Practice Paper F
Marking Scheme - Paper I Section A

1.
$$\sqrt{(5-2)^2+(4-(-1))^2}$$
$$=\sqrt{3^2+5^2}$$
$$=\sqrt{34}$$

Answer: B

2.
$$f'(x)=4x+k$$
$$x=-2$$
$$-8+k=-11$$
$$k=-3$$

Answer: A

3.
$$\left[2x^3-x\right]_0^k=2k^2-k-0=15$$
$$2k^2-k-15=0$$
$$\Rightarrow (2k+5)(k-3)=0$$
$$k=3$$

Answer: D

4.
$$r^2=3^2+1^2-c=16$$
$$10-c=16$$
$$c=-6$$

Answer: B

5.
$$(x+5)^2-25+29$$
$$=(x+5)^2+4$$
Turning point $(-5, 4)$

Answer: C

6.
$$\frac{d}{dx}x^{\frac{4}{3}}$$
$$=\frac{4}{3}x^{\frac{1}{3}}=\frac{4}{3}\sqrt[3]{x}$$

Answer: D

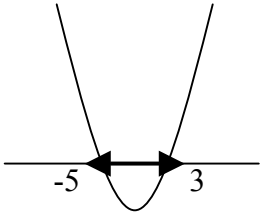
7.
$$b^2-4ac=16-4.k.-5=0$$
$$16+20k=0$$
$$20k=-16$$
$$k=-\frac{4}{5}$$

Answer: A

8.
$$2\cos^2 75°-1=\cos(2\times 75°)$$
$$=\cos 150°=-\cos 30°=-\frac{\sqrt{3}}{2}$$

Answer: B

9.
$$(x+5)(x-3)<0$$
$$-5<x<3$$



Answer: D

10.
$$\begin{pmatrix}-5\\10\\-3\end{pmatrix}-\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-5\\9\\3\end{pmatrix}$$
$$-5-x=-5;\ x=0$$
$$10-y=9;\quad y=1$$
$$-3-z=3;\quad z=-6$$
$$P(0, 1, -6)$$

Answer: C

11.
$$\boldsymbol{a}=\begin{pmatrix}1\\1\\2\end{pmatrix}\text{ and }\boldsymbol{b}=\begin{pmatrix}3\\-1\\0\end{pmatrix}$$
$$2\boldsymbol{a}-\boldsymbol{b}=\begin{pmatrix}2\\2\\4\end{pmatrix}-\begin{pmatrix}3\\-1\\0\end{pmatrix}=\begin{pmatrix}-1\\3\\4\end{pmatrix}$$

Answer: B

12.
$$4(5-2x)^3.-2$$
$$=-8(5-2x)^3$$

Answer: C

13.

$$\log_a 6 + \log_a x = \log_a 24$$
$$6x = 24$$
$$x = 4$$

Answer: D

14.

$$\frac{dy}{dx} = 2\cos 2x$$
$$x = \frac{\pi}{3}$$
$$\frac{dy}{dx} = 2\cos\frac{2\pi}{3}(120°)$$
$$= 2 \times -\frac{1}{2} = -1$$

Answer: D

15.

$$\log_4 8 \times 2 - 3$$
$$= 2 - 3 = -1$$

Answer: B

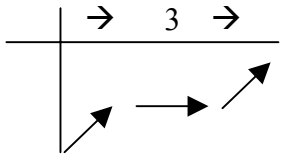
16.

$$\sqrt{x+2} > 0$$
$$x + 2 > 0$$
$$x > -2$$

Answer: C

17.

point of inflexion



Answer: C

18.

$$k = \sqrt{(\sqrt{3})^3 + 1^2} = \sqrt{4} = 2$$

$$\tan\alpha = \frac{\sqrt{3}}{-1}$$

S	A
T	C

Quadrant II

$$\tan^{-1}\sqrt{3} = 60°$$

Answer: B

19.

$$m = \tan\theta = \tan 120°$$
$$= -\tan 60°$$
$$= -\sqrt{3}$$

Answer: A

20.

$$L = \frac{-2}{1 - \frac{1}{7}} = \frac{-2}{\frac{6}{7}}$$
$$= -2 \times \frac{7}{6} = -\frac{14}{6} = -\frac{7}{3}$$

