NATIONAL QUALIFICATIONS

Mathematics
Practice Paper E
Paper 1
Assessing Units 1, 2 & 3

Time allowed - 1 hour 30 minutes

Read carefully

Calculators may NOT be used in this paper.

Section A - Questions 1 - 20 (40 marks)

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

Section B (30 marks)

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g\ ,-f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a\ ,b)$ and radius r.

Scalar Product: $a \cdot b = |a||b|\cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$
$$\sin 2A = 2\sin A\cos A$$

Table of standard

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

All questions should be answered

Section A

1. Given that $x = 4$ is a root of the equation $2x^3 + 15x^2 - 104x + 48 = 0$, the other roots are

A. 12 and $-\frac{1}{2}$

B. 6 and -1

C. 1 and -6

D. $\frac{1}{2}$ and -12

2. $\frac{d}{dx}\cos 5x$ equals

A. $\frac{1}{5}\sin 5x$

B. $-5\sin 5x$

C. $5\sin 5x$

D. $-\frac{1}{5}\sin 5x$

3. The point A$(-3, 9)$ lies on the curve with equation $y = 3x^2 + 2x - 12$. The gradient of the tangent to the curve at A is

A. -16

B. 9

C. 56

D. 63

4. If A the point $(1, 5, -7)$, B the point $(3, 4, 1)$ and C$(7, 2, z)$ are collinear, the value of z is

A. 9

B. 17

C. 5

D. -11

5. Two functions f and g are defined by $f(x) = 3x - 1$ and $g(x) = x^2 + 4$. A formula for $g(f(x))$ is

A. $9x^2 - 6x + 5$

B. $9x^2 + 5$

C. $3x^2 + 5$

D. $3x^3 + 3x^2 - 1$

6. A line has equation $5y + 3x = 1$. The line makes an angle of θ with the positive direction of the x - axis. The value of tan θ is

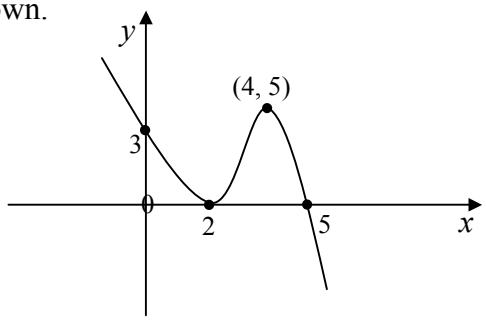
A. -3

B. $-0 \cdot 6$

C. $0 \cdot 6$

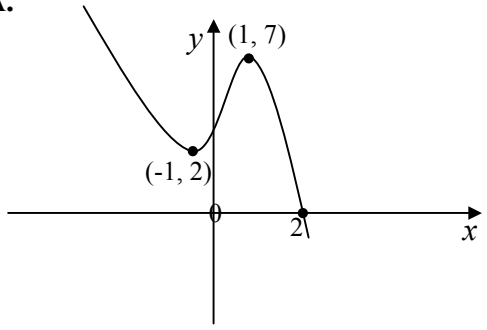
D. 3

7. The graph of $f(x)$ is shown.

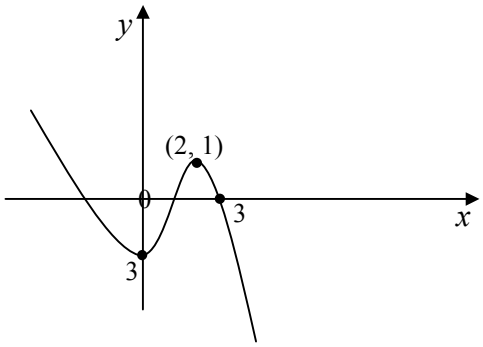


Which of these graphs shows the related function $y = f(x-3) + 2$?

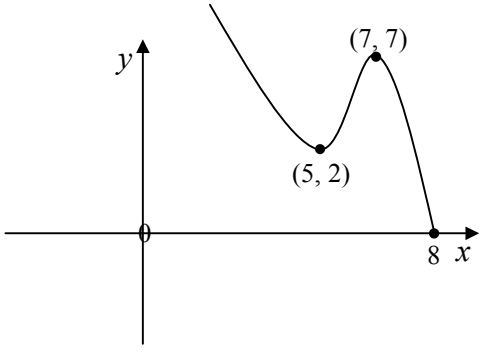
A.



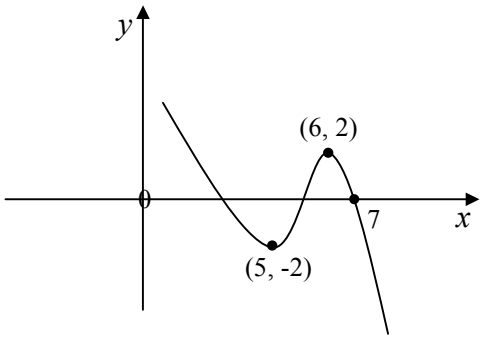
B.



C.



D.



