Practice Paper D
Marking Scheme - Paper I Section A

1. There are 2 cycles so $a = 2$

Amplitude is 3 - graph has been

Moved up 2 units so $b = 2$

Answer: A

2. $2\sin 5x \times \frac{1}{5} + C$

$= \frac{2}{5}\sin 5x + C$

Answer: B

3. $m_{PQ} = \frac{7-5}{2-(-4)} = \frac{2}{6} = \frac{1}{3}$

Answer: B

4. $\boldsymbol{m} \cdot \boldsymbol{n} = 28 - 42 + 24 = 10$

$|m| = \sqrt{49+36+36}$ $\quad |n| = \sqrt{16+49+16}$

$= \sqrt{121} = 11$ $\quad = \sqrt{81} = 9$

$\cos\theta = \frac{10}{9 \times 11} = \frac{10}{99}$

Answer: C

5. ***Answer: B***

6.

$\overrightarrow{AB} = \begin{pmatrix}10\\3\\5\end{pmatrix} - \begin{pmatrix}2\\-1\\7\end{pmatrix} = \begin{pmatrix}8\\4\\-2\end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix}12\\0\\7\end{pmatrix} - \begin{pmatrix}10\\3\\5\end{pmatrix} = \begin{pmatrix}2\\-3\\2\end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix}12\\0\\7\end{pmatrix} - \begin{pmatrix}2\\-1\\7\end{pmatrix} = \begin{pmatrix}10\\1\\0\end{pmatrix}$

$\overrightarrow{AC}.\overrightarrow{BC} = 0$

statement I is incorrect

statement II is correct

Answer: C

7. $m_{PQ} = \frac{-1-5}{-3-5} = \frac{-6}{-8} = \frac{3}{4}$

$m_{perp} = -\frac{4}{3}$

Answer: D

8. $f(-2) = 1 = 2(-2)^2 = 1 + 8 = 9$

$g(x) = 3(9) + 4 = 31$

Answer: A

9. $L = \frac{8}{1-(-0\cdot 4)} = \frac{8}{1\cdot 4}$

$= 8 \times \frac{10}{14} = \frac{80}{14} = \frac{40}{7}$

Answer: B

10. $\frac{dy}{dx} = \frac{4}{3}x^{\frac{1}{3}}$

$x = 8$

$\frac{dy}{dx} = \frac{4}{3}(8)^{\frac{1}{3}}$

$= \frac{4}{3} \times 2 = \frac{8}{3}$

Answer: D

11. $(x-4)^2 - 16 + 5$

$= (x-4)^2 - 11$

$b = -11$

Answer: C

12.

$$\begin{array}{c|cccc} 3 & 1 & -2 & -k & 6 \\ & & 3 & 3 & 9-3k \\ \hline & 1 & 1 & 3-k & 15-3k \end{array}$$

$$15-3k=0$$
$$-3k=-15$$
$$k=5$$

<u>***Answer: A***</u>

13.

$$\int \frac{1}{2}x^{-\frac{1}{2}} = \frac{\frac{1}{2}x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= x^{\frac{1}{2}} + C = \sqrt{x} + C$$

<u>***Answer: C***</u>

14.

$$\int 9x^2 + 8x - 1\, dx$$
$$y = 3x^3 + 4x^2 - x + C$$
$$-7 = -3 + 4 + 1 + C$$
$$-7 = 2 + C$$
$$C = -9$$
$$y = 3x^3 + 4x^2 - x - 9$$

<u>***Answer: D***</u>

15. Centre (6, 3)

$$r = \sqrt{3^2 + 5^2} = \sqrt{34}$$

<u>***Answer: C***</u>

16. log graph has been moved 2 right

so $a = -2$

(7, 1) would have been (5, 1) so

$b = 5$

<u>***Answer: A***</u>

17.

$$\log_{10} 72$$
$$= \log_{10} 2^3 + \log_{10} 3^2$$
$$(72 = 2 \times 2 \times 2 \times 3 \times 3)$$

<u>***Answer: B***</u>

18.

$$f\,'(x) = 3\cos 3x$$
$$f\,'(\frac{\pi}{3}) = 3\cos \pi$$
$$= 3 \times -1 = -3$$

<u>***Answer: D***</u>

19.

