightarrow{CA}$ represents $\boldsymbol{t}$ and $\overrightarrow{DA}$ represents $\boldsymbol{u}$



One of these statements is false, which one?

A $\boldsymbol{p} = -\boldsymbol{q} + \boldsymbol{s} - \boldsymbol{u}$

B $\boldsymbol{q} = -\boldsymbol{p} + \boldsymbol{s} + \boldsymbol{u}$

C $\boldsymbol{r} = -\boldsymbol{p} - \boldsymbol{t} + \boldsymbol{u}$

D $\boldsymbol{s} = \boldsymbol{p} + \boldsymbol{q} + \boldsymbol{u}$

14. P divides AB in the ratio 3:2 where A is the point (–3, 2, 6) and B is the point (7, –3, 1). What is the y-coordinate of P?

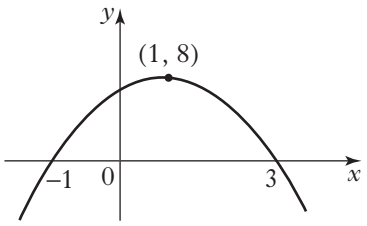
A -1

B 0

C 1

D 3

15. The diagram shows a graph with equation of the form $y = k(x - a)(x - b)$



What is the equation of the graph?

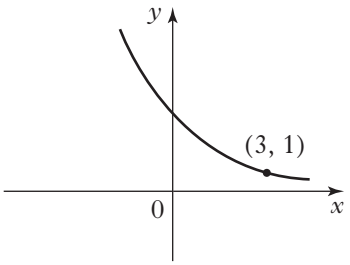
A $y = -2(x + 1)(x - 3)$

B $y = -2(x - 1)(x + 3)$

C $y = 8(x + 1)(x - 3)$

D $y = 8(x - 1)(x + 3)$

16. The graph shown in the diagram has equation of the form $y = k \times 2^{-x}$



What is the value of k?

A $-\frac{3}{2}$

B $\frac{1}{8}$

C $\frac{1}{3}$

D 8

17. $3x^2 - 6x + 11$ is expressed in the form $3(x + a)^2 + b$
What is the value of b?

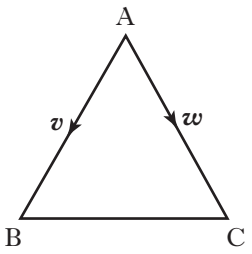
A 1

B 6

C 8

D 11

18. ABC is an equilateral triangle with side length 3 units. $\overrightarrow{AB}$ represents $\boldsymbol{v}$ and $\overrightarrow{AC}$ represents $\boldsymbol{w}$.



Find the value of $\boldsymbol{v}$. $(\boldsymbol{v} - \boldsymbol{w})$

A 0

B $\frac{9}{2}$

C $\frac{9\sqrt{3}}{2}$

D 9

19. A function f is defined by $f(x) = \frac{5}{2(x^2-3x+2)}$. A suitable domain for f is the set of Real numbers apart from which values?

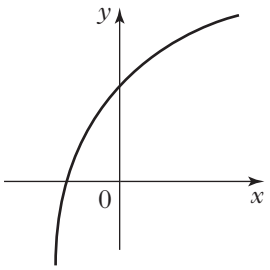
A $x = -2$ and $x = -1$

B $x = 0$

C $x = 1$ and $x = 2$

D $x = 2$ and $x = 4$

20. The graph $y = 2\log_5(x + 3)$ is shown in the diagram.



At what point does this graph intersect the x-axis?

A $(-4, 0)$

B $(-3, 0)$

C $\left(-\frac{3}{2}, 0\right)$

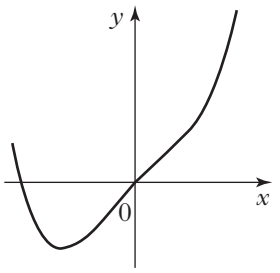
D $(-2, 0)$

[End of section A]

SECTION B

Marks

21. The diagram shows a sketch of the curve with equation $y = \frac{1}{16}x^4 - \frac{1}{8}x^2 + x$. The line $y = x + c$ is a tangent to this curve.



Find the possible values for c and for each value find the coordinates of the point of contact of the tangent. **7**

22. A function f is defined by the formula $f(x) = x^3 + 3x^2 - 4$.

(a) Find the coordinates of the stationary points on the graph with equation $y = f(x)$ and determine their nature **6**

(b) (i) Show that $(x + 2)$ is a factor of $x^3 + 3x^2 - 4$

(ii) Hence or otherwise factorise $x^3 + 3x^2 - 4$ fully **5**

(c) Find the coordinates of the points where the curve $y = f(x)$ crosses the x and y-axes and hence sketch the curve **4**

23. Functions f and g are defined by $f(x) = 2x - 1$ and $g(x) = \log_{12} x$ suitable domains

(a) Show that the equation $f(g(x)) + g(f(x)) = 0$ has a solution $x = 2$ **6**

(b) Show that the equation has no other real solutions **2**

[End of section B]

[End of question paper]