NATIONAL
QUALIFICATIONS

Mathematics

Practice Paper F

Paper 2

Assessing Units 1, 2 & 3

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $a \cdot b = |a||b|\cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

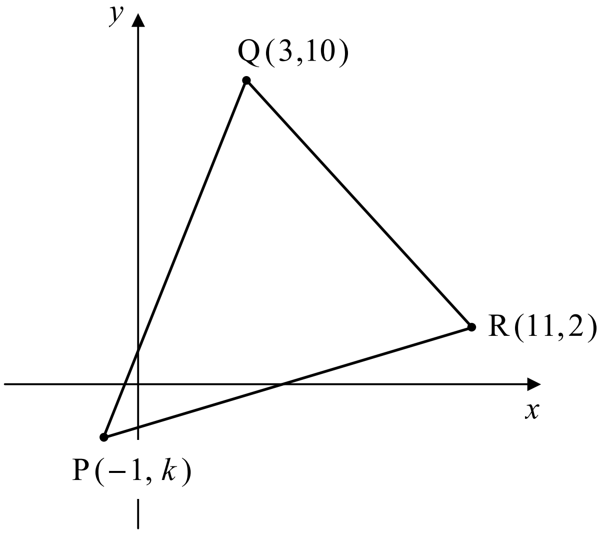
$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A \\
\sin 2A &= 2\sin A \cos A
\end{aligned}$$

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

All questions should be attempted

1. Triangle PQR has vertices $P(-1, k)$, $Q(3,10)$ and $R(11,2)$ as shown.



(a) Given that the gradient of side PQ is 3, find the equation of PQ. **2**

(b) Hence find *k*, the *y*-coordinate of vertex P. **1**

(c) Find the equation of the median from P to QR. **3**

(d) Show that this median is at right-angles to side QR.
What type of triangle is PQR? **3**

2. Evaluate $f'(4)$ when $f(x) = \dfrac{x - 2\sqrt{x}}{x^2}$. **5**

3. Two functions are defined as $f(x) = ax^2 - 2b$ and $h(x) = \dfrac{2x - 6b}{3}$,
where a is a constant .

(a) Given that $f(2) = h(2)$, show clearly that $a = \frac{1}{3}$. **3**

(b) If $b = px - 6$, show that $f(x) = \frac{1}{3}x^2 - 2px + 12$. **1**

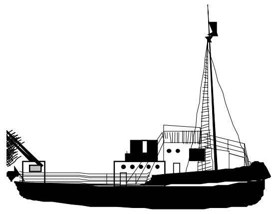
(c) Hence state the values of p for which $f(x) = 0$ has no real roots. **4**

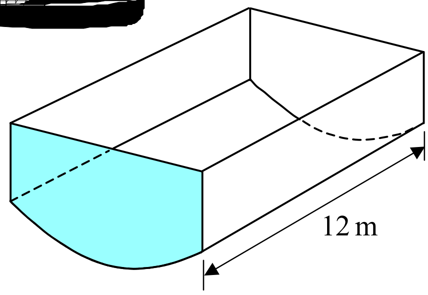
4. Solve algebraically the equation

$$3\sin x^o = 1 - \sqrt{7}\cos x^o \quad , \quad \text{where} \quad 0 \le x < 180 .$$ **7**

5. A fishing boat's fish hold is in the shape of the prism shown opposite.

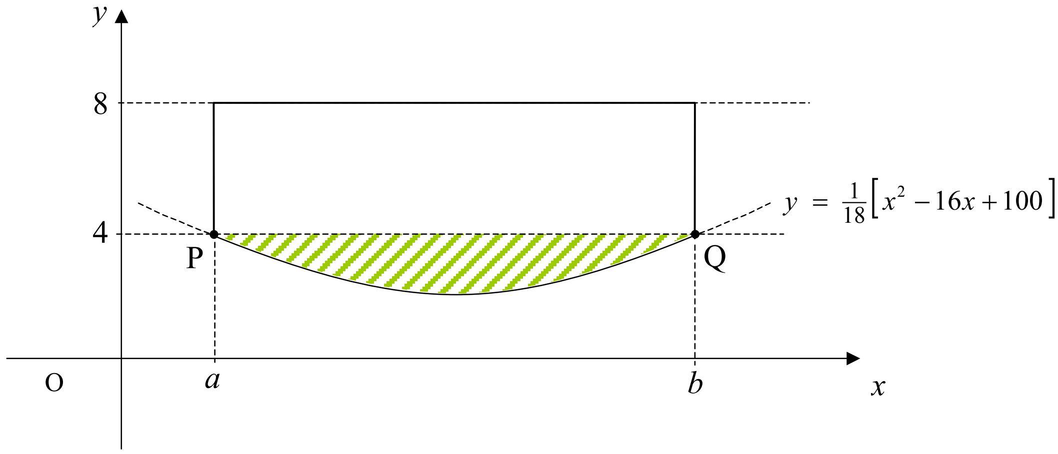
The length of the hold is 12 metres.





