NATIONAL QUALIFICATIONS

Mathematics
Practice Paper E
Paper 2
Assessing Units 1, 2 & 3

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g\ ,-f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a\ ,b)$ and radius r.

Scalar Product: $a \cdot b = |a||b|\cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

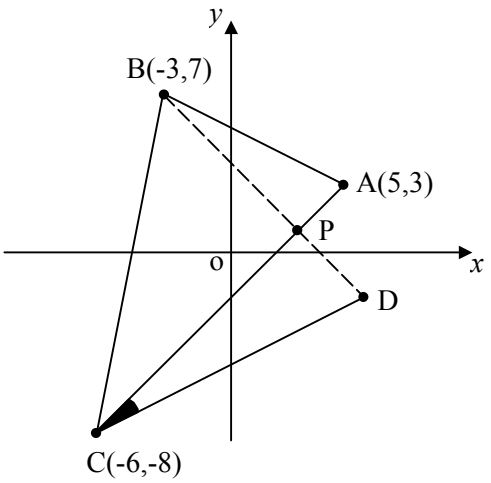
$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$
$$\sin 2A = 2\sin A\cos A$$

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

All questions should be attempted

1. Triangle ABC has vertices A(5,3), B(-3,7) and C(-6,-8) as shown.
The altitude through B meets AC at P.



(a) Find the equation of side AC and the equation of the altitude BP. **4**

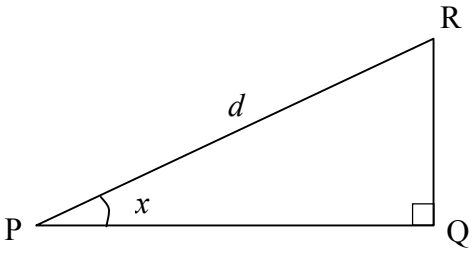
(b) Hence find the coordinates of P. **3**

(c) BP is produced in such a way that PD = $\frac{1}{2}$ BP. Establish the coordinates of D. **1**

(d) By considering gradients, calculate the size of angle DCP to the nearest degree. **3**

2. Triangle PQR is right-angled at Q.

The hypotenuse is d units long and angle QPR = x radians.

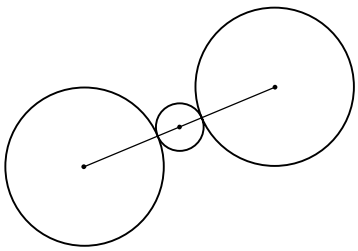


(a) Explain why PQ = $d\cos x$ and QR = $d\sin x$ units long. **2**

(b) Hence show that the perimeter (P) of the triangle can be expressed in the form

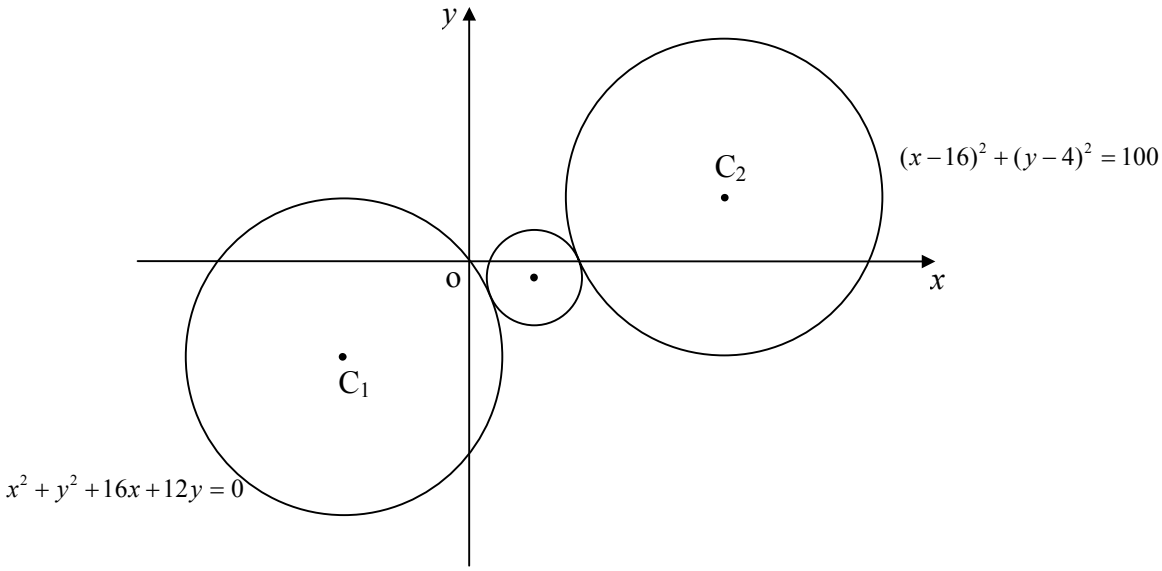
$$P = d + \sqrt{2}\,d\cos\left(x - \tfrac{\pi}{4}\right)$$ **7**

3. Three wheels are positioned in such a way that their centres are collinear.



When placed on a set of rectangular axes the equations of the two larger circles are

$x^2 + y^2 + 16x + 12y = 0$ and $(x-16)^2 + (y-4)^2 = 100$, as shown.