8. Given that $f(x) = \dfrac{2x^3 - 3x^2 - 1}{x}$, the value of $f'(-1)$ is

A. -6

B. 13

C. 0

D. 1

9. The exact value of $\sin\dfrac{2\pi}{3}$ is

A. $-\dfrac{\sqrt{3}}{2}$

B. $-\dfrac{1}{2}$

C. $\dfrac{\sqrt{3}}{2}$

D. $\dfrac{1}{2}$

10. The line $y + 2x = 5$ is a tangent to the parabola $y = 4 - x^2$. The coordinates of the point of contact are

A. $(-1,\ 7)$

B. $(1,\ -3)$

C. $(-1,\ 3)$

D. $(1,\ 3)$

11. The range of values of k for which $x^2 - 3x + k = 0$ has real roots is

A. $k \le \dfrac{9}{4}$

B. $k < \dfrac{9}{4}$

C. $k \ge \dfrac{9}{4}$

D. $k > \dfrac{9}{4}$

12. The remainder when $2x^3 + 4x + 3$ is divided by $(x + 2)$ is

A. 27

B. 3

C. -21

D. 19

13. $\int_{-1}^{2} x^{-3}\, dx$ equals

A. $\frac{3}{16}$

B. $\frac{3}{8}$

C. $-\frac{5}{16}$

D. $-\frac{5}{8}$

14. A sequence is defined by the recurrence relation $U_{n+1} = aU_n + b$. If $U_0 = 5$ and $U_1 = 10$. An expression for *a* in terms of *b* is

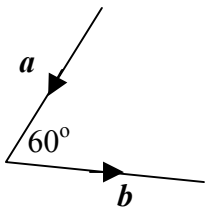
A. $a = \frac{1}{2} + \frac{1}{10}b$

B. $a = \frac{1}{2} - \frac{1}{10}b$

C. $a = 2 + \frac{1}{5}b$

D. $a = 2 - \frac{1}{5}b$

15. In this diagram $|\boldsymbol{a}| = 2$ and $|\boldsymbol{b}| = \sqrt{3}$



The value of **a** . **b** is

A. $\sqrt{3}$

B. 3

C. $-\sqrt{3}$

D. -3

16. The gradient of the tangent to the curve $y = \cos x$ at the point where $x = 210°$ is

A. $-\frac{\sqrt{3}}{2}$

B. $-\frac{1}{2}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{1}{2}$

17. $4\log 2 - 3\log 3$ expressed as the log of a single number is

A. $\log\frac{16}{27}$

B. $-\log 11$

C. $\log 1$

D. $-\log 1$

18. $\int 4\sin 3x \, dx$ equals

A. $\frac{4}{3}\cos 3x + C$

B. $12\cos 3x + C$

C. $-\frac{4}{3}\cos 3x + C$

D. $-12\cos 3x + C$

19. When $5\cos x - \sin x$ is expressed in the form $k\sin(x + \alpha)$, the value of k and $\tan\alpha$ are

A. $\sqrt{24}$ and $\tan\alpha = 5$

B. $\sqrt{24}$ and $\tan\alpha = \frac{1}{5}$

C. $\sqrt{26}$ and $\tan\alpha = -5$

D. $\sqrt{26}$ and $\tan\alpha = -\frac{1}{5}$

20. A circle has equation $(x-2)^2+(y+1)^2=29$. The gradient of the tangent to the circle at the point $(-3, 1)$ is

A. $-\frac{2}{5}$

B. $\frac{2}{5}$

C. $\frac{5}{2}$

D. $-\frac{5}{2}$

Section B

21. A sequence is defined by the recurrence relation $U_{n+1} = 0 \cdot 6U_n + 8$.

(a) Explain why this sequence has a limit as $n \to \infty$. **1**

(b) Find the limit of this sequence. **2**

(c) Given that $L - U_1 = 3$, where L is the limit of this sequence, establish the value of U_0 , the initial value. **3**

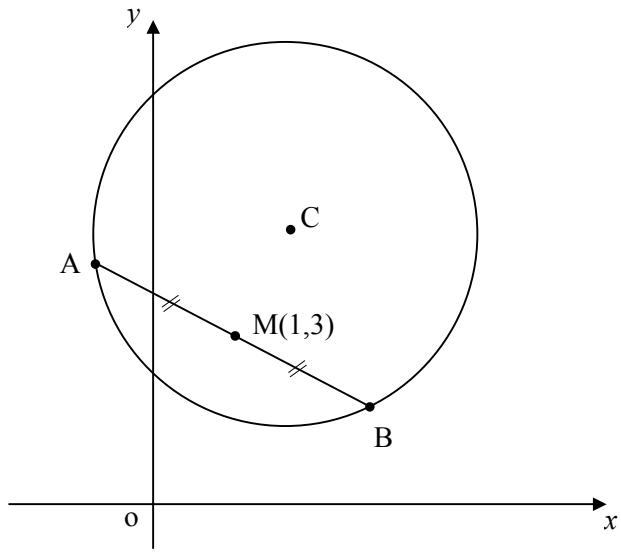
22. Given that $x = -2$ and $x = 1$ are two roots of the equation $x^3 + px^2 - 6x + q = 0$, establish the values of p and q and hence find the third root of the equation. **5**

23. A function is defined as $g(\theta) = 2\cos^2\theta - 2\cos 2\theta$.

Show that $g'(\theta)$ can be written in the form

$$g'(\theta) = 2\sin 2\theta$$ **5**

24. The circle below, centre C, has as its equation $x^2 + y^2 - 4x - 10y + 19 = 0$.
M(1,3) is the mid-point of the chord AB.



(a) Write down the coordinates of C, the centre of the circle. **1**

(b) Show that the equation of the chord AB can be written as $x = 7 - 2y$. **3**

(c) Hence find algebraically the coordinates of A and B.

4

25. (a) Given that $2\log_x y = \log_x 2y + 2$, show that x and y are related in such a way that

$$y = 2x^2.$$ **4**

(b) Hence find y when $x = \frac{1}{4}y$ and $y > 0$. **2**

[END OF QUESTION PAPER]