Practice Paper H
Marking Scheme - Paper I Section A

1. M is the midpoint of QR

i.e. (0, 6)

Answer: B

2.
$$U_{n+1} = aU_n + 6$$
$$L = \frac{6}{1-a} = 10$$
$$10 - 10a = 6$$
$$-10a = -4$$
$$a = 0\cdot 4$$

Answer: C

3.
$$f(-2) = 3 - \frac{6}{-2} = 3 + 3 = 6$$
$$f(6) = 3 - \frac{6}{6} = 2$$

Answer: B

4. ***Answer: C***

5.
$$\begin{array}{r|rrrr} -1 & 1 & -4 & 1 & 6 \\ & & -1 & 5 & -6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$
$$(x+1)(x^2 - 5x + 6)$$
$$= (x+1)(x-2)(x-3)$$

Answer: C

6. ***Answer: A***

7.
$$\sin(\frac{11\pi}{6})(330°)$$
$$= -\sin(\frac{\pi}{6})(30°)$$
$$= -\frac{1}{2}$$

Answer: B

8.
$$U_1 = 1\cdot 5 \times 12 - 2 = 16$$
$$U_2 = 1\cdot 5 \times 16 - 2 = 22$$
$$U_3 = 1\cdot 5 \times 22 - 2 = 31$$

Answer: C

9.
$$k = \log_3 81$$
$$k = 4$$

Answer: D

10.
$$2\sin 75° \cos 75° = \sin 150°$$
$$= \sin 30°$$
$$= \frac{1}{2}$$

Answer: B

11.
$$\int (1+4x)^{\frac{1}{2}}\ dx$$
$$= \frac{(1+4x)^{\frac{3}{2}}}{\frac{3}{2}} \times \frac{1}{4} + C$$
$$= \frac{2}{3}(1+4x)^{\frac{3}{2}} \times \frac{1}{4} + C$$
$$= \frac{1}{6}(1+4x)^{\frac{3}{2}} + C$$

Answer: C

12.
$$|g| = \sqrt{7^2 + (3\sqrt{5})^2 + (5\sqrt{2})^2}$$
$$= \sqrt{49 + 45 + 50} = \sqrt{144} = 12$$

Answer: A

13.

$4(4-x)^3.-1$

$=-4(4-x)^3$

Answer: B

14.

$$\begin{array}{r|rrrr} -3 & 1 & 0 & -5a & -3 \\ & & -3 & 9 & 15a-27 \\ \hline & 1 & -3 & 9-5a & 15a-30 \end{array}$$

$15a-30=0$

$15a=30$

$a=2$

Answer: C

15.

$(x-5)^2-25+16$

$=(x-5)^2-9$

Answer: D

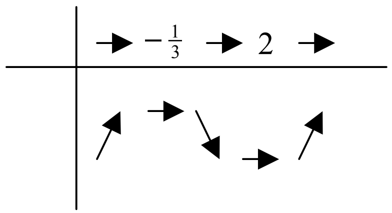
16.

$\frac{dy}{x}=6x^2-10x-4=0$

$2(3x^2-5x-2)=0$

$2(3x+1)(x-2)=0$

$x=-\frac{1}{3};\ 2$



Answer: B

17.

$(x+3)^2-9-3$

$=(x+3)^2-12$

Minimum value is -12

Maximum value is $-\frac{1}{12}$

Answer: D

18.

$-2\cos(3x+1)\times\frac{1}{3}+C$

$=-\frac{2}{3}\cos(3x+1)+C$

Answer: C

19.

$(-4,\ -1,\ -2)$ $(5,\ 8,\ 7)$

$5:4$

$x_T=\frac{-16+25}{9}=1$

$y_T=\frac{-4+40}{9}=4$

$z_T=\frac{-8+35}{9}=3$

$T(1,\ 4,\ 3)$

Answer: B

20.

