Higher Grade - Paper 1 (Paper J) **ANSWERS - Section A**

1	D
2	B
3	C
4	C
5	B
6	D
7	A
8	C
9	D
10	B
11	B
12	C
13	D
14	A
15	D
16	C
17	C
18	D
19	B
20	A

	A	B	C	D
1	□	□	□	■
2	□	■	□	□
3	□	□	■	□
4	□	□	■	□
5	□	■	□	□
6	□	□	□	■
7	■	□	□	□
8	□	□	■	□
9	□	□	□	■
10	□	■	□	□
11	□	■	□	□
12	□	□	■	□
13	□	□	□	■
14	■	□	□	□
15	□	□	□	■
16	□	□	■	□
17	□	□	■	□
18	□	□	□	■
19	□	■	□	□
20	■	□	□	□

** Please note .... at time of production the worked solutions to this multiple choice section were not available.

Higher Grade Paper 1 (Paper J) — Marking Scheme - Section B

	Give 1 mark for each •	Illustration(s) for awarding each mark
21(a)	**ans: proof (3 marks)**	
	•[1] finds expressions for 2 areas	•[1] $A_{rect} = x(2k-2)$; $A_{tri} = x(x+k)$
	•[2] adds 4 to area of triangle and equates	•[2] $x(x+k)+4 = x(2k-2)$
	•[3] reorganises to given form	•[3] $x^2+(2-k)x+4=0$
(b)	**ans: $k = 6$ (3 marks)**	
	•[1] knows condition for equal roots	•[1] $b^2-4ac=0$ [stated or implied]
	•[2] substitutes values	•[2] $(2-k)^2-4\times1\times4=0$
	•[3] solves and discards	•[3] $(k+2)(k-6)=0; k=-2$ or 6; $k=6$
(c)	**ans: $x = 2$; 20cm^2; 16cm^2 (3 marks)**	
	•[1] substitutes value of k to form quadratic	•[1] $x^2-4x+4=0$
	•[2] solves to x	•[2] $(x-2)^2=0; x=2$
	•[3] finds areas	•[3] $A_{rect}=20cm^2; A_{tri}=16cm^2$
22(a)	**ans: $3y + x = -30$ (2 marks)**	
	•[1] identifies required gradient	•[1] $m_{\text{CB}} = -\frac{1}{3}$
	•[2] substitutes into general equation	•[2] $y+11=-\frac{1}{3}(x-3)$ [or equivalent]
(b)	**ans: D(-3,-9) (3 marks)**	
	•[1] knows to use systems of equations	•[1] evidence
	•[2] finds value for x	•[2] $x = -3$
	•[3] finds value for y and states coordinates	•[3] $y = -9$; (-3,-9)
(c)	**ans: C(-9,-7) (1 mark)**	
	•[1] states coordinates of C	•[1] C(-9,-7)
(d)	**ans: $(x + 3)^2 + (y - 1)^2 = 100$ (4 marks)**	
	•[1] identifies diameter	•[1] AC is diameter [$\angle$ADC is right-angled]
	•[2] finds centre	•[2] midpoint of AC is (-3,1)
	•[3] finds radius or r^2	•[3] $r = 10$ or $r^2 = 100$
	•[4] subs into general equation	•[4] $(x+3)^2+(y-1)^2=100$

	Give 1 mark for each •	Illustration(s) for awarding each mark
23(a)	**ans: $(x-4)^2 - 15$; $p = -4$, $q = -15$ (4 marks)**	
	•[1] finds derivative •[2] starts to complete square •[3] completes •[4] states values of p and q	•[1] $f(x) = x^2 - 8x + 1$ •[2] $(x-4)^2$ …….. •[3] ……… -15 •[4] $p = -4$, $q = -15$
(b)	**ans: -15 when $x = 4$ (2 marks)**	
	•[1] states minimum rate of change •[2] states value of x	•[1] rate of change is -15 •[2] $x = 4$
24	**ans: $\frac{2\pi}{3}$, 0 (5 marks)**	
	•[1] collects terms to LHS and equates to 0 •[2] factorises quadratic •[3] finds values for $\cos a$ •[4] finds one value for a •[5] finds second value for a	•[1] $2\cos^2 a - \cos a - 1 = 0$ •[2] $(2\cos a + 1)(\cos a - 1) = 0$ •[3] $\cos a = -\frac{1}{2}$ or $\cos a = 1$ •[4] $\frac{2\pi}{3}$ •[5] 0
		Total: 30 marks