Answer: D

Practice Paper F - Paper 1 Section B **Marking Scheme**

	Give 1 mark for each •	Illustration(s) for awarding each mark
21a	**ans:** $y = 2x$ **4 marks**	
	•[1] For mid-point of AB	•[1] M(-2,-4)
	•[2] For gradient of AB	•[2] $m_{AB} = -\frac{1}{2}$
	•[3] For gradient of perpendicular	•[3] $m_1 \times m_2 = -1$, $m_{bis.} = 2$
	•[4] For equation of bisector	•[4] $y + 4 = 2(x + 2) \Rightarrow y = 2x$
b	**ans:** C(4,8) **2 marks**	
	•[1] For knowing to use x_A	•[1] $x_C = x_A = 4$
	•[2] For sub. in equation to answer	•[2] $y = 2(4) = 8 \ \therefore C(4,8)$
c	**ans:** $(x-4)^2 + (y-8)^2 = 225$ **2 marks**	
	•[1] For finding radius	•[1] CA is vertical = radius = 15 units
	•[2] For sub. in standard equ. to answer	•[2] C(4,8) , $r = 15$ in equ……. $(x-a)^2 + (y-b)^2 = r^2$
22a	**ans:** $k = \frac{1}{2}$, $c = 30$ **3 marks**	
	•[1] Setting up a system of equ.	•[1] $U_3 = kU_2 + c \Rightarrow 65 = 70k + c$ $U_4 = kU_3 + c \Rightarrow 62 \cdot 5 = 65k + c$
	•[2] Finding k	•[2] $5k = 2 \cdot 5 \Rightarrow k = \frac{1}{2}$
	•[3] Finding c	•[3] $c = 30$
b	**ans:** 60 **2 marks**	
	•[1] Knowing how to find limit	•[1] $L = \frac{b}{1-a}$, or equivalent
	•[2] Calculating limit	•[2] $L = 30 / (1 - \frac{1}{2}) = 60$
c	**ans:** 2% **2 marks**	
	•[1] For calculating U_5	•[1] $U_5 = \frac{1}{2}(62 \cdot 5) + 30 = 61 \cdot 25$
	•[2] For percentage calculation	•[2] $\frac{1 \cdot 25}{60} \times 100 \approx 2\%$

	Give 1 mark for each •	Illustration(s) for awarding each mark
23a	**ans:** proof **4 marks**	
	•[1] For differentiating first term	•[1] $-2\sin 2\theta$ .....
	•[2] For differentiating second term	•[2] …….. + $6\sin\theta\cos\theta$
	•[3] common factor (isolating double angle)	•[3] ……..+ $3(2\sin\theta\cos\theta)$
	•[4] for double angle + simplifying	•[4] $-2\sin\theta + 3(\sin 2\theta) = \sin 2\theta$
b	**ans:** $\frac{1}{2}$ **1 mark**	
	•[1] answer	•[1] $f'(\frac{\pi}{12}) = \sin(2\times\frac{\pi}{12}) = \sin\frac{\pi}{6} = \frac{1}{2}$
24a	**ans:** S(1,0,0) **2 marks**	
	•[1] For finding displacement $\vec{QR}$	•[1] $\vec{QR} = r - a = \begin{pmatrix}7\\4\\3\end{pmatrix} - \begin{pmatrix}10\\1\\1\end{pmatrix} = \begin{pmatrix}-3\\3\\2\end{pmatrix}$
	•[2] For establishing coordinates of S	•[2] $s = \begin{pmatrix}4\\-3\\-2\end{pmatrix} + \begin{pmatrix}-3\\3\\2\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$, or equiv.
b	**ans:** proof **3 marks**	
	•[1] For knowing $\vec{SR}.\vec{SP} = 0$, for R.A.	•[1] For right-angle $\vec{SR}.\vec{SP} = 0$
	•[2] For both displacements	•[2] $\vec{SR}.\vec{SP} = \begin{pmatrix}6\\4\\3\end{pmatrix}\begin{pmatrix}3\\-3\\-2\end{pmatrix} =$ .....
	•[3] For scalar product calculation to zero	•[3] $= 18-12-6 = 0$, $\therefore r-angled$
25a	**ans:** 3 **1 mark**	
	•[1] answer	•[1] $f(3) = 0$, $h(0) = 3$ (or equiv.)
b	**ans:** $f(h(x)) = 12x + 4x^2$ **2 marks**	
	•[1] For substitition	•[1] $f(h(x)) = (3+2x)^2 - 9$
	•[2] Simplifying to answer	•[2] $f(h(x)) = 12x + 4x^2$
c	**ans:** $x = -3$, $x = -1$ **2 marks**	
	•[1] For equating	•[1] $12x + 4x^2 = x^2 - 9$
	•[2] For solving to answers	•[2] $3x^2 + 12x + 9 = 0$ $3(x+3)(x+1) = 0$ … x = -3 or -1

