NATIONAL QUALIFICATIONS

Mathematics
Practice Paper C
Paper 1
Assessing Units 1, 2 & 3

Time allowed - 1 hour 30 minutes

Read carefully

Calculators may NOT be used in this paper.

Section A - Questions 1 - 20 (40 marks)

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

Section B (30 marks)

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g\ ,-f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a\ ,b)$ and radius r.

Scalar Product: $a \cdot b = |a||b|\cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$
$$\sin 2A = 2\sin A\cos A$$

Table of standard

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

All questions should be answered

Section A

1. When $4x^3 + 3x - 2$ is divided by $(x - 3)$, the remainder is

A. 25

B. 115

C. 43

D. -119

2. A line has as its equation $3y = x + 6$. Any line perpendicular to this line will have gradient

A. -3

B. -1

C. $-\frac{1}{3}$

D. $\frac{1}{3}$

3. Which of the following is/are true for the circle with equation $x^2 + y^2 - 4x + 6y - 3 = 0$?

(1) It passes through the point (2, 1).

(2) It has the point $(2, -3)$ as its centre.

(3) It has radius 6 units.

A. (1) and (2)

B. (1) and (3)

C. (2) and (3)

D. (1), (2) and (3)

4. If $f(x) = \frac{2}{x^4}$ and $x \neq 0$, $f'(x)$ is

A. $\frac{2}{4x^3}$

B. $-\frac{8}{x^3}$

C. $-\frac{8}{x^5}$

D. $-\frac{2}{3x^3}$

5. Two functions defined on suitable domains are given as $f(x) = x^2 - 3$ and $g(x) = 3 - x$.

$f(g(x))$ is equivalent to

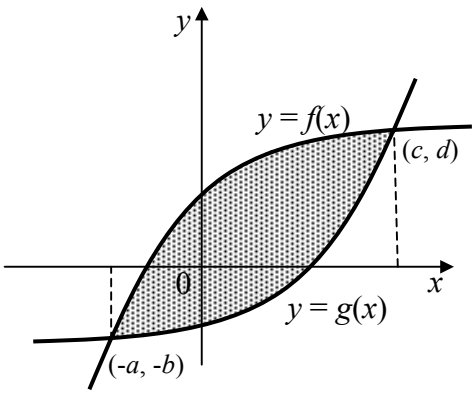
A. $9 - 6x + x^2$

B. $6 - x^2$

C. $6 - 6x + x^2$

D. $6 + x^2$

6. The diagram shows the graphs of 2 functions, $y = f(x)$ and $y = g(x)$, which intersect at the points $(-a, -b)$ and (c, d).