Higher Grade Paper 2 (Paper J) **Marking Scheme**

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	**ans: $2y - x = -2$ (3 marks)**	
	•1 finds midpoint of BC	•1 midpoint BC: (10,4)
	•2 establishes gradient of AM	•2 $m_{AM} = \frac{4+2}{10+2} = \frac{1}{2}$
	•3 substitutes in general equation	•3 $y - 4 = \frac{1}{2}(x - 10)$
(b)	**ans: D(4,1) (3 marks)**	
	•1 realising $y = 1$	•1 $y = 1$
	•2 substitutes into equation	•2 $2(1) - x = -2;\ x = 4$
	•3 states coordinates of D	•3 D(4,1)
(c)	**ans: proof (3 marks)**	
	•1 finds gradient of BD	•1 $m_{BD} = -2$
	•2 knows condition for perp. lines	•2 $m_1 \times m_2 = -1$ [stated or implied]
	•3 makes statement re perpendicular	•3 $\frac{1}{2} \times -2 = -1$ so AM and BD are perp.
2(a)	**ans: P(1,-$\frac{25}{2}$) (4 marks)**	
	•1 knows to take derivative and equate to 0	•1 $\frac{dy}{dx} = 0$
	•2 takes derivative	•2 $3x^2 - 15x + 12 = 0$
	•3 solves to find x - coordinate	•3 $x = 1$ [or 4]
	•4 substitutes to find y - coordinate	•4 $y = 1^3 - \frac{15}{2}(1) + 12(1) - 18 = -\frac{25}{2}$
(b)	**ans: Q(6,0) (3 marks)**	
	•1 knows to make $y = 0$	•1 $y = 0$
	•2 uses synthetic division to find x	•2 $\begin{array}{c\|cccc} 6 & 1 & -\frac{15}{2} & 12 & -18 \\ & & 6 & -9 & 18 \\ \hline & 1 & -\frac{3}{2} & 3 & 0 \end{array}$
	•3 states coordinates of Q	•3 Q(6,0)

	Give 1 mark for each •	Illustration(s) for awarding each mark
3(a)	**ans: 32·6 gigatonnes** **(3 marks)**	
	●[1] correct multiplier ●[2] completes calculation ●[3] calculation and correct rounding	●[1] 0·96 ●[2] $0{\cdot}96^5 \times 40$ ●[3] 32·6 gigatonnes
(b)	**ans: 31 gigatonnes** **(3 marks)**	
	●[1] sets up recurrence relation ●[2] knows to calculate 3 figures ●[3] final answer	●[1] $U_{n+1} = 0\cdot 96^5 U_n + 3\cdot 8$ ●[2] 1^{st} year: 36·4; 2^{nd} year: 33·4795 ●[3] 3^{rd} year: 31 gigatonnes
(c)	**ans: upper 20·6; lower 16·8** **(3 marks)**	
	●[1] knows limit exists ●[2] finds upper limit ●[3] finds lower limit	●[1] limit exists since $-1 < 0{\cdot}96^5 < 1$ ●[2] $L = \dfrac{3\cdot 8}{1-(0\cdot 96)^5} = 20{\cdot}6$ ●[3] $20{\cdot}6 - 3{\cdot}8 = 16{\cdot}8$
4(a)	**ans: $a = -2$** **(2 marks)**	
	●[1] finds expression for $f(g(-2))$ ●[2] equates to -1 and solves for a	●[1] $f(g(-2)) = f(-1) = 1 + a$ ●[2] $a = -2$
(b)	**ans: $x = -2, 0, 2$** **(5 marks)**	
	●[1] substitutes ●[2] simplifies ●[3] equates to 2 ●[4] factorises ●[5] solves for x	●[1] $f(f(x)) = (x^2-2)^2 - 2$ ●[2] $x^4 - 4x^2 + 2$ ●[3] $x^4 - 4x^2 + 2 = 2; x^4 - 4x^2 = 0$ ●[4] $x^2(x^2-4) = 0$ ●[5] $x = -2, 0, 2$
5(a)	**ans: $x = 1$** **(2 marks)**	
	●[1] realises y = 0; equates to 0 ●[2] solves for x	●[1] $3x^2 - 6x + 3 = 0$ ●[2] $3(x-1)^2 = 0; x = 1$
(b)	**ans: $b = 2$** **(5 marks)**	
	●[1] integrates expression ●[2] substitutes values ●[3] simplifies, equates to 1, rearranges ●[4] uses synthetic division to solve ●[5] realises one solution; discards $b^2 - b + 1$	●[1] $[x^3 - 3x^2 + 3x]_1^b$ ●[2] $\left(b^3 - 3b^2 + 3b\right) - \left(1 - 3 + 3\right)$ ●[3] $b^3 - 3b^2 + 3b - 2 = 0$ ●[4] $\begin{array}{r\|rrrr} 2 & 1 & -3 & 3 & -2 \\ & & 2 & -2 & 2 \\ \hline & 1 & -1 & 1 & 0 \end{array}$ ●[5] $b = 2$

