Practice Paper E
Marking Scheme - Paper I Section A

1.

$$\begin{array}{c|cccc} 4 & 2 & 15 & -104 & 48 \\ & & 8 & 92 & -48 \\ \hline & 2 & 23 & -12 & 0 \end{array}$$

$(x-4)(2x^2+23x-12)=0$

$(x-4)(2x-1)(x+12)=0$

$x=4,\ \frac{1}{2},\ -12$

Answer: D

2.

$-\sin 5x \times 5$

$-5\sin 5x$

Answer: B

3.

$\frac{dy}{dx}=6x+2$

$x=-3$

$\frac{dy}{dx}=6(-3)+2=-16$

Answer: A

4.

$$\overrightarrow{AB}=\begin{pmatrix}3\\4\\1\end{pmatrix}-\begin{pmatrix}1\\5\\-7\end{pmatrix}=\begin{pmatrix}2\\-1\\8\end{pmatrix}$$

$$\overrightarrow{BC}=\begin{pmatrix}7\\2\\z\end{pmatrix}-\begin{pmatrix}3\\4\\1\end{pmatrix}=\begin{pmatrix}4\\-2\\z-1\end{pmatrix}$$

$z-1=16$

$z=17$

Answer: B

5.

$g(x-1)=(3x-1)^2+4$

$=9x^2-6x+1+4$

$=9x^2-6x+5$

Answer: A

6.

$5y=-3x+1$

$y=-\frac{3}{5}x+\frac{1}{5}$

$m=-\frac{3}{5}$

$\tan\theta=m$

$\tan\theta=-\frac{3}{5}=-0\cdot 6$

Answer: B

7. graph has to be moved 3 right and 2 up

(4, 5) ……..(7, 7)

Answer C

8.

$f(x)=2x^2-3x-x^{-1}$

$f'(x)=4x-3+x^{-2}$

$f'(x)=4x-3+\frac{1}{x^2}$

$x=-1$

$f'(-1)=4(-1)-3+\frac{1}{(-1)^2}$

$=-4-3+1=-6$

Answer: A

9.

$\sin(\frac{2\pi}{3})(120°)=\sin\frac{\pi}{3}(60°)$

$=\frac{\sqrt{3}}{2}$

Answer: C

10.

$$y = -2x + 5$$
$$y = 4 - x^2$$
$$-2x + 5 = 4 - x^2$$
$$x^2 - 2x + 1 = 0$$
$$(x-1)^2 = 0$$
$$x = 1; \quad y = -2(1) + 5 = 3$$

Answer: D

11.

$$x^2 - 3x + k = 0$$
$$b^2 - ac \geq 0 \text{ for real roots}$$
$$9 - 4.1.k \geq 0$$
$$-4k \geq -9$$
$$k \leq \frac{9}{4}$$

Answer: A

12.

-2	2	0	4	3
		-4	8	-24
	2	-4	12	-21

Answer: C

13.

$$\left[\frac{x^{-2}}{-2}\right]_{-1}^{2} = \left[-\frac{1}{2x^2}\right]_{-1}^{2}$$
$$= (-\frac{1}{8}) - (-\frac{1}{2}) = \frac{3}{8}$$

Answer: B

14.

$$U_1 = 5a + b = 10$$
$$5a = 10 - b$$
$$a = 5 - \frac{1}{5}b$$

Answer: D

15.

$$\boldsymbol{a} \,.\, \boldsymbol{b} = |a||b|\cos 60°$$
$$= 2 \times \sqrt{3} \times -\frac{1}{2} = -\sqrt{3}$$

Answer: C

16.

$$\frac{dy}{dx} = -\sin x$$
$$x = 210°$$
$$= -\sin 210°$$
$$= -(-\sin 30°)$$
$$= \frac{1}{2}$$

Answer: D

17.

$$\log 2^4 - \log 3^3$$
$$= \log 16 - \log 27$$
$$= \log \frac{16}{27}$$

Answer: A

18.

$$-4\cos 3x \times \frac{1}{3} + C$$
$$-\frac{4}{3}\cos 3x + C$$

Answer: C

19.