The shaded area is given by

A. $\int_{-a}^{c} (f(x) - g(x))\, dx$

B. $\int_{-b}^{d} (f(x) - g(x))\, dx$

C. $\int_{-a}^{c} (f(x) + g(x))\, dx$

D. $\int_{-b}^{d} (f(x) + g(x))\, dx$

7. P and Q are the points $(a, 7)$ and $(1, 4)$ respectively on the curve with equation $y = x^2 + 3$.

Given that $a > 0$, the gradient of PQ is

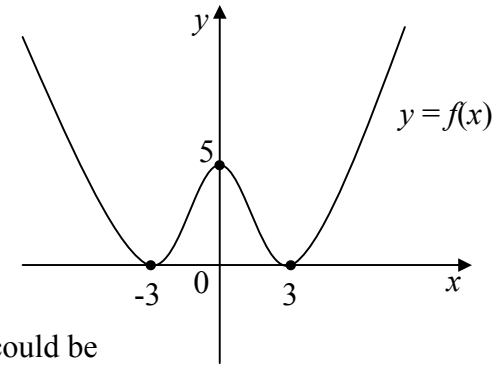
A. 3

B. -3

C. $\frac{1}{3}$

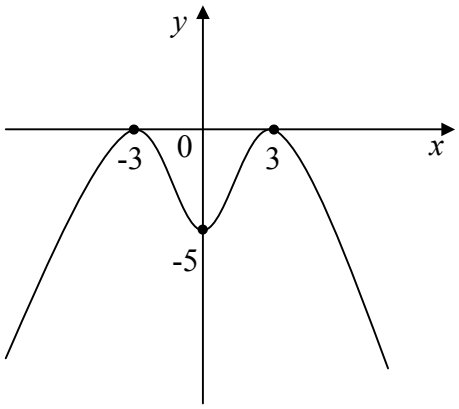
D. $-\frac{1}{3}$

8. Part of the graph of $y = f(x)$ is shown in the diagram.

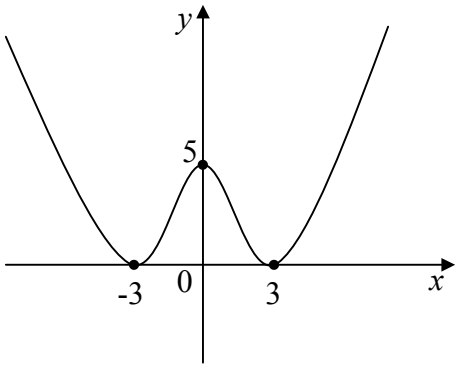


The graph of $f'(x)$ could be

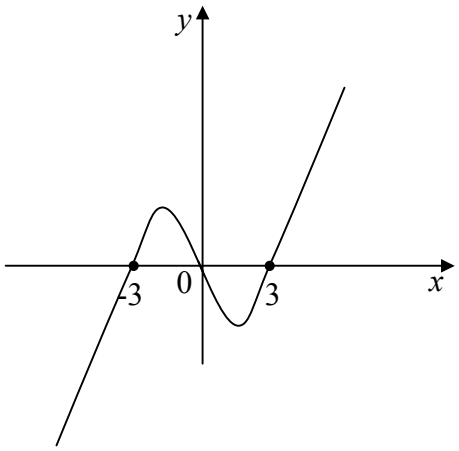
A.



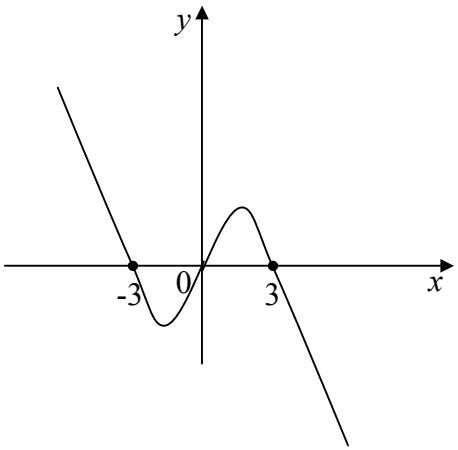
B.



C.



D.



9. Given that $\cos x° = \frac{\sqrt{3}}{2}$ and $0 < x < 90$, the exact value of $\cos 2x°$ will be

A. $\sqrt{3}$

B. $\frac{1}{2}$

C. $\frac{5}{2}$

D. $\frac{\sqrt{3}}{4}$

10. The tangent to the curve $y = x^3 - 2$ at the point where $x = -1$ has gradient

A. 9

B. 3

C. -3

D. 1

11. The exact value of $\tan\frac{5\pi}{6}$ is

A. $-\sqrt{3}$

B. $\sqrt{3}$

C. $-\frac{1}{\sqrt{3}}$

D. $\frac{1}{\sqrt{3}}$

12. The vectors $\boldsymbol{u} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} -1 \\ 4 \\ z \end{pmatrix}$ are perpendicular. The value of z is

A. $\frac{7}{2}$

B. $\frac{1}{2}$

C. $\frac{2}{7}$

D. 2

13. $\int \sin 3x \, dx$ is

A. $3\cos 3x + C$

B. $-3\cos 3x + C$

C. $\cos 3x + C$

D. $-\frac{1}{3}\cos 3x + C$

14. A recurrence relation is defined by $U_{n+1} = 0 \cdot 3U_n - 21$. The limit of this sequence as $n \to \infty$ is

A. 70

B. -30

C. -70

D. $6 \cdot 3$

15. $\log_5 125 + \log_4 16$ is equal to

A. 5

B. 6

C. $2 \cdot 5$

D. $3 \cdot 5$

16. k and α are given by $k \sin \alpha = -1$ and $k \cos \alpha = 3$. The value of k and the range of values for α are

