Practice Paper B Marking Scheme - Paper I Section A

1.

$$y = x^3 - 4$$

$$\frac{dy}{dx} = 3x^2$$

when $x = -2$

$$\frac{dy}{dx} = 3(-2)^2 = 12$$

Answer: A

2.

$$\overrightarrow{AB} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ -6 \end{pmatrix}$$

$$\overrightarrow{BC} = \begin{pmatrix} x \\ 4 \\ -5 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} x+1 \\ 2 \\ 4 \end{pmatrix}$$

$$x + 1 = -2$$

$$x = -3$$

Answer: C

3.

$$\int x^6 + x^{-5}\, dx$$

$$= \frac{x^7}{7} + \frac{x^{-4}}{-4} + C$$

$$= \frac{1}{7}x^7 - \frac{1}{4x^4} + C$$

Answer: B

4.

$$(x+4)^2 - 16 - 2$$

$$= (x+4)^2 - 18$$

Turning point $(-4, -18)$

Answer: D

5.

$$\cos 2\alpha = 2\cos^2 \alpha - 1$$

$$= 2 \times (\frac{\sqrt{55}}{8})^2 - 1$$

$$= 2 \times \frac{55}{64} - 1$$

$$= \frac{110}{64} - \frac{64}{64} = \frac{46}{64} = \frac{23}{32}$$

Answer: C

6.

Answer: B

7.

Midpoint AB $\left(\frac{7-1}{2}, \frac{5+1}{2}\right)$

$= (3, 3)$

Answer: B

8.

3	1	-6	p	-6
		3	-9	$3p-27$
	1	-3	$p-9$	$3p-33$

$$3p - 33 = 0$$

$$p = 11$$

Answer: C

9.

$$2x^2 + x - p = 0$$

$$a = 2;\ b = 1;\ c = -p$$

$$b^2 - 4ac = 1^2 - 4.2.-p < 0$$

$$1 + 8p < 0$$

$$8p < -1$$

$$p < -\frac{1}{8}$$

Answer: B

10.

$$2x - 3y = 4$$

$$-3y = -2x + 4$$

$$y = \frac{2}{3}x - \frac{4}{3}$$

$$m = \frac{2}{3}$$

Answer: C

11. $\mathbf{C}(-3, 2)$

$r = \sqrt{(-3)^2 + 2^2 + 3}$

$= \sqrt{9+4+3} = \sqrt{16} = 4$

Answer: D

12. $\dfrac{1}{(1+x)(1-x)}$

$x \neq 1$ or -1

Answer: D

$x^2 - 4x + 4 - 13 = 0$

$x^2 - 4x - 9 = 0$

13. $b^2 - 4ac = 16 - 4.1.-9$

$= 16 + 36$

$= 52$

Roots are real and unequal

Answer: B

14. Minimum value of $6\sin\ldots\ldots\ldots = -6$

$-6 + 4 = -2$

Answer: C

$2(x^2 - 8x) + 11$

$= 2[(x-4)^2 - 16] + 11$

15. $= 2(x-4)^2 - 32 + 11$

$= 2(x-4)^2 - 21$

$p = -4$

Answer: D

$x^2 + k^2 - 9 \geq 0$

$0^2 - 4.1.(k^2 - 9) \geq 0$

$-4(k^2 - 9) \geq 0$

16. $-4k^2 + 36 \geq 0$

$-4k^2 \geq -36$

$k^2 \leq 9$

$-3 \leq k \leq 3$

Answer: C

$\overrightarrow{DT} = \dfrac{1}{2}(\overrightarrow{DA} + \overrightarrow{AB})$

17.

