NATIONAL QUALIFICATIONS

Mathematics
Practice Paper C
Paper 2
Assessing Units 1, 2 & 3

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g\ ,-f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a\ ,b)$ and radius r.

Scalar Product: $a \cdot b = |a||b|\cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

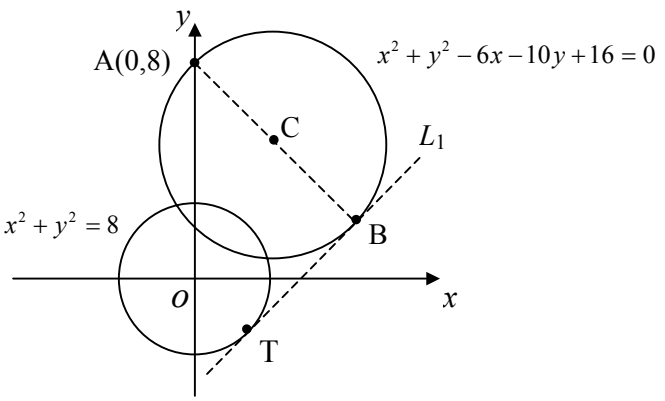
$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$
$$\sin 2A = 2\sin A\cos A$$

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

All questions should be attempted

1. The diagram below shows two overlapping circles.
The larger of the two has as its equation $x^2 + y^2 - 6x - 10y + 16 = 0$ and the smaller $x^2 + y^2 = 8$.



(a) Write down the coordinates of C, the centre of the larger circle. **(1)**

(b) Hence find the coordinates of B, given that AB is a diameter of this circle. **(1)**

(c) The line L_1 is the tangent to the circle at B. Find the equation of L_1. **(3)**

(d) Show that the line L_1 is also a tangent to the smaller circle and establish the coordinates of T, the point of tangency. **(4)**

2. Solve the equation $2(2\cos 2x^\circ + \cos x^\circ) = -3$ in the interval $0 \le x \le 360$. **(5)**

3. In a marine tank the amount of salt in the water is crucial for the health of the fish. Recommended limits give a salt solution of between 41 and 55 grammes per gallon (g/gallon).

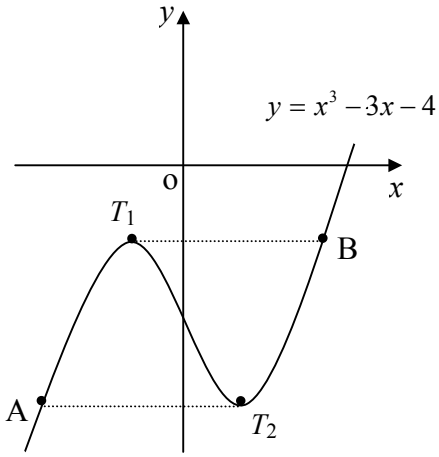
It is known that the strength of the salt solution decreases by 15% every day.
To combat this, salt is added at the **end** of each day, which effectively increases the strength of the solution by 8 g/gallon, thus creating a closed system.

To allow the plants to acclimatise the initial strength in the tank has to be 45 g/gallon.

(a) For how many days should the system be run before the introduction of fish ? **(3)**

(b) In the long term will the strength of the solution remain within safe limits ? Give reasons. **(3)**

4. The diagram shows a sketch of the graph of $y = x^3 - 3x - 4$.



The tangents at the turning points of the curve meet the curve again at the points A and B as shown.

(a) Find the coordinates of the two stationary points T_1 and T_2. **(4)**

(b) Establish the coordinates of A and B. **(4)**

(c) Show that the tangents to the curve at A and B are parallel. **(2)**

5. The functions $f(x) = \dfrac{1}{\frac{1}{2}x + 1}$ and $g(x) = 2x^2 - 4$ are defined on suitable domains.

(a) Given that $h(x) = f\big(g(x)\big)$, show that $h(x)$ can be written as

$$h(x) = \frac{1}{(x-1)(x+1)}\ .$$ **(2)**

(b) State a suitable domain for $h(x)$. **(1)**

(c) Show that there are two values of x for which the functions f and h have the same image but that they are both irrational. **(4)**

6. The diagram shows a sketch of the curve $y = 4x - x^2$ and the line $y = 3$.

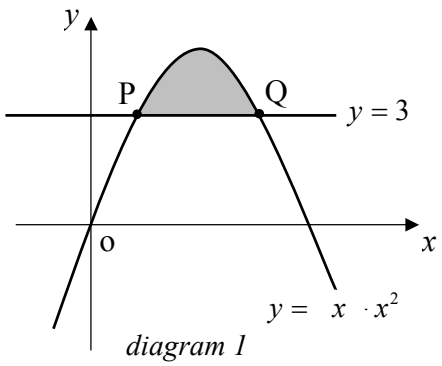


diagram 1

(a) Establish the coordinates of the points P and Q. **(2)**

(b) Calculate the shaded area in *diagram* 1. **(5)**

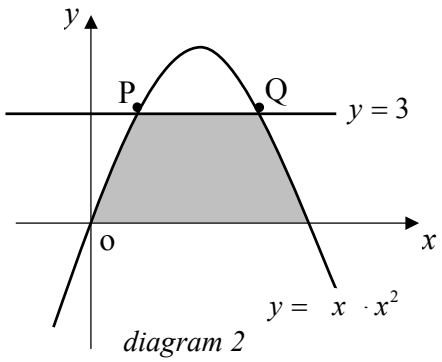
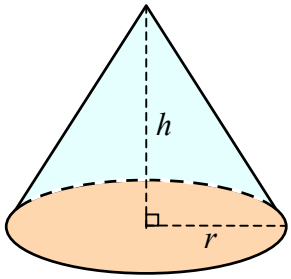


diagram 2

(c) Hence calculate the shaded area in *diagram* 2. **(3)**

7. A cone is such that the **sum** of its base **diameter** and its vertical height is 18cm.

(a) For this cone, write down an expression for the height (h) in terms of the radius (r). **(1)**



(b) Given that the formula for the volume of any cone is $V = \frac{1}{3}\pi r^2 h$, show that a function for the voulme of this cone can be expressed as

$$V(r) = 6\pi r^2 - \frac{2}{3}\pi r^3$$ **(2)**

(c) Hence find the value of r which will maximise the volume of the cone, and calculate this maximum volume in cubic centimetres. **(5)**

8. Two forces are represented by the vectors $F_1 = 2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$ and $F_2 = \sqrt{3}\,\underset{\sim}{i} + \underset{\sim}{k}$.

Calculate the angle between these two forces. **(5)**

[END OF QUESTION PAPER]