$k^2=3^2+1^2=10$

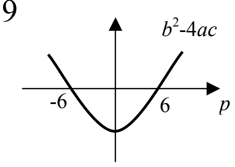
$k=\sqrt{10}$

$\tan\alpha=\frac{3}{1}$

Quadrant I

Answer: D

Practice Paper H - Paper 1 Section B   Marking Scheme

	Give 1 mark for each •	Illustration(s) for awarding each mark
21a	**ans:** $y = 3x - 7$ (or equiv.) **3 marks**	
	•1 for gradient of line •2 for gradient of AB •3 for equation	•1 $m = -\frac{1}{3}$ •2 $m_{AB} = 3$ •3 $y - 2 = 3(x - 3)$
b	**ans:** A(0,-7) **1 mark**	
	•1 answer (y intercept)	•1 A(0,-7)
c	**ans:** Area = 15 square units **4 marks**	
	•1 y intercept of top line •2 length of base •3 perpendicular length •4 calculation to answer	•1 (0,3) •2 distance between y intercepts = 10 •3 y -axis to B … 3 units •4 $A = \frac{1}{2}bh$ $= \frac{1}{2} \times 10 \times 3 = 15\ units^2$
22.	**ans:** $-6 < p < 6$ (or equivalent) **6 marks**	
	•1 dealing with the fractions •2 manipulation to quad. form •3 discriminant statement •4 for a, b and c •5 finding discriminant •6 solution from quad. inequat.	•1 strategy …. $\times\ px$ (or equiv.) •2 $x^2 + 9 = px$; $x^2 - px + 9 = 0$ •3 for no real roots $b^2 - 4ac < 0$ •4 $a = 1$, $b = -p$, $c = 9$ •5 $p^2 - 36 < 0$ •6 $-6 < p < 6$ 
23a	**ans:** proof; length of OA = AB **3 marks**	
	•1 magnitudes of the two position vectors •2 magnitude of $\overrightarrow{AB}$ •3 statement	•1 $\lvert\overrightarrow{OA}\rvert = \sqrt{1+4+9} = \sqrt{14}$ $\lvert\overrightarrow{OB}\rvert = \sqrt{16+1+25} = \sqrt{42}$ •2 $\overrightarrow{AB} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$, $\lvert\overrightarrow{AB}\rvert = \sqrt{9+1+4} = \sqrt{14}$ •3 2 sides the same isosceles (or equiv.)

	Give 1 mark for each •	Illustration(s) for awarding each mark
23b	**ans:** 30° **3 marks**	
	•[1] knows method	•[1] $\cos\theta = \dfrac{\overrightarrow{OA}\,.\,\overrightarrow{OB}}{\lvert\overrightarrow{OA}\rvert.\lvert\overrightarrow{OB}\rvert}$
	•[2] finds scalar product	•[2] $\overrightarrow{OA}\,.\,\overrightarrow{OB} = \begin{pmatrix}1\\2\\3\end{pmatrix}.\begin{pmatrix}4\\1\\5\end{pmatrix} = 4+2+15=21$
	•[3] substitutes s.p. and magnitudes to answer	•[3] $\cos\theta = \dfrac{21}{\sqrt{14}\times\sqrt{42}}$ ... $\theta = 30^\circ$
24a	**ans:** proof **4 marks**	
	•[1] know to solve a system	•[1] set up a system
	•[2] combining equations & simplify to quad.	•[2] $(2y)^2 + y^2 - 6(2y) - 18y + 45 = 0$ $5y^2 - 30y + 45 = 0$
	•[3] for 1st coordinate	•[3] $5(y-3)(y-3) = 0 \;\therefore\; y = 3$
	•[4] for 2nd coordinate	•[4] $x = 2(3) = 6$ (or equivalent)
b	**ans:** B(9,-3) , $(x-9)^2 + (y+3)^2 = 45$ **6 marks**	
	•[1] knowing T mid-pt between centres	•[1] strategy
	•[2] drawing out centre of top circle	•[2] A(3,9)
	•[3] finding B	•[3] A(3,9) $\rightarrow$ T(6,3) $\rightarrow$ B(9,-3)
	•[4] knowing r the same	•[4] stated or implied $r_1 = r_2$
	•[5] finding r^2	•[5] $r^2 = \sqrt{9+81-45} = 45$
	•[6] writing down equation of lower circle	•[6] $(x-9)^2 + (y+3)^2 = 45$

