Mathematics Higher

Practice Papers
for SQA Exams

Exam K
Higher
Paper 1
Non-calculator

You are allowed 1 hour, 30 minutes to complete this paper.

You must not use a calculator.

Full marks will only be awarded where your answers include relevant working.

You will not receive any marks for answers derived from scale drawings.

FORMULAE LIST

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Circle

The equation $x^2 + y^2 + 2nx + 2py + c = 0$ represents a circle centre $(-n, -p)$ and radius $\sqrt{n^2 + p^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Table of standard derivatives

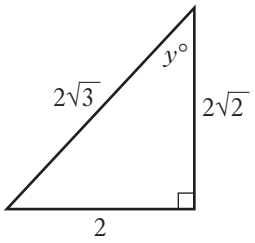
$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Scalar Product $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

SECTION A

1. The diagram shows a right-angled triangle with sides 2, $2\sqrt{2}$ and $2\sqrt{3}$. What is the value of $\sin 2y°$?



A $\frac{2}{\sqrt{3}}$

B $\frac{1}{3}$

C $\frac{2\sqrt{2}}{3}$

D $2\sqrt{3}$

2. If $y = \frac{x^4 + 1}{x}$ what is $\frac{dy}{dx}$?

A $4x + 1$

B $x^4 + 1 + x^{-1}$

C $4x^3 + 1$

D $3x^2 - \frac{1}{x^2}$

3. Which of the following describes the stationary point on the curve with equation $y = 2(x + 2)^2 - 1$?

A minimum at (2, –1)

B maximum at (2, –1)

C minimum at (–2, –1)

D maximum at (–2, –1)

4. Functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 3 - x$ for $x \geq 0$.
Which of the following is an expression for $f(g(x))$?

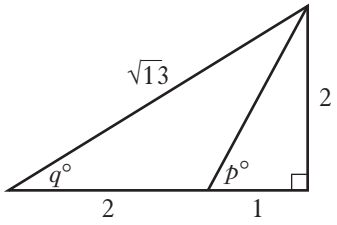
A $3\sqrt{x} - x\sqrt{x}$

B $\sqrt{3-x}$

C $\sqrt{x}+3-x$

D $3-x\sqrt{x}$

5. p and q are angles as shown in the diagram
What is the value of $\cos(p+q)°$?



A $\dfrac{8}{\sqrt{65}}$

B $-\dfrac{1}{\sqrt{65}}$

C $\dfrac{1}{\sqrt{5}}+\dfrac{3}{\sqrt{13}}$

D $\dfrac{1}{\sqrt{5}}-\dfrac{3}{\sqrt{13}}$

6. Find $\int \dfrac{3}{\sqrt{x}}dx$

A $6\sqrt{x}+\text{c}$

B $-\dfrac{3}{x}+\text{c}$

C $\dfrac{3}{2\sqrt{x}}+\text{c}$

D $\dfrac{3}{x^2}+\text{c}$

7. A circle has equation $x^2 + y^2 - 2x + 6y - 1 = 0$.
What is the radius of this circle?

A $\sqrt{3}$

B $\sqrt{11}$

C $\sqrt{37}$

D $\sqrt{39}$

8. What is the distance PQ where P is the point (–1, –3, 5) and Q is the point (0, 5, –2)?

A $\sqrt{12}$

B $\sqrt{14}$

C $\sqrt{64}$

D $\sqrt{114}$

9. A sequence is defined by the recurrence relation

$u_{n+1} = u_n^2 - 1, u_0 = -2$

What is the value of u_2?

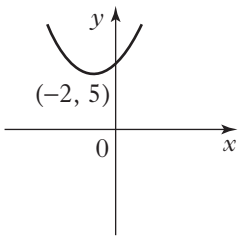
A –26

B –11

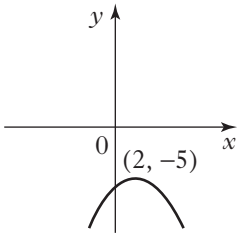
C 8

D 64

10. The diagram shows a sketch of the graph with equation $y = f(x)$. Which of the diagrams below shows a sketch of $y = -f(x - 4)$?



A



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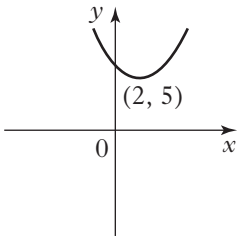

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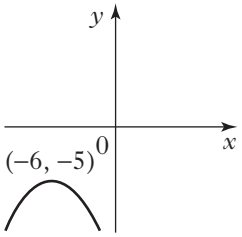

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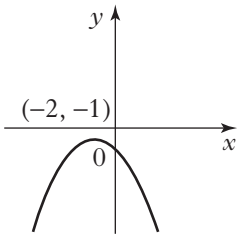


11. The equation $3x^2 + x + m = 0$ has equal roots. What is the value of m?

A $-\frac{1}{3}$

B $-\frac{1}{12}$

C $\frac{1}{12}$

D 12

12. The point A(2, 3) lies on the circle with equation $x^2 + y^2 + 2x - 4y - 5 = 0$. What is the gradient of the tangent at A?