A. $\sqrt{10}$ and $\frac{\pi}{2} < \alpha < \pi$

B. $\sqrt{10}$ and $\frac{3\pi}{2} < \alpha < 2\pi$

C. $2\sqrt{2}$ and $\frac{\pi}{2} < \alpha < \pi$

D. $2\sqrt{2}$ and $\frac{3\pi}{2} < \alpha < 2\pi$

17. The derivative of $\frac{2}{(x^2+1)}$ with respect to x is

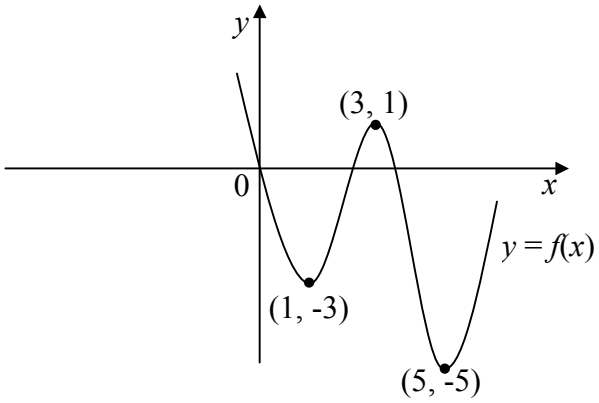
A. $\frac{2x}{(x^2+1)^2}$

B. $-\frac{2}{(x^2+1)^2}$

C. $-\frac{4x}{(x^2+1)^2}$

D. $\frac{2}{(x^2+1)}$

18. The diagram shows part of the graph of $y = f(x)$.



Which of the following are true for the above graph?

(1) $f'(0) < 0$

(2) $f'(4) > 0$

(3) $f'(5) = 0$

A. (1) and (2)

B. (1) and (3)

C. (2) and (3)

D. (1), (2) and (3)

19. The quadratic equation $x^2 - 2kx + k = 0$ has equal roots ($k \neq 0$). The value(s) of k is/are

A. 1

B. 0

C. 1 and 0

D. 2

20. $f(x) = ax^2 - 3x + 2$ has a stationary value where $x = -2$. The value of a is

A. $-\frac{3}{4}$

B. $\frac{3}{4}$

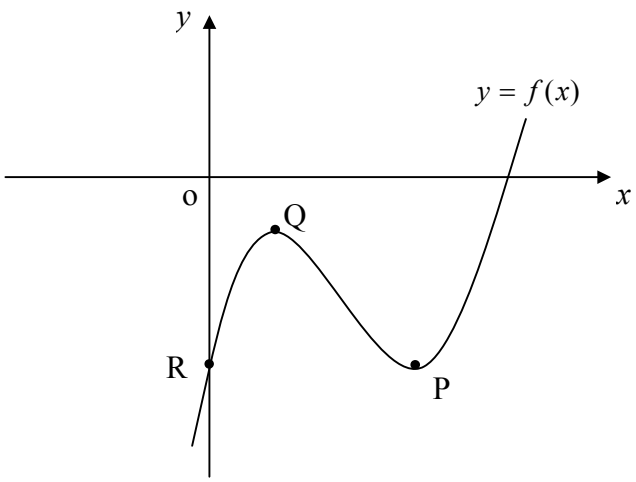
C. -2

D. 1

Section B

21. Find $f'(x)$ when $f(x) = \dfrac{x^2 - \sqrt{x}}{x}$. **(4)**

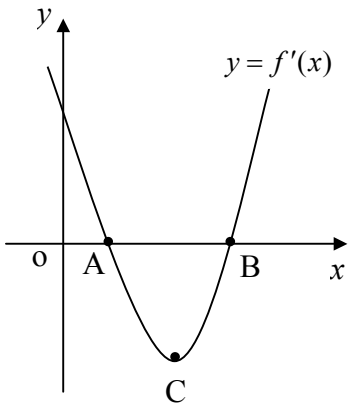
22. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x - 5$ is shown below.
The graph has two turning points at P and Q and a y-intercept point at R.