$$\boldsymbol{p} \cdot (\boldsymbol{p} + \boldsymbol{q})$$
$$= \boldsymbol{p} \cdot \boldsymbol{p} + \boldsymbol{p} \cdot \boldsymbol{q}$$
$$= 9 + \boldsymbol{p} \cdot \boldsymbol{q} = 18$$
$$\boldsymbol{p} \cdot \boldsymbol{q} = 9$$

<u>***Answer: C***</u>

20.

$$\cos 2x = 2 \times (\frac{12}{13})^2 - 1$$
$$= \frac{288}{169} - \frac{169}{169} = \frac{119}{169}$$

<u>***Answer: C***</u>

Practice Paper D - Paper 1 Section B Marking Scheme

	Give 1 mark for each •	Illustration(s) for awarding each mark
21a	**ans:** $x^2 + y^2 - 10y = 0$ **4 marks**	
	•1 for radius (5 units)	•1 $r = 5$
	•2 for strategy	•2 $(x-a)^2 + (y-b)^2 = r^2$
	•3 for substituting in formula	•3 $(x-0)^2 + (y-5)^2 = 25$
	•4 for expanding	•4 $x^2 + y^2 - 10y + 25 - 25 = 0$ ....
b	**ans:** $k = 2$ **5 marks**	
	•1 knowing to substitute point in equ.	•1 $4^2 + k^2 - 10k = 0$
	•2 simplifying to quadratic	•2 $k^2 - 10k + 16 = 0$
	•3 solving to answers	•3 $(k-8)(k-2) = 0$
	•4 discarding $k = 8$	•4 $\therefore\ k = 8$
	•5 answer	•5 $k = 2$
c	**ans:** $3y = 4x - 10$ **3 marks**	
	•1 for gradient of radius	•1 $m_r = \frac{2-5}{4-0} = -\frac{3}{4}$
	•2 for gradient of tangent	•2 $m_{\tan} = \frac{4}{3}$
	•3 sub. to answer	•3 $y - 2 = \frac{4}{3}(x-4)$
22.	**ans:** $a = \frac{2}{3}$ **7 marks**	
	•1 for setting up integral	•1 $A = \int_1^{1+a} (6x-2)\ dx$
	•2 integrating correctly	•2 $= \left[3x^2 - 2x \right]_1^{1+a}$
	•3 making integral equal 4	•3 $\left[3x^2 - 2x \right]_1^{1+a} = 4$
	•4 substituting	•4 $(3(1+a)^2 - 2(1+a)) - (1) = 4$
	•5 simplifying to quadratic equ.	•5 $3a^2 + 4a - 4 = 0$
	•6 factorising	•6 $(3a-2)(a+2) = 0$
	•7 solving to answer	•7 $\therefore a = \frac{2}{3}$ (note: -2 is a discard)

	Give 1 mark for each •	Illustration(s) for awarding each mark
23a	**ans:** proof **3 marks**	
	•[1] for area strategy	•[1] $A = \frac{1}{2}bh$
	•[2] for substitution	•[2] $A = \frac{1}{2}bh = \frac{1}{2}\times x \times 4\sqrt{3}$
	•[3] for answer	•[3] $8\sqrt{3} = x \times 2\sqrt{3} \quad \therefore\ x = 4$
b	**ans:** $\theta = \frac{\pi}{9}$ **3 marks**	
	•[1] for strategy and writing .. $\tan 3\theta =$	•[1] $\tan 3\theta = \dfrac{4\sqrt{3}}{4} = \sqrt{3}$
	•[2] for knowing exact value	•[2] $If\ \ \tan 3\theta = \sqrt{3}\ \ then\ \ 3\theta = \frac{\pi}{3}$
	•[3] calculating answer	•[3] $\therefore\ \theta = \frac{\pi}{9}$
24.	**ans:** $k = -1$ **5 marks**	
	•[1] making 2 a power	•[1] $\log_2(x+3) = \log_2 x^2 + 2$
	•[2] taking logs to the one side	•[2] $\log_2(x+3) - \log_2 x^2 = 2$
	•[3] combining the logs	•[3] $\log_2\left(\dfrac{x+3}{x^2}\right) = 2$
	•[4] combining the logs	•[4] $2^2 = \dfrac{x+3}{x^2}$
	•[5] solving quadratic equ. to answer	•[5] $4x^2 - x - 3 = 0$ $(4x+3)(x-1) = 0$ $x = -\frac{3}{4} \quad or \quad 1$