A -10

B -3

C 0

D $\frac{1}{3}$

13. $7 - 8x - x^2$ is expressed in the form $a - (x + b)^2$. What is the value of a?

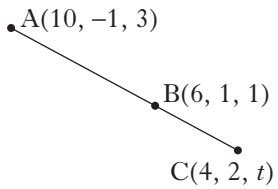
A -23

B -9

C 9

D 23

14. The points A(10, –1, 3), B(6, 1, 1) and C (4, 2, t) are collinear as shown in the diagram. What is the value of t?



A 0

B 2

C 4

D 6

15. For a curve $y = f(x)$ it is known that $\frac{dy}{dx} = 4x^3 - x^2 - 1$ and that it passes through the origin. What is the equation of the curve?

A $y = x^4 - \frac{1}{3}x^3 - x$

B $y = 12x^4 - 2x^3$

C $y = 12x^2 - 2x$

D $y = 4x^4 - x^3 - x + 1$

16. If $\log_2 9 = 3 - \log_2 x$, what is the value of x?

A $\frac{3}{81}$

B $\frac{8}{9}$

C 1

D 6

17. What is the value of $\int_o^{\pi/3} \cos\frac{1}{2}x \, dx$?

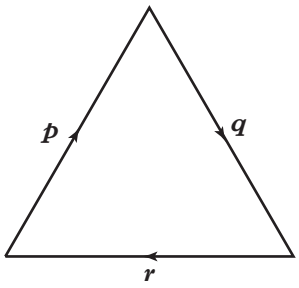
A $-\sqrt{3}$

B $-\frac{1}{2}$

C 1

D $\sqrt{3}$

18. The vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are represented by the sides of an equilateral triangle as shown in the diagram.



Here are two statements about these vectors:

(1) $\boldsymbol{q}.(\boldsymbol{p} - \boldsymbol{r}) = 0$

(2) $\boldsymbol{q}.(\boldsymbol{p} + \boldsymbol{r}) = 0$

Which of the following is true?

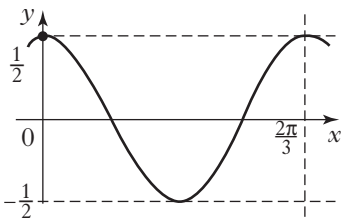
A neither statement is true

B only statement (1) is correct

C only statement (2) is correct

D both statements are correct

19. What is the equation of the graph shown in the diagram?



A $y = \frac{1}{2}\cos\frac{1}{3}x$

B $y = \cos 3x + \frac{1}{2}$

C $y = \frac{2}{3}\cos\frac{1}{2}x$

D $y = \frac{1}{2}\cos 3x$

20. If $\frac{\log_k 4}{\log_e 2} = 2e^0$, what is the value of k?

A $k = 1$

B $k = \sqrt{2}$

C $k = \sqrt[4]{2e^2}$

D $k = e$

[End of section A]

SECTION B

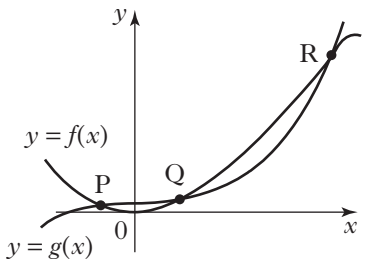
Marks

21. (*a*) Find the stationary points on the curve with equation $y = x^3 - 3x^2 + 4$ and justify their nature. **7**

(*b*) (i) Show that $(x + 1)(x - 2)^2 = x^3 - 3x^2 + 4$

(ii) Hence sketch the graph of $y = x^3 - 3x^2 + 4$ **4**

22. Two cubic graphs, $y = f(x)$ and $y = g(x)$, where $f(x) = 2x^3 + 3x + 12$ and $g(x) = 2 + 16x^2 - x^3$, are shown in the diagram.



Determine the x–coordinates of each of P, Q and R, the three points of intersection of the two graphs. **8**

23. The islanders living in Tarbert on the island of Harris are planning to build a new sewage processing plant. Central to the plant is the seepage pit which allows most of the week's sewage to seep harmlessly through the soil and drain away. Sewage is pumped into the pit at the start of each week. These are two possible sites with the following specifications:

	Seepage Rate	**Pumping capacity**
Upland Site:	65% of 1 week's sewage	2000 litres at start of week
Lowland Site:	75% of 1 week's sewage	2500 litres at start of week

(*a*) Write down a recurrence relation for each site using u_n to represent the amount of litres of sewage stored at the Upland site immediately after pumping at the start of the n^{th} week and let v_n be the equivalent volume at the Lowland site. Clearly label each relation with the site name. **2**

(*b*) The size of the storage tank at each site is determined by the maximum volume of sewage that will remain at the site in the long term. Which site requires the smaller tank in the long term? **4**

24. Solve the equation $\cos\theta(\cos\theta - 1) = \sin^2\theta$ for $\pi < \theta < 2\pi$ **5**

[End of section B]

[End of question paper]