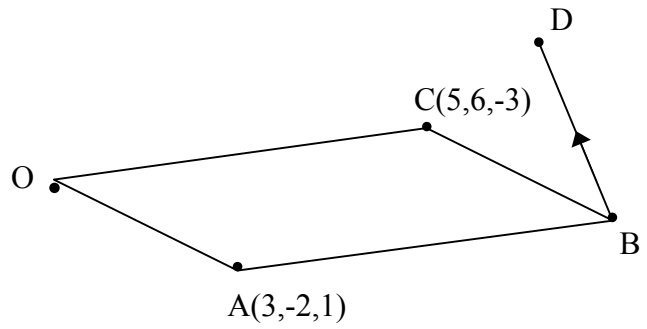
(a) Find the equation of the tangent to the curve at R. **(4)**

(b) The sketch of the derived function ($y = f'(x)$) from the above graph, is shown opposite.

Find the coordinates of A, B and C. **(4)**



23. The parallelogram OABC, where O is the origin, has two more of its vertices at A(3,-2,1) and C(5,6,-3), as shown in the diagram.

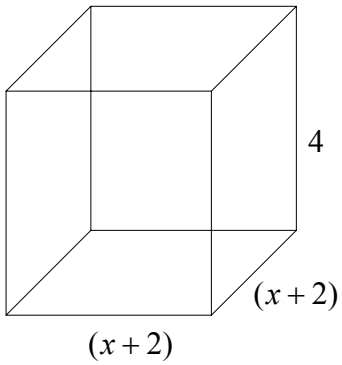


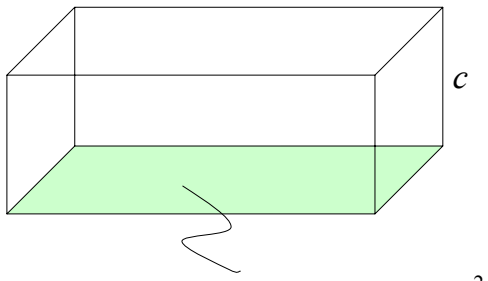
(a) Find $\overrightarrow{AC}$ in component form. **(1)**

(b) D is a point such that $\overrightarrow{BD} = \begin{pmatrix} -2 \\ 6 \\ -3 \end{pmatrix}$.

Show that A, C and D are collinear. **(4)**

24. The two cuboids below have equal volumes.
All lengths are in centimetres.



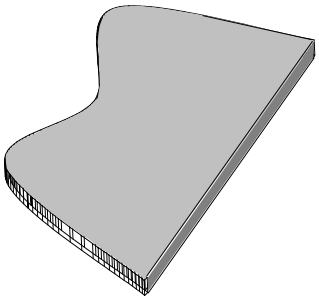


(a) By equating the two volumes show that the following equation can be constructed .....

$$4x^2 + (16-8c)x + (16-5c) = 0$$ **(2)**

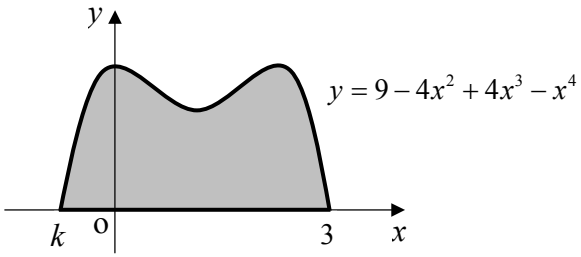
(b) Given that $c > 0$, find the value of c for which the equation $4x^2 + (16-8c)x + (16-5c) = 0$ has **equal** roots. **(4)**

25. A modern table top is an abstract bow shape.



The table top can be modelled by a straight line and a curve.

When rotated and placed on a set of rectangular axes the straight edge can be placed along the x-axis and the curve given the equation $y = 9 - 4x^2 + 4x^3 - x^4$, as shown below.



(a) Given that one root of the equation $9 - 4x^2 + 4x^3 - x^4 = 0$ is 3, find the value of the other root k. **(3)**

(b) Calculate the area of the table top in square units. **(4)**

[END OF QUESTION PAPER]