$$k = \sqrt{5^2 + (-1)^2} = \sqrt{26}$$
$$\tan\alpha = \frac{5}{-1} = -5$$

Answer: C

20. Centre $(2, -1)$, $(-3, 1)$

$$m = \frac{-1-1}{2+3} = -\frac{2}{5}$$
$$m_{\tan} = \frac{5}{2}$$

Answer: C

Practice Paper E - Paper 1 Section B **Marking Scheme**

	Give 1 mark for each •	Illustration(s) for awarding each mark
21a	**ans: statement** **1 mark**	
	•[1] Explanation	•[1] Because $-1 < a < 1$ (or equiv.)
b	**ans: 20** **2 marks**	
	•[1] Method used	•[1] $L = \dfrac{b}{1-a}$
	•[2] For calculating limit	•[2] $L = \dfrac{8}{1-0\cdot 6} = \dfrac{8}{0\cdot 4} = \dfrac{80}{4} = 20$
c	**ans: $U_0 = 15$** **3 marks**	
	•[1] For initial equating and finding U_1	•[1] $20 - U_1 = 3 \;\; \therefore \;\; U_1 = 17$
	•[2] For recurrence with U_1 in place	•[2] $17 = 0\cdot 6U_0 + 8$
	•[3] For answer	•[3] $9 = 0\cdot 6U_0 \;\Rightarrow\; U_0 = \dfrac{9}{0\cdot 6} = \dfrac{90}{6} \;\; U_0 = 15$
22.	**ans: p = -3 , q = 8 : x = 4** **5 marks**	
	•[1] Setting up synth. division	•[1] 1 | 1 p -6 q
	•[2] Obtaining first equation	•[2] $p + q = 5$
	•[3] Obtaining second equation	•[3] $4p + q = -4$
	•[4] Solving system for p and q	•[4] $p = -3$, $q = 8$
	•[5] Sub. (say p in quotient) for 3rd root	•[5] $x^2 - 2x - 8 = 0 \;\Rightarrow\; (x+2)(x-4) = 0$ $x = 4$ is missing root
23	**ans: proof** **5 marks**	
	•[1] For diff. power in first term	•[1] .... $(4\cos\theta)$ ....
	•[2] For diff. $\cos\theta$ in first term	•[2] $-\sin\theta$ i.e. $-4\cos\theta\sin\theta$
	•[3] For differentiating second term	•[3] .... $(-4\sin 2\theta)$
	•[4] For extracting $2\sin\theta\cos\theta$ for replace.	•[4] $-2(2\sin\theta\cos\theta) - (-4\sin 2\theta)$
	•[5] Simplifying to given answer	•[5] $-2\sin 2\theta + 4\sin 2\theta = 2\sin 2\theta$

	Give 1 mark for each •	Illustration(s) for awarding each mark
24a	**ans: C(2,5) 1 mark**	
	$\bullet^1$ For answer	$\bullet^1$ C(2,5)
b	**ans: proof 3 marks**	
	$\bullet^1$ For gradient of CM	$\bullet^1$ $M_{cm} = \frac{5-3}{2-1} = 2$
	$\bullet^2$ For perp. gradient of chord AB	$\bullet^2$ $M_{AB} = -\frac{1}{2}$
	$\bullet^3$ For equ. of chord and rearranging	$\bullet^3$ $y-3 = -\frac{1}{2}(x-1)$, $2y-6 = -x+1$, $x = 7-2y$
c	**ans: A(-1,4) , B(3,2) 4 marks**	
	$\bullet^1$ For attempting to solve a system	$\bullet^1$ $(7-2y)^2 + y^2 - 4(7-2y) - 10y + 19 = 0$
	$\bullet^2$ For expanding and simplifying	$\bullet^2$ $5y^2 - 30y + 40 = 0$
	$\bullet^3$ For factorising and finding *y* coords.	$\bullet^3$ $(5(y-4)(y-2) - 0 \quad \therefore \ y = 4 \ , \ y = 2$
	$\bullet^4$ For completing points	$\bullet^4$ $y = 4$ *then* $x = -1$, $y = 2$ *when* $x = 3$
25a	**ans: proof 4 marks**	
	$\bullet^1$ For logs to same side	$\bullet^1$ $2\log_x y - \log_x 2y = 2$
	$\bullet^2$ For moving the power	$\bullet^2$ $\log_x y^2 - \log_x 2y = 2$
	$\bullet^3$ For combining logs	$\bullet^3$ $\log_x \frac{y^2}{2y} = 2$
	$\bullet^4$ For changing to index form	$\bullet^4$ $x^2 = \frac{1}{2}y \ \Rightarrow \ y = 2x^2$
b	**ans: $y = 8$ 2 marks**	
	$\bullet^1$ For substitution	$\bullet^1$ $y = 2(\frac{1}{4}y)^2$
	$\bullet^2$ Manipulation and answer	$\bullet^2$ $y = 8$