Total 30 marks

Practice Paper H - Paper 2 **Marking Scheme**

	Give 1 mark for each •	Illustration(s) for awarding each mark
1a	**ans:** $k=2$ $k=2$ **4 marks**	
	•[1] gradient of AB	•[1] $m_{AB}=\frac{-3-5}{11+5}=-\frac{1}{2}$
	•[2] gradient of AP	•[2] $m_{AP}=\frac{k-5}{6}$
	•[3] equating gradients	•[3] $\frac{k-5}{6}=-\frac{1}{2}$
	•[4] finding k	•[4] $k=2$
b	**ans:** $y=2x$ **2 marks**	
	•[1] gradient of PQ	•[1] $m_{PQ}=\frac{8-2}{4-1}=2$
	•[2] equation	•[2] $y-2=2(x-1)$
c	**ans:** R(2,4) **5 marks**	
	•[1] coordinates of M	•[1] M(9,3)
	•[2] equation of median	•[2] $y-3=-\frac{1}{7}(x-9)$
	•[3] setting up a system	•[3] $7y=-x+30$; $y=2x$
	•[4] finding first coordinate	•[4] $x=2$
	•[5] finding 2nd coordinate	•[5] $y=4$
2a	**ans:** $1011\cdot 33$ bats (ignore rounding) **3 marks**	
	•[1] first two lines of calculation	•[1] …… Low High $U_1=0\cdot 75(2100)=1575+200\quad =1775$ $U_2=0\cdot 75(1775)=1331\cdot 25+200=1531\cdot 25$
	•[2] lines 3 and 4 of calculations	•[2] $U_3=0\cdot 75(1531\cdot 25)=1148\cdot 24+200=1348\cdot 44$ $U_4=0\cdot 75(1348\cdot 44)=1011\cdot 33$
	•[3] answer	•[3] $1011\cdot 33$
b	**ans:** Colony is in danger. 600 prior to breeding week is less than 700 bats **4 marks**	
	•[1] knows to calculate limit + knows formula	•[1] $L=\frac{b}{1-a}$
	•[2] calculates limit correctly	•[2] $L=\frac{200}{1-0\cdot 75}=800$
	•[3] knows to subtract 200	•[3] low population 800 - 200 =600
	•[4] explanation	•[4] 600 prior to breeding week is less than 700 bats so colony in danger

	Give 1 mark for each •	Illustration(s) for awarding each mark
3a	**ans:** proof **4 marks**	
	•[1] area of rectangle	•[1] $A_{rec} = (2x+2)(x-4p)$ $= 2x^2 + 2x - 8px - 8p$
	•[2] area of triangle	•[2] $A_{tri} = \frac{1}{2}(x+6)\times 2x = x^2 + 6x$
	•[3] subtracting areas	•[3] $A_1 - A_2 =$ $= 2x^2 + 2x - 8px - 8p\ -(x^2+6x)$
	•[4] tidy up and common factor	•[4] $A_1 - A_2 = x^2 - (8p+4)x - 8p$
b	**ans:** $p = -\frac{1}{4}$ **6 marks**	
	•[1] equating to zero	•[1] $x^2 - (8p+4)x - 8p - 1 = 0$
	•[2] discriminant statement	•[2] $b^2 - 4ac = 0$ for equal roots
	•[3] a, b and c	•[3] $a = 1,\ b = -(8p+4)\ ,\ c = -8p - 1$
	•[4] substitution	•[4] $(8p+4)^2 - 4(-8p-1) = 0$
	•[5] to quadratic form	•[5] $64p^2 + 96p + 20 = 0$
	•[6] answer	•[6] $4(4p+5)(4p+1) = 0$ $p = -\frac{5}{4}\ \ or\ \ p = -\frac{1}{4}$
c	**ans:** $x = 1$ **2 marks**	
	•[1] substitution	•[1] $x^2 - (8(-\frac{1}{4})+4)x - 8(-\frac{1}{4}) - 1 = 0$
	•[2] solving to answer	•[2] $(x-1)(x-1) = 0\ ,\ x = 1$
4a	**ans:** A(2,6) **5 marks**	
	•[1] knowing and preparing to differentiate	•[1] $y = \frac{1}{2}x^2 + 8x^{-1}$
	•[2] differentiating	•[2] $\frac{dy}{dx} = x - 8x^{-2} = x - \frac{8}{x^2}$
	•[3] solving to zero	•[3] $x - \frac{8}{x^2} = 0$
	•[4] x coordinate	•[4] $x^3 - 8 = 0\ \ \therefore\ \ x = 2$
	•[5] y coordinate	•[5] $y = 6$