The cross-section of the hold is represented in the coordinate diagram below.

All the units are in metres, with the floor of the hold represented by the curve $y = \frac{1}{18}\left[x^2 - 16x + 100\right]$.



(a) Find the values of a and b, the x-coordinates of P and Q. **3**

(b) Show clearly that the area between the line PQ and the curve $y = \frac{1}{18}\left[x^2 - 16x + 100\right]$

can be calculated by evaluating the integral: $A = \frac{1}{18}\int_a^b (16x - x^2 - 28)\,dx$. **2**

(c) Calculate this area in square metres. **4**

(d) Hence calculate the **volume** of the hold, in cubic metres, by first establishing the **total** cross-sectional area of the hold. **2**

6. Two vectors are defined as $V_1 = \sqrt{2}\boldsymbol{i} + 3\boldsymbol{j} - \sqrt{5}\boldsymbol{k}$ and $V_2 = \sqrt{3}\boldsymbol{i} + \sqrt{6}\boldsymbol{j}$.

Calculate the angle between these two vectors to the nearest degree. **5**

7. Certain radioisotopes are used as *tracers*, to track down diseased tissue within the body, and then be absorbed, to act as a long-term radio-therapy treatment. Their passage through the body and mass is ascertained by means of a Geiger-Müller counter.

During trials of a particular radioisotope the following information was obtained.

- *the isotope loses* 3% *of its mass every hour*
- *the maximum recommended mass in the bloodstream is* 165mgs
- 100mgs *is the smallest mass detectable by the Geiger-Müller counter*

(a) An intial dose of 150mgs of the isotope is injected into a patient.

Would the mass remaining after 12 hours still be detectable by the Geiger-Müller counter? **3**

Your answer must be accompanied by appropriate working.

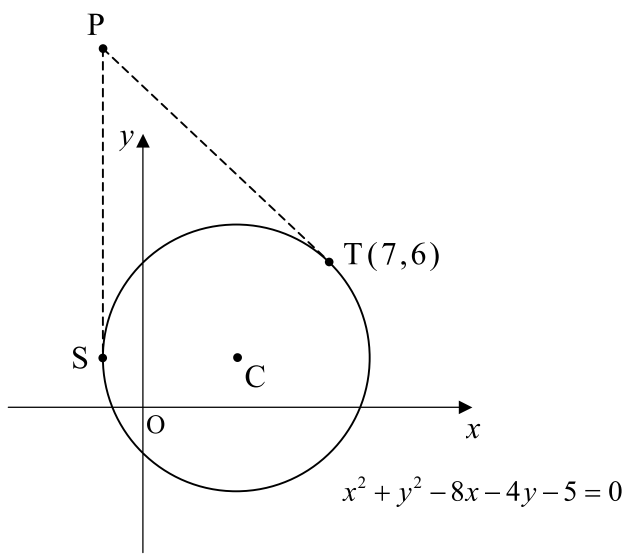
(b) **After the initial dose**, top-up injections of 50mgs are given every 12 hours.

Comment on the long-term suitabilty of this plan. **4**

Your answer must be accompanied by appropriate working.

8. The diagram shows a circle, centre C, with equation $x^2 + y^2 - 8x - 4y - 5 = 0$.

Two common tangents have been drawn from the point P to the points S and T$(7,6)$ on the circle.



(a) Find the centre and radius of the circle. **2**

(b) Hence find the equation of the tangent PT. **3**

(c) Given now that the tangent PS is parallel to the *y*-axis, determine the coordinates of S and P. **3**

[END OF QUESTION PAPER]