Total 30 marks

Practice Paper E - Paper 2 **Marking Scheme**

	Give 1 mark for each •	Illustration(s) for awarding each mark
1a	**ans:** $y = x - 2$, $y = -x + 4$ **4 marks**	
	•[1] For gradient of AC	•[1] $m_{AC} = \frac{3+8}{5+6} = 1$
	•[2] For equation of AC	•[2] $y - 3 = 1(x - 5)$
	•[3] For finding gradient of altitude	•[3] $m_{alt} = -1$
	•[4] For equation of altitude	•[4] $y - 7 = -1(x + 3)$.
b	**ans:** P(3,1) **3 marks**	
	•[1] Knowing to solve a system	•[1] $x - 2 = -x + 4$
	•[2] Finding first coordinate	•[2] $2x = 6 \Rightarrow x = 3$
	•[3] Finding second coord.	•[3] $y = 3 - 2 = 1$
c	**ans:** D(6,-2) **1 mark**	
	•[1] Answer	•[1] $\rightarrow 6 \downarrow 6 \therefore \rightarrow 3 \downarrow 3$ from P, D(6,-2)
d	**ans:** 18° **3 marks**	
	•[1] For knowing and using $\tan\theta = m$	•[1] $m_{AC} = 1 \therefore \tan^{-1} 1 = \theta = 45^\circ$
	•[2] For angle between CD and horoz.	•[2] $m_{CD} = \frac{-2+8}{6+6} = 0{\cdot}5 \therefore \tan^{-1} 0{\cdot}5 = 26{\cdot}6^\circ$
	•[3] For 45^o and subtraction to ans.	•[3] $45 - 26{\cdot}6 = 18^\circ$
2a	**ans:** proof **2 marks**	
	•[1] For knowing to involve trig ratios	•[1] $\sin x = \frac{o}{d} \Rightarrow$ ... etc
	•[2] For manipulation	•[2] $QR = d\sin x$ *and* $PQ = d\cos x$
b	**ans:** proof **7 marks**	
	•[1] Writing down an expression for P	•[1] $P = d + d\cos x + d\sin x$
	•[2] Know to involve $k\cos(x - \alpha)$	•[2] $P = d + d(\cos x + \sin x)$
	•[3] Select expansion and expand	•[3] $\cos x + \sin x = k\cos(x - \alpha) =$ $= k\cos x\cos\alpha + k\sin x\sin\alpha$
	•[4] Compare coefficients	•[4] $k\sin\alpha = 1$, $k\cos\alpha = 1$
	•[5] Find k	•[5] $k = \sqrt{1^2 + 1^2} = \sqrt{2}$
	•[6] Interpret comparison and find angle	•[6] $\tan\alpha = \frac{1}{1} = 1 \quad \therefore \quad \alpha = \pi/4$
	•[7] Manipulation to answer	•[7] $P = d + d[\sqrt{2}\cos(x - \frac{\pi}{4})]$