	Give 1 mark for each •	Illustration(s) for awarding each mark
4b	**ans:** B(4,10) **6 marks**	
	•1 know to form a system	•1 $y=\frac{x^2}{2}+\frac{8}{x}$; $2y=7x-8$
	•2 combining equations	•2 $7x-8=x^2+\frac{16}{x}$
	•3 manipulation to polynomial form	•3 $x^3-7x^2+8x+16=0$
	•4 sets up synthetic division	•4 $\begin{array}{c\|cccc} & 1 & -7 & 8 & 16 \\ \hline & & & & 0 \end{array}$
	•5 finds *x* coordinate	•5 $\begin{array}{c\|cccc} 4 & 1 & -7 & 8 & 16 \\ & & 4 & -12 & -16 \\ \hline & 1 & -3 & -4 & 0 \end{array}$ $x=4$
	•6 for *y* coordinate	•6 $2y=7(4)-8 \quad \therefore \; y=10$
5.	**ans:** $V=\frac{1}{9}p^{\frac{1}{2}}$ **5 marks**	
	•1 for knowing original form $V=kp^n$	•1 original data in form $V=kp^n$ (stated <u>or</u> implied)
	•2 calculating gradient	•2 $m=\frac{2}{4}=\frac{1}{2}$
	•3 for gradient is power	•3 power = gradient $\therefore \; n=\frac{1}{2}$
	•4 for finding *k*	•4 y-intercept, $\log_3 k=-2$, $3^{-2}=k=\frac{1}{9}$
	•5 for final equation	•5 $V=\frac{1}{9}p^{\frac{1}{2}}$
6.	**ans:** $7\frac{1}{3}$ cm^2 **6 marks**	
	•1 setting up integral	•1 $A=\int_2^4 x^2-6x+12 \; dx$
	•2 integrating	•2 $=\left[\frac{x^3}{3}-3x^2+12x\right]_2^4$
	•3 substituting limits	•3 $=(\frac{64}{3}-3(16)+12(4))-(\frac{8}{3}-12+24)$
	•4 area under curve	•4 $=6\frac{2}{3}$ cm^2
	•5 area of rectangle	•5 $A_{rec}=2\times 7=14$
	•6 subtraction to answer	•6 $A=14-6\frac{2}{3}=7\frac{1}{3}$ cm^2

	Give 1 mark for each •	Illustration(s) for awarding each mark
7a	**ans:** proof **3 marks**	
	•[1] strategy with displacements (or equiv.)	•[1] $\overrightarrow{OT} = \overrightarrow{OQ} + \overrightarrow{QT} = \overrightarrow{OQ} + \frac{1}{2}\overrightarrow{QP}$ (or equiv)
	•[2] calculating vector algebra	•[2] $\boldsymbol{t} = \boldsymbol{q} + \frac{1}{2}(\boldsymbol{p} - \boldsymbol{q})$
	•[3] simplify to required form	•[3] $\boldsymbol{t} = \frac{1}{2}(\boldsymbol{p} + \boldsymbol{q})$
b	**ans:** proof **5 marks**	
	•[1] setting up replacement in scalar product	•[1] $\boldsymbol{p}.\boldsymbol{t} = \boldsymbol{p} \,.\, \frac{1}{2}(\boldsymbol{p} + \boldsymbol{q})$
	•[2] simplifying scalar product	•[2] $\boldsymbol{p}.\boldsymbol{t} = \frac{1}{2}\,\boldsymbol{p}.\boldsymbol{p} + \frac{1}{2}\,\boldsymbol{p}.\boldsymbol{q}$
	•[3] evaluating scalar product $\boldsymbol{p}.\boldsymbol{q}$	•[3] $\boldsymbol{p}.\boldsymbol{q} = \lvert\boldsymbol{p}\rvert\lvert\boldsymbol{q}\rvert\cos 120^\circ = 3\times 6\times -\frac{1}{2} = -9$
	•[4] substituting in and knowing mag. squar. for $\boldsymbol{p}.\boldsymbol{p}$	•[4] $\boldsymbol{p}.\boldsymbol{t} = \frac{1}{2}\lvert\boldsymbol{p}\rvert^2 + \frac{1}{2}(-9)$
	•[5] answer of zero and subsequent statement	•[5] $\boldsymbol{p}.\boldsymbol{t} = \frac{1}{2}(9) + \frac{1}{2}(-9) = 0$ $\therefore$ angle TOP is 90° since scalar product is zero (or equivalent)

Total 60 marks