Total 30 marks

Practice Paper D - Paper 2 Marking Scheme

	Give 1 mark for each •	Illustration(s) for awarding each mark
1a	**ans:** $3y = 2x + 3$ **2 marks**	
	•1 for gradient	•1 $m = \frac{5+1}{6+3} = \frac{2}{3}$
	•2 for sub. to answer	•2 $y - 5 = \frac{2}{3}(x - 6)$
b	**ans: C(3, 3)** **3 marks**	
	•1 realising mid-point gives $x = 3$	•1 $mid_{AB} = \frac{-3+9}{2} = 3$
	•2 knowing to sub. in equation	•2 $\therefore\ 3y = 2(3) + 3$
	•3 calculating y correctly then answer	•3 $3y = 9 \quad \therefore \quad y = 3 \Rightarrow$ C(3,3)
c	**ans:** $\angle BCD \approx 67^o$ **3 marks**	
	•1 for knowing to use $\tan\theta = m$	•1 $\tan\theta = m$
	•2 equating and calculating an angle	•2 $\tan D\hat{A}B = \frac{2}{3} \quad \therefore \angle DAB \approx 33\cdot 7^o$
	•3 working towards and finding angle	•3 $\angle BCD \approx 67^o$
2.	**ans:** $\{5^o\ ,\ 9^o\}$ **6 marks**	
	•1 strategy … knows to change form	•1 $\sin 30t + \sqrt{3}\cos 30t = k\cos(30t - \alpha)^\circ$ $= k\cos 30t\cos\alpha + k\sin 30t\sin\alpha$
	•2 finding k	•2 $k = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$
	•3 finding α	•3 $\tan\alpha = \frac{1}{\sqrt{3}} \quad \therefore \quad \alpha = 30^\circ$
	•4 solving to number	•4 $2\cos(30t - 30)^\circ + 3 = 2$ $\therefore\ \cos(30t - 30)^\circ = -\frac{1}{2}$
	•5 first answer	•5 $30t - 30 = 120 \quad \therefore \quad t = 5^\circ$
	•6 second answer	•6 $30t - 30 = 240 \quad \therefore \quad t = 9^\circ$

	Give 1 mark for each •	Illustration(s) for awarding each mark
3a	**ans:** $y = 3x^2 - x^3$ **3 marks**	
	•[1] dealing with the composite function	•[1] $g(f(x)) = 3(x-1)^2 - 3$
	•[2] simplifying the composite function	•[2] $g(f(x)) = 3x^2 - 6x$
	•[3] subtracting $h(x)$ to answer	•[3] $y = 3x^2 - 6x - (x^3 - 6x) = 3x^2 - x^3$
b	**ans:** (2,4) **4 marks**	
	•[1] knowing to differentiate and solve to 0	•[1] $\frac{dy}{dx} = 6x - 3x^2 = 0$
	•[2] finding the two x values	•[2] $3x(2-x) = 0 \quad \therefore \quad x = 0 \;\; or \;\; x = 2$
	•[3] finding corresponding y values	•[3] $(0,0)$, $y = 3(2^2) - 2^3 = 4 \;\; \therefore \;\; (2,4)$
	•[4] statement/conclusion	•[4] justification table (or 2nd deriv.) .
4a	**ans:** P(4,6) **4 marks**	
	•[1] startegy + substituting	•[1] $(2y-8)^2 + y^2 - 4(2y-8) - 20y + 84 = 0$
	•[2] simplifying to quadratic equation	•[2] $5y^2 - 60y + 180 = 0$
	•[3] factorising + first coordinate	•[3] $5(y-6)(y-6) = 0 \quad \therefore \quad y = 6$
	•[4] second coordinate	•[4] $x = 2(6) - 8 = 4$
b	**ans:** $(x-14)^2 + (y-16)^2 = 20$ **5 marks**	
	•[1] stepping out strategy	•[1] From P to Q …. 12 along , 6 up
	•[2] finding original centre	•[2] $C_1(2,10)$
	•[3] establishing the new centre	•[3] $C_2(2+12,10+6) = C_2(14,16)$
	•[4] calculating radius *(may use pyth.)*	•[4] $r = \sqrt{(-4)^2 + (-10)^2 - 84} = \sqrt{20}$
	•[5] substituting in general equ. to answer	•[5] $(x-14)^2 + (y-16)^2 = 20$
5a	**ans:** $\sin\theta = \frac{2}{\sqrt{6}}$, $\cos\theta = \frac{\sqrt{2}}{\sqrt{6}}$ **3 marks**	
	•[1] drawing a R.A. triangle	•[1] drawing triangle
	•[2] calculating hypotenuse	•[2] $h^2 = 2 + 4 = 6 \quad \therefore \quad h = \sqrt{6}$
	•[3] lifting answers	•[3] $\sin\theta = \frac{2}{\sqrt{6}}$, $\cos\theta = \frac{\sqrt{2}}{\sqrt{6}}$
b	**ans: proof** **5 marks**	
	•[1] expanding	•[1] $\sin(\theta + \frac{\pi}{3}) = \sin\theta\cos\frac{\pi}{3} + \cos\theta\sin\frac{\pi}{3}$
	•[2] putting in all exact values	•[2] $= \frac{2}{\sqrt{6}}(\frac{1}{2}) + \frac{\sqrt{2}}{\sqrt{6}}(\frac{\sqrt{3}}{2})$
	•[3] simplifying	•[3] $= \frac{1}{\sqrt{6}} + \frac{1}{2}$
	•[4] rationalising the denominator	•[4] $= \frac{\sqrt{6}}{6} + \frac{1}{2}$
	•[5] take out common factor to answer	•[5] $\sin(\theta + \frac{\pi}{3}) = \frac{1}{6}(\sqrt{6} + 3)$