	Give 1 mark for each •	Illustration(s) for awarding each mark
3a	**ans:** $C_1(-8,-6)$, $C_2(16,4)$ **2 marks**	
	•[1] For first centre	•[1] $C_1(-8,-6)$
	•[2] For second centre	•[2] $C_2(16,4)$
b	**ans:** $r_1 = 10$, $r_2 = 10$, $d = 26$ **4 marks**	
	•[1] Finding r of C_1	•[1] $r = \sqrt{(-8)^2 + (-6)^2 - 0} = \sqrt{100} = 10$
	•[2] Finding r of C_2	•[2] $r = \sqrt{100} = 10$
	•[3] For method (dist. form , pyth, etc.)	•[3] $d = \sqrt{(x_2 - x_1)^2}$ ..... etc
	•[4] For correct distance	•[4] $d = \sqrt{676} = 26$
c	**ans:** $(x-4)^2 + (y+1)^2 = 9$ **3 marks**	
	•[1] For centre	•[1] Centre must be mid-pt $C_3(4,-1)$
	•[2] For radius	•[2] $r = (26 - 20) \div 2 = 3$
	•[3] For sub. into equ. to answer	•[3] $(x-4)^2 + (y+1)^2 = 9$
4.	**ans:** $\{19 \cdot 5 , 160 \cdot 5 , 210 , 330\}$ **5 marks**	
	•[1] For correct substitution	•[1] $\sin x - 3(1 - 2\sin^2 x) + 2 = 0$
	•[2] For re-arranging to quadratic	•[2] $6\sin^2 x + \sin x - 1 = 0$
	•[3] Factorising to two roots	•[3] $\sin x = \frac{1}{3}$ *or* $\sin x = -\frac{1}{2}$
	•[4] Two ans. from one root	•[4] $19 \cdot 5^o$, $160 \cdot 5^o$
	•[5] Two ans. from second root	•[5] 210^o , 330^o
5a	**ans:** proof **3 marks**	
	•[1] For expanding original functions	•[1] $f(x) = x^2 + 3x + 2$, $g(x) = x^2 - 2x$
	•[2] For correct substitution	•[2] $h(x) = (x^2 - 2x)^2 + 3(x^2 - 2x) + 2$
	•[3] For expanding to answer	•[3] $h(x) = x^4 - 4x^3 + 4x^2 + 3x^2 - 6x + 2$
b	**ans:** x = 1 , proof **4 marks**	
	•[1] Knowing to use synthetic division	•[1] 1 | -4 7 -6 2
	•[2] Finding the root , $x = 1$	•[2] evidence
	•[3] Using x = 1 again !	•[3] 1 again leaves quot. $x^2 - 2x + 2$
	•[4] Showing remaining quotient has no roots	•[4] for $b^2 - 4ac = -4$ $\therefore$ no more roots

	Give 1 mark for each •	Illustration(s) for awarding each mark
6a	**ans:** P(1,-2) , R(3,0) **6 marks**	
	•[1] Preparing to differentiate	•[1] $f(x) = x^{\frac{3}{2}} - 3x^{\frac{1}{2}}$
	•[2] Knowing to solve deriv. to zero	•[2] S.P. when $f'(x) = 0$ (stated or impl.)
	•[3] Differentiating	•[3] $f'(x) = \frac{3}{2}x^{\frac{1}{2}} - \frac{3}{2}x^{-\frac{1}{2}}$ (or equivalent)
	•[4] Solving to answer for x coord. of P	•[4] $\frac{3\sqrt{x}}{2} - \frac{3}{2\sqrt{x}} = 0 \quad (\times 2\sqrt{x})$ $3x - 3 = 0 \quad \therefore \ x = 1$
	•[5] Finding y coord. of P	•[5] $y = \sqrt{1}(1-3) = -2$
	•[6] Finding root (coords. of R)	•[6] $x - 3 = 0 \ , \ x = 3$
b	**ans:** Area = $2 \cdot 55$ units^2 **5 marks**	
	•[1] For setting up correct integral	•[1] $A = -\int x^{\frac{3}{2}} - 3x^{\frac{1}{2}} \ dx$
	•[2] For integrating first term	•[2] ..... $\frac{2}{5}x^{\frac{5}{2}}$ (or equivalent)
	•[3] Integrating 2nd term	•[3] ..... $2x^{\frac{3}{2}}$ (or equivalent)
	•[4] Substituting limits	•[4] $A = -\left[\frac{2}{5}(3^{\frac{5}{2}}) - 2(3^{\frac{3}{2}})\right] - \left[\frac{2}{5} - 2\right]$
	•[5] Calculations to answer	•[5] $A = -[6 \cdot 24 - 10 \cdot 39] - [-1 \cdot 6] = 2 \cdot 56$
7a	**ans:** proof **4 marks**	
	•[1] For attempting to use pythagoras	•[1] $OP^2 = a^2 + b^2$ (stated or implied)
	•[2] For length x	•[2] $OP^2 = x^2 + ...$
	•[3] For length (4-x)	•[3] $OP^2 = ... + (4-x)^2$
	•[4] For expansion to answer	•[4] $OP^2 = x^2 + 16 - 8x + x^2$ $= 2x^2 - 8x + 16$
b	**ans:** $x = 2$, $OP_{min} = \sqrt{8}$ **4 marks**	
	•[1] For removing common factor	•[1] $2(x^2 - 4x) + 16$
	•[2] Completing the square with $x^2 - 4x$	•[2] $[(x-2)^2 - 4]$
	•[3] Tidying to final form	•[3] $OP^2 = 2(x-2)^2 + 8$
	•[4] Answer for replacement and minimum	•[4] minimum when x = 2 minimum value of $OP^2 = 8$ $\therefore OP_{min} = \sqrt{8}$

Total 60 marks