Total 30 marks

Practice Paper F - Paper 2 **Marking Scheme**

	Give 1 mark for each •	Illustration(s) for awarding each mark
1a	**ans:** $y = 3x + 1$ **2 marks**	
	•[1] For using Q	•[1] Q(3,10) , $m = 3$
	•[2] For answer	•[2] $y - 10 = 3(x - 3)$
b	**ans:** $k = -2$ **1 mark**	
	•[1] For subst. to answer	•[1] $y = 3(-1) + 1 = -2 = k$
c	**ans:** $y = x - 1$ **3 marks**	
	•[1] For mid-point of QR	•[1] $M_{QR}(7,6)$
	•[2] For calculating gradient	•[2] $m_{med} = \frac{6-(-2)}{7-(-1)} = 1$
	•[3] answer	•[3] $y - 6 = 1(x - 7)$
d	**ans:** proof , isosceles **3 marks**	
	•[1] For knowing $m_1.m_2 = -1$	•[1] If perp… $m_{QR} \times m_{Pm} = -1$ (*stated <u>or</u> implied*)
	•[2] For calculation to prove	•[2] $-1 \times 1 = -1$
	•[3] For isosceles (no explanation required)	•[3] isosceles
2	**ans:** $f'(4) = \frac{1}{32}$ **5 marks**	
	•[1] For preparing to differentiate	•[1] $f(x) = x^{-2}(x - 2x^{\frac{1}{2}})$ $= x^{-1} - 2x^{-\frac{3}{2}}$
	•[2] Differentiating first term	•[2] $f'(x) = -x^{-2}$ .....
	•[3] Differentiating second term	•[3] $f'(x) =$ ..... $3x^{-\frac{5}{2}}$
	•[4] Subst. x = 4 in derivative	•[4] $f'(x) = -\frac{1}{4^2} + \frac{3}{4^{\frac{5}{2}}}$
	•[5] Calculating answer	•[5] $f'(4) = -\frac{2}{32} + \frac{3}{32} = \frac{1}{32}$

	Give 1 mark for each •	Illustration(s) for awarding each mark
3a	**ans:** proof **3 marks**	
	•[1] For sub 2 in f and h	•[1] $f(2) = 4a - 2b$, $h(2) = \dfrac{4-6b}{3}$
	•[2] For equating	•[2] $4a - 2b = \dfrac{4-6b}{3}$
	•[3] For solving to required answer	•[3] $3(4a-2b) = 4-6b$ $12a - 6b = 4 - 6b$ .....$a = \frac{1}{3}$
b	**ans:** proof **1 marks**	
	•[1] For sub. for a and b and adjusting to required answer	•[1] $f(x) = \frac{1}{3}x^2 - 2(px - 6)$ $f(x) = \frac{1}{3}x^2 - 2px + 12$
c	**ans:** $-2 < p < 2$ **4 marks**	
	•[1] For discr. statement (or implied)	•[1] for no real roots $b^2 - 4ac < 0$
	•[2] For values of a, b and c	•[2] $a = \frac{1}{3}$, $b = -2p$, $c = 12$
	•[3] For subst. and factorising	•[3] $(-2p)^2 - (4.\frac{1}{3}.12) < 0$ $4(p-2)(p+2) < 0$
	•[4] For final statement (worded ans. o.k.)	•[4] p has to lie between -2 and 2
4.	**ans:** $90{\cdot}0^o$ **7 marks**	
	•[1] For re-arranging and realising … = 1	•[1] $\sin x + \sqrt{8}\cos x = 1$
	•[2] For expansion & equating coefficients	•[2] $= k\cos x\cos\alpha + k\sin x\sin\alpha$ $k\cos\alpha = \sqrt{8}$, $k\sin\alpha = 1$
	•[3] Tan ratio	•[3] $\tan\alpha = \dfrac{1}{\sqrt{8}}$
	•[4] For α	•[4] $\alpha = 19{\cdot}5^\circ$
	•[5] For k	•[5] $k = \sqrt{9} = 3$
	•[6] For solving to one-third	•[6] $\cos(x - 19{\cdot}5) = \frac{1}{3}$
	•[7] For angle	•[7] $x - 19{\cdot}5 = 70{\cdot}5$ ∴ $x = 90{\cdot}0^o$
5a	**ans:** $a = 2$, $b = 14$ **3 marks**	
	•[1] For sub. 4 for y in order to solve	•[1] $4 = \frac{1}{18}(x^2 - 16x + 100)$
	•[2] manipulating equation to zero	•[2] $x^2 - 16x + 28 = 0$
	•[3] factorising and answers	•[3] $(x-2)(x-14) = 0 \Rightarrow x = 2$, $x = 14$