	Give 1 mark for each •	Illustration(s) for awarding each mark
6	**ans: 58** **(4 marks)** •[1] knows to take logs •[2] releases the power •[3] makes t the subject •[4] answer + rounding	•[1] $\log 2^{-0.04t} = \log 0\cdot 2$ •[2] $-0\cdot 04t\log 2 = \log 0\cdot 2$ •[3] $t = \dfrac{\log 0\cdot 2}{(-0\cdot 04\log 2)}$ (or equivalent) •[4] $t = 58$
7(a)	**ans: S(6,3,-12)** **(3 marks)** equiv.) •[1] for vector algebra •[2] for substituting values •[3] answer	•[1] $\mathbf{p} - \mathbf{q} = 2(\mathbf{s} - \mathbf{r}) \Rightarrow \mathbf{p} - \mathbf{q} + 2\mathbf{r} = 2\mathbf{s}$ (or •[2] $\begin{pmatrix} 6 \\ -1 \\ 4 \end{pmatrix} - \begin{pmatrix} 10 \\ 3 \\ -2 \end{pmatrix} + \begin{pmatrix} 16 \\ 10 \\ -22 \end{pmatrix} = 2\,\mathbf{s}$ •[3] S(6,3,-12)
(b)	**ans: proof** **(3 marks)** $\vec{PS}.\vec{SR} = 0$) •[1] knows scalar product = 0 (stated or implied) •[2] finds both displacements •[3] calculation angle	•[1] statement made <u>or</u> implied (•[2] $\vec{PS} = \begin{pmatrix} 0 \\ 4 \\ -8 \end{pmatrix}, \vec{SR} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ (or equivalent) •[3] $\vec{PS}.\vec{SR} =$ 0 + 8 + (-8) = 0 $\therefore$ right
8(a)	**ans: Proof** **(1 mark)** •[1] uses trig ratios and equates	•[1] $\cos x = \dfrac{AB}{4}$, $\sin x = \dfrac{BC}{6}$ $4\cos x^\circ + 6\sin x^\circ = 6\cdot 72$
(b)	**ans: 35°** **(5 marks)** •[1] recognising as $k$cos(… equation •[2] finds k •[3] finds α •[4] finds first solution $x = 77\cdot 6^\circ$ •[5] finds 2nd solution and decides correct ans.	•[1] $4\cos x + 6\sin x = k\cos(x - \alpha)$ (or equivalent) •[2] $k^2 = 4^2 + 6^2 \;\therefore\; k = \sqrt{52}$ •[3] $\tan\alpha = \dfrac{6}{4}$, $\alpha = 56\cdot 3^\circ$ •[4] $\cos(x - 56\cdot 3) = \frac{6\cdot 72}{\sqrt{52}} \;\therefore\; x - 56\cdot 3 = 21\cdot 3$ •[5] *or* $x - 56\cdot 3 = 338\cdot 7 \;\therefore\; x = 395^\circ = 35^\circ$ then decides 35° as < 45

	Give 1 mark for each •	Illustration(s) for awarding each mark
9(a)	**ans: m = 2400ml** **(4 marks)**	
	•[1] knows to differentiate and equate to 0	•[1] $H'(m) = 0$
	•[2] differentiates	•[2] $4 - \frac{1}{600}m = 0$
	•[3] solves for x	•[3] $m = 2400$
	•[4] justifies maximum	•[4] table of values; second derivative
(b)	**ans: 4800 feet** **(1 mark)**	
	•[1] knows to sub into function and evaluate	•[1] $4(2400) - \frac{(2400)^2}{1200} = 4800\text{feet}$

Total: 60 marks