NATIONAL QUALIFICATIONS

Higher Mathematics
Practice Paper J

Paper 1
Assessing Units 1, 2 & 3

Time allowed - 1 hour 30 minutes

Read carefully

Calculators may <u>NOT</u> be used in this paper.

Section A - Questions 1 - 20 (40 marks)

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

Section B (30 marks)

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product: $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3 \text{ where } \boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

SECTION A
ALL questions should be attempted

1. A line has as its equation $3y = x + 6$.

Any line parallel to this line will have as its gradient

A -3

B 1

C $-\frac{1}{3}$

D $\frac{1}{3}$

2. If $f(x) = \frac{1}{x^3}$ and $x \neq 0$, then $f'(x)$ is

A $\frac{1}{3x^2}$

B $-\frac{3}{x^4}$

C $-\frac{3}{x^2}$

D $-\frac{1}{2x^2}$

3. The remainder when $2x^3 + x^2 - 1$ is divided by $x - 2$ is

A 9

B 5

C 19

D -13

4. Which of the following is/are true of the circle with equation $x^2 + y^2 - 36 = 0$?

1 It passes through the origin.
2 It has a radius of 6.
3 It has the origin as its centre.

A 1 only

B 2 only

C 2 and 3 only

D some other combination of responses

5. Given that $\cos x^\circ = \frac{1}{\sqrt{3}}$ and $0 < x < 90$, then the exact value of $\cos 2x^\circ$ will be

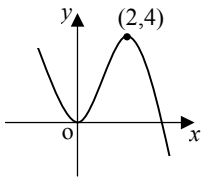
A $\frac{2}{\sqrt{3}}$

B $-\frac{1}{3}$

C $\frac{1}{3}$

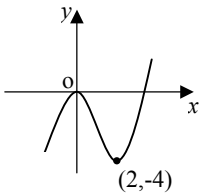
D $\frac{1}{2\sqrt{3}}$

6. Part of the graph of $y = f(x)$ is shown below.



The graph of $y = f'(x)$ could be represented by

A



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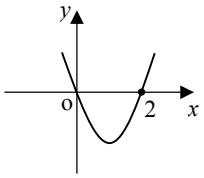
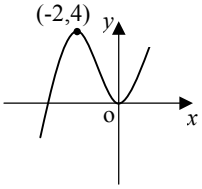


C



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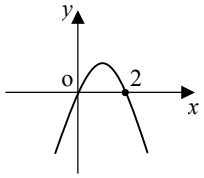


7. Which one of the following points lies on the graph of $y = \log_3 x$?

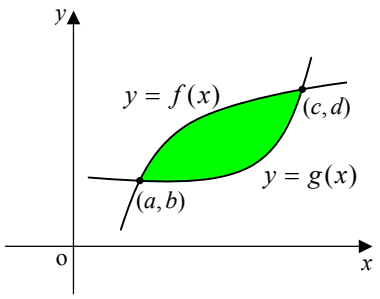
A (9,2)

B (3,27)

C (2,9)

D (0,0)

8.



The shaded area above is given by

A $\int_b^d \big(f(x) - g(x)\big)\, dx$

B $\int_a^c \big(f(x) + g(x)\big)\, dx$

C $\int_a^c \big(f(x) - g(x)\big)\, dx$

D $\int_a^d \big(f(x) - g(x)\big)\, dx$

9. Two functions, defined on suitable domains, are given as $f(x) = 3x^2 - 2$ and $g(x) = 1 - x$.

The value of $f(g(2))$ is

A -9

B -5

C -1

D 1

10. The value of $\cos\frac{5\pi}{6}$ is

A $-\frac{1}{2}$

B $-\frac{\sqrt{3}}{2}$

C $\frac{\sqrt{3}}{2}$

D $\frac{1}{2}$

11. Given that $\boldsymbol{v} = \begin{pmatrix} \sqrt{2} \\ 2 \\ \sqrt{3} \end{pmatrix}$, then $|\boldsymbol{v}|$ is

A $2+\sqrt{5}$

B 3

C 9

D $\sqrt{7}$

12. A circle has as its equation $x^2 + y^2 + 4x - 2y - 4 = 0$.

Which of the following correctly states the coordinates of its centre and the value of its radius?

A $(-2,1)$, $r = 1$

B $(2,-1)$, $r = 3$

C $(-2,1)$, $r = 3$

D $(2,-1)$, $r = 1$

13. $\int_0^{\frac{\pi}{4}} 4(\cos 2x)\, dx$ is equal to

A 0

B 4

C -2

D 2

14. A recurrence relation is defined by $U_{n+1} = 0 \cdot 4U_n - 24$. The limit of this sequence is

A -40

B -24

C $0 \cdot 03$

D 50

15. If x and y are integers the value of $(x+y)^2-(x-y)^2$ is always

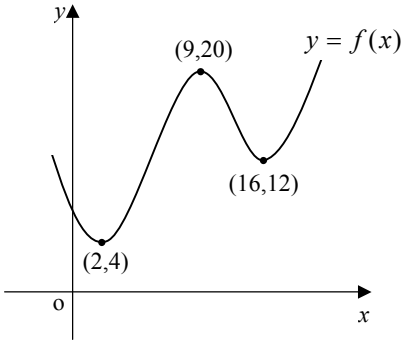
A negative

B positive

C a perfect square

D a multiple of 4

16.



The diagram shows part of the graph of $y=f(x)$.

Which of the following is/are true for the function above?