(a) Write down the coordinates of the two centres C_1 and C_2. **2**

(b) Calculate the radii of the two larger circles and the distance between the two centres C_1 and C_2. **4**

(c) Hence establish the centre and radius of the small circle and write down its equation. **3**

4. Solve algebraically the equation

$$\sin x^\circ - 3\cos 2x^\circ + 2 = 0, \qquad 0 \le x < 360.$$ **5**

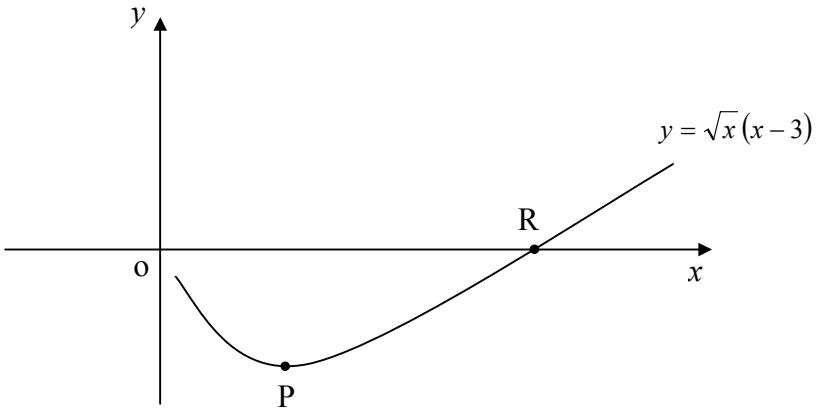
5. Two functions are defined as $f(x) = (x+2)(x+1)$ and $g(x) = x(x-2)$.

(a) Given that $h(x) = f\big(g(x)\big)$, show clearly that $h(x) = x^4 - 4x^3 + 7x^2 - 6x + 2$. **3**

(b) Hence solve the equation $h(x) = 0$ showing that it has, in fact, only one real root. **4**

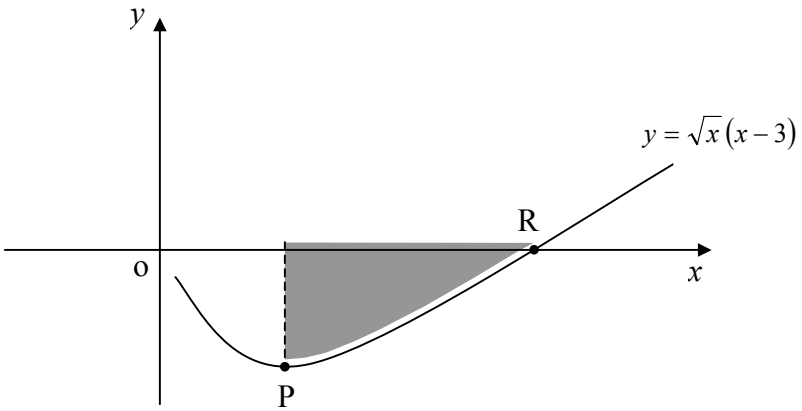
6. A function is defined as $f(x) = \sqrt{x}(x-3)$, where only the positive value of $\sqrt{x}$ is taken for each value of $x > 0$.

Part of the graph of $y = f(x)$ is shown below.



(a) Find the coordinates of the turning point at P and the root at R. **6**

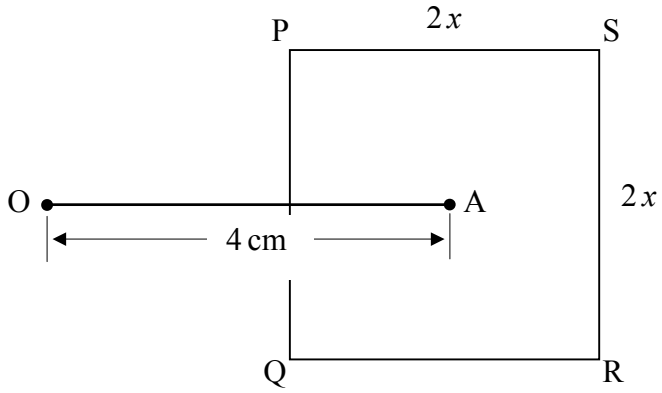
(b) Hence calculate the shaded area below giving your answer **correct to 2 decimal places**.



5

7. In the diagram below PQRS is a square of side $2x$ cm.

A straight line OA, measuring 4 cm, has been drawn in such a way that A lies at the centre of the square and OA is parallel to PS.



(a) Show that $OP^2 = 2x^2 - 8x + 16$. **4**

(b) Hence, by completing the square, or otherwise, find x for which the length of OP is at a minimum and state the minimum length of OP. **4**

[END OF QUESTION PAPER]