	Give 1 mark for each •	Illustration(s) for awarding each mark
6a	**ans:** Area = $1\frac{1}{3}$ m^2 **7 marks**	
	•1 for setting up integral	•1 $A = \int_2^4 (5 + 2x - \frac{1}{4}x^2)\ dx$
	•2 integrating	•2 $= \left[5x + x^2 - \frac{1}{12}x^3 \right]_2^4$
	•3 substituting in limits	•3 $= (20 + 16 - 5\frac{1}{3}) - (10 + 4 - \frac{2}{3})$
	•4 calculating area	•4 $= 17\frac{1}{3}$ square metres
	•5 finding y coordinate at $x = 2$	•5 $y = 5 + 2(2) - \frac{1}{4}(2^2) = 8$
	•6 calculating area of rectangle	•6 $A_{rec} = 8 \times 2 = 16$ square metres
	•7 subtracting to work out shaded area	•7 $A_{sh} = 17\frac{1}{3} - 16 = 1\frac{1}{3}$ sq. m
b	**ans:** 32 m^3 **2 marks**	
	•1 for knowing how to calculate volume	•1 $V = \textit{face area} \times \textit{depth}$
	•2 for calculations to answer	•2 $V = 1\frac{1}{3} \times 6 = 8$ .... $V_{tot} = 8 \times 4 = 32$ m^3
7a	**ans:** $h(d) = \left[\dfrac{-4}{(d-2)^2 + 1} \right] + 6$ **2 marks**	
	•1 starting to complete the square	•1 $\left[(d-2)^2 - 4 \right] + 5$
	•2 complete the square	•2 $h(d) = \left[\dfrac{-4}{(d-2)^2 + 1} \right] + 6$
b	**ans:** $h_{\min} = 2$ @ $d = 2$ **2 marks**	
	•1 finds minimum value	•1 $h_{\min} = 2$
	•2 finds corresponding value for d	•2 $d = 2$
c	**ans:** P(300,200) **1 mark**	
	•1 find coordinates of P	•1 P(300,200)

	Give 1 mark for each •	Illustration(s) for awarding each mark
8a	**ans:** $\vec{PR} = a - b$ **1 mark**	
	●[1] answer	●[1] $\vec{PR} = a - b$
b	**ans:** 3 **2 marks**	
	●[1] for using formula	●[1] $a.b = \lvert a \rvert \lvert b \rvert \cos\theta$
	●[2] substituting and answer	●[2] $a.b = 2 \times 3 \times \frac{1}{2} = 3$
c	**ans:** 2 **5 marks**	
	●[1] for strategy	●[1] $v.u = 2a.(a-b)$
	●[2] expanding brackets	●[2] $v.u = 2a.a - 2a.b$
	●[3] knowing $a.a$ = magnitude squared	●[3] $v.u = 2\lvert a \rvert^2 - 2a.b$
	●[4] substitutes	●[4] $v.u = 2(2^2) - 2(3)$
	●[5] answer	●[5] $v.u = 2$

Total 60 marks