1 $f'(0)<0$
2 $f'(6)<0$
3 $f'(9)=0$
4 $f'(12)>0$

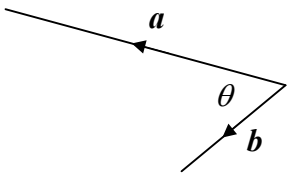
A 2 and 3 only

B 3 only

C 1 and 3 only

D 1, 2, 3 and 4

17. Consider the diagram and information below.



$\cos\theta=\frac{1}{2}$

If the magnitude of vector **a** is 2 and the magnitude of vector **b** is 1 then the value of $\mathbf{a}.(\mathbf{a}+\mathbf{b})$ is

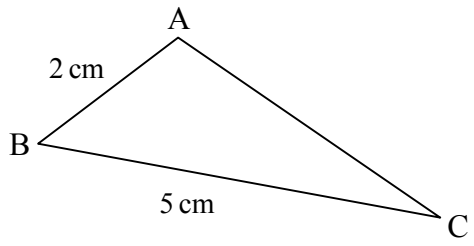
A 6

B $\sqrt{5}$

C 5

D 3

18.



If $\tan \text{ABC} = \frac{3}{4}$ then the area of triangle ABC in square centimetres is

A 5

B 4

C $\frac{15}{4}$

D 3

19. The quadratic equation $4kx^2 - 8x + k = 0$ has equal roots.

The value of k, where $k > 0$ is

A 4

B 2

C 0

D -2

20. $f(x) = ax^2 - 2x - 5$ has a stationary value when $x = 3$.

The value of a is

A $\frac{1}{3}$

B $-\frac{1}{3}$

C $\frac{7}{6}$

D $\frac{11}{9}$

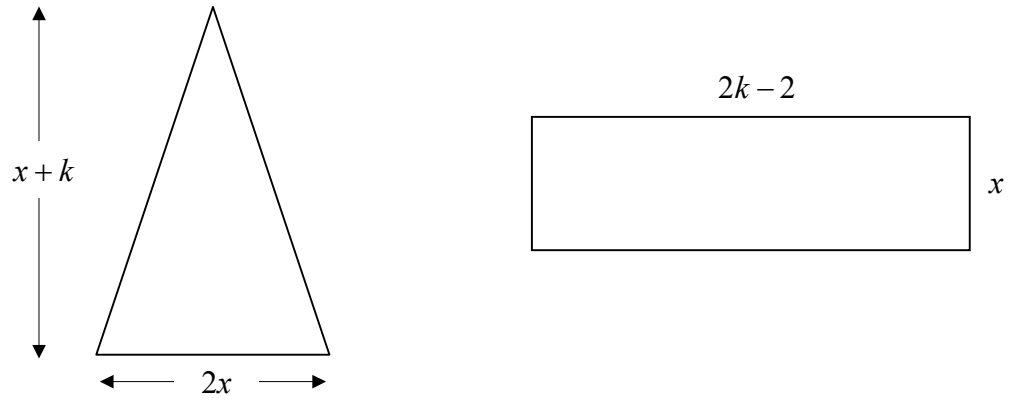
[END OF SECTION A]

SECTION B

ALL questions should be attempted

21. Consider the isosceles triangle and the rectangle below.

The triangle has a base measuring $2x$ and a vertical height of $x + k$.
The rectangle has dimensions $2k - 2$ by x as shown.
All dimensions are in centimetres.



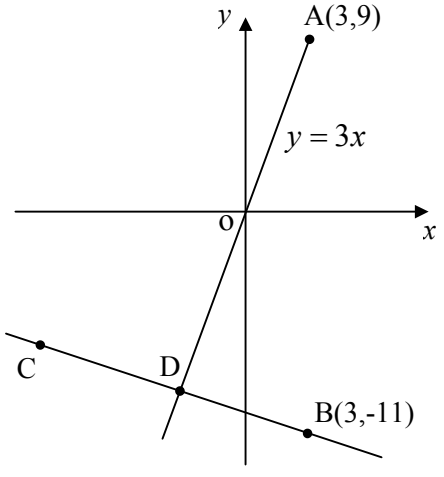
(a) Given that the **area of the rectangle** is 4cm^2 **more than** the area of the triangle, **show clearly** that the following equation can be formed.

$$x^2 + (2-k)x + 4 = 0$$ **3**

(b) Hence find k, given that the equation $x^2 + (2-k)x + 4 = 0$ has equal roots and $k > 0$. **3**

(c) Find x when k takes this value and calculate the area of each shape. **3**

22. In the diagram A has coordinates (3,9) and the point B has coordinates (3,-11) as shown.
A lies on the line with equation $y = 3x$.



(a) If line BC is perpendicular to the line AD, establish the equation of BC. **2**

(b) Hence find the coordinates of D. **3**

(c) If D is the mid-point of BC, write down the coordinates of C. **1**

(d) Find the equation of the circle passing through the points A, D and C. **4**

23. A function is defined on a suitable domain as $f(x) = \frac{1}{3}x^3 - 4x^2 + x$.

(a) Show that its derivative can be expressed in the form

$f'(x) = (x + p)^2 + q$, and state the values of p and q. **4**

(b) Hence state the minimum rate of change of this function and the corresponding value of x. **2**

24. Find the solution(s) of the equation $2\cos^2 a = \cos a + 1$ for $0 \le a \le \pi$. **5**

[END OF SECTION B]

[END OF QUESTION PAPER]