	Give 1 mark for each •	Illustration(s) for awarding each mark
5b	**ans:** proof **2 marks**	
	•[1] Strategy of line minus curve	•[1] $A = \int 4 - \frac{1}{18}(x^2 - 16x - 100)\ dx$
	•[2] Constant out + tidy to required answer	•[2] $A = \frac{1}{18}\int 72 - (x^2 - 16x + 100)\ dx$ $A = \frac{1}{18}\int_2^{14} (16x - x^2 - 28)\ dx$
c	**ans:** 16 m^2 **4 marks**	
	•[1] For integrating (all 3 terms) •[2] For substituting	•[1] $A = \frac{1}{18}\left[8x^2 - \frac{x^3}{3} - 28x\right]_2^{14}$ •[2] $A = \frac{1}{18}\left[(8(14^2) - \frac{14^3}{3} - 28(14)) - (8(2^2) - \frac{2^3}{3} - 28(2))\right]$
	•[3] For simplifying each part •[4] For calculating answer	•[3] $A = \frac{1}{18}[1200 - 912]$ •[4] $A = \frac{1}{18}[288] = 16$
d	**ans:** 768 m^3 **2 marks**	
	•[1] For total surface area •[2] For volume	•[1] $A = 16 + (4 \times 12) = 64\ m^2$ •[2] $V = 64 \times 12 = 768\ m^3$
6.	**ans:** 35^o **5 marks**	
	•[1] consruct appropriate vectors	•[1] $V_1 = \begin{pmatrix} \sqrt{2} \\ 3 \\ -\sqrt{5} \end{pmatrix}$, $V_2 = \begin{pmatrix} \sqrt{3} \\ \sqrt{6} \\ 0 \end{pmatrix}$
	•[2] strategy of $\cos\theta = ...$	•[2] $\cos\theta = ...$ (formula may only appear when numbers are subst.)
	•[3] calculate scalar product	•[3] $V_1.V_2 = \sqrt{6} + 3\sqrt{6} + 0 = 4\sqrt{6}$
	•[4] process denominator (magnitudes)	•[4] $\lvert V_1\rvert \times \lvert V_2\rvert = \sqrt{16} \times \sqrt{9} = 12$
	•[5] calculate angle	•[5] $\cos\theta = \frac{4\sqrt{6}}{12}$ $\therefore$ $\theta = 35 \cdot 3^o \approx 35^o$

	Give 1 mark for each •	Illustration(s) for awarding each mark
7a	**ans:** Yes , $104\cdot08 > 100$ **3 marks**	
	•[1] For taking $a = 0\cdot97$	•[1] $a = 0\cdot97$
	•[2] For calculation	•[2] $U_{12} = (0\cdot97)^{12} \times 150 = 104\cdot08$
	•[3] For consistent answer	•[3] Yes , since $U_{12} > 100$
b	**ans:** Plan o.k., over the long-term between $113\cdot3$ *and* $163\cdot3$ mgs **4 marks**	
	•[1] For attempting lines of working	•[1] $U_1 \rightarrow U_5$ below …… 154·08, 156·91, 158·87, 160·23, 161·17
	•[2] For finding the limit	•[2] $L = \dfrac{50}{1-(0\cdot97)^{12}} = 163\cdot31$
	•[3] For being aware of the lower limit as well as the upper limit	•[3] lower limit = $163\cdot31 - 50 = 113\cdot31$
	•[4] Consistent comment on findings	•[4] Over the long-term the amount present would always be between 113·31 and 163·31 which is ideal.
8a	**ans:** C(4,2) , $r = 5$ **2 marks**	
	•[1] For centre	•[1] C(4,2)
	•[2] For radius	•[2] $r = \sqrt{4^2 + 2^2 - (-5)} = \sqrt{25} = 5$
b	**ans:** $4y + 3x = 45$ (or equiv.) **3 marks**	
	•[1] For gradient of radius	•[1] $m_{CT} = \dfrac{6-2}{7-4} = \dfrac{4}{3}$
	•[2] For gradient of tangent	•[2] $m_{\tan} = -\frac{3}{4}$
	•[3] For sub. using m_{tan} & T(7,6)	•[3] $y - 6 = -\frac{3}{4}(x-7)$
c	**ans:** S(-1,2) , P(-1,12) **3 marks**	
	•[1] For x_c - 5 = x_s = -1 , then coord. of S	•[1] $4 - 5 = -1 \therefore S(-1,2)$ … same y as C
	•[2] For knowing to sub. x = -1 into equat.	•[2] x-coordinate of P is same as S. Also on line $4y + 3x = 45$. $4y + 3(-1) = 45$
	•[3] For coordinates of P	•[3] $y = 12 \ \therefore \ P(-1,12)$

Total 60 marks