$= \dfrac{1}{2}(\boldsymbol{u} - \boldsymbol{v})$

Answer: C

18 $\boldsymbol{u} = \begin{pmatrix} \frac{1}{3} \\ p \\ 0 \end{pmatrix}$

$\left(\dfrac{1}{3}\right)^2 + p^2 = 1$

$\dfrac{1}{9} + p^2 = 1$

$p^2 = \dfrac{8}{9}$

$p = \dfrac{2\sqrt{2}}{3}$

Answer: A

$y = k(x+3)(x+3)(x-2)$

$9 = k(3)(3)(-2)$

19. $k = -\dfrac{1}{2}$

$y = -\dfrac{1}{2}(x+3)^2(x-2)$

Answer: D

$g'(x) = 6x - 4x^3$

20. $= 6(-1) - 4(-1)^3$

$= -6 + 4 = -2$

Answer: C

Practice Paper B - Paper 1 Section B **Marking Scheme**

	Give 1 mark for each •	Illustration(s) for awarding each mark
21a	**ans:** $x = 90^\circ$ **5 marks**	
	•[1] using correct replacement	•[1] $1 - 2\sin^2 x^\circ$
	•[2] substitute and simplify	•[2] $-2(\sin^2 x^\circ + 3\sin x^\circ - 4) = 0$
	•[3] factorise and solve	•[3] $\sin x^\circ = -4$ *or* $\sin x^\circ = 1$
	•[4] discarding due to no solution	•[4] $\sin x^\circ = -4$
	•[5] for final answer	•[5] $x = 90^\circ$
22a	**ans:** $f_{\max} = \frac{1}{2}p$ **4 marks**	
	•[1] for completing the square	•[1] $\left[(x-9)^2 - 81 \right] + 87$
	•[2] substituting into formula	•[2] $f(x) = \dfrac{3p}{(x-9)^2 + 6}$
	•[3] stating or implying minimum value	•[3] $(x-9)^2 + 6 \rightarrow \min = 6$
	•[4] finding maximum value	•[4] $\therefore \quad f_{\max} = \dfrac{3p}{6} = \frac{1}{2}p$
b	**ans:** proof **2 marks**	
	•[1] start proof	•[1] $f_{\max} = \dfrac{1}{2} \cdot \dfrac{2}{\sqrt{2}+1} = \dfrac{1}{\sqrt{2}+1}$
	•[2] complete proof	•[2] $f_{\max} = \dfrac{1}{\sqrt{2}+1} \cdot \dfrac{\sqrt{2}-1}{\sqrt{2}-1} = \dfrac{\sqrt{2}-1}{1} = ans$
23a	**ans:** $k = \frac{\pi}{4}$ **1 mark**	
	•[1] find value of k	•[1] $k = \frac{\pi}{4}$
b	**ans:** $\sqrt{2} - 1$ *sq.units* **5 marks**	
	•[1] sets up integral	•[1] $A = \int_0^{\frac{\pi}{4}} (\cos\theta - \sin\theta)\, dx$
	•[2] integrates	•[2] $= \left[\sin\theta + \cos\theta\right]_0^{\frac{\pi}{4}}$
	•[3] substitutes into integral	•[3] $= \left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) - (\sin 0 + \cos 0)$
	•[4] starts to evaluate	•[4] $A = \dfrac{2}{\sqrt{2}} - 1$
	•[5] finds area	•[5] $A = \sqrt{2} - 1$ *sq.units*

	Give 1 mark for each •	Illustration(s) for awarding each mark
24a	**ans:** $U_2 = 4a^2 - 8a - 8$ **2 marks**	
	•[1] finds U_1 in terms of a	•[1] $U_1 = a(4) - 8 = 4a - 8$
	•[2] finds U_2 in terms of a	•[2] $U_2 = a(4a - 8) - 8 = 4a^2 - 8a - 8$
b	**ans:** $a = 6$ **3 marks**	
	•[1] sets up equation	•[1] $4a^2 - 8a - 8 = 88$
	•[2] equates to zero and factorise	•[2] $4(a-6)(a+4) = 0$
	•[3] correct answer (discarding a =- 4)	•[3] $\therefore \quad a = 6$
c	**ans:** $S_3 = 624$ **2 marks**	
	•[1] evaluates U_1 and U_3	•[1] $U_1 = 16$ *and* $U_3 = 520$
	•[2] evaluates S_3	•[2] $S_3 = 16 + 88 + 520 = 624$
25a	**ans:** $\cos 2\theta = \frac{1}{3}$ **4 marks**	
	•[1] calculates hypotenuse	•[1] $h = \sqrt{3}$
	•[2] evaluates $\cos\theta$	•[2] $\cos\theta = \frac{\sqrt{2}}{\sqrt{3}}$
	•[3] uses correct formula	•[3] $\cos 2\theta = 2\cos^2\theta - 1$
	•[4] evaluates $\cos 2\theta$	•[4] $\cos 2\theta = \frac{1}{3}$
b	**ans:** $k = 3\sqrt{2}$ **2 marks**	
	•[1] equating	•[1] $\cos 2\theta = \frac{1}{3} = \frac{\sqrt{2}}{k}$
	•[2] finds value of k	•[2] $k = 3\sqrt{2}$

Total 30 marks

Practice Paper B - Paper 2 **Marking Scheme**

	Give 1 mark for each •	Illustration(s) for awarding each mark
1a	**ans:** R(0,4,-6) and S(6,10,-15) **2 marks**	
	•[1] finds coordinates of R •[2] finds coordinates of S	•[1] R(0,4,-6) •[2] S(6,10,-15)
b	**ans: proof** **2 marks**	
	•[1] finds two displacements	•[1] $\vec{PR} = \begin{pmatrix} 8 \\ 8 \\ -12 \end{pmatrix}$ *and* $\vec{RS} = \begin{pmatrix} 6 \\ 6 \\ -9 \end{pmatrix}$
	•[2] show they are collinear	•[2] $\vec{PR} = \frac{4}{3}\vec{RS}$
c	**ans: proof** **1 mark**	
	•[1] finds two displacements	•[1] $\vec{RP} = \begin{pmatrix} -8 \\ -8 \\ 12 \end{pmatrix}$ *and* $\vec{RQ} = \begin{pmatrix} 6 \\ -14 \\ -9 \end{pmatrix}$
d	**ans: angle PRQ** = $98 \cdot 7^\circ$ **5 marks**	
	•[1] knowing to use scalar product	•[1] $\cos\theta = \dfrac{\vec{RP}.\vec{RQ}}{\lvert RP\rvert.\lvert RQ\rvert}$
	•[2] for scalar product	•[2] $\vec{RP}.\vec{RQ} = -44$
	•[3] calculates magnitudes	•[3] $\sqrt{272}$ *and* $\sqrt{313}$
	•[4] starts to solve scalar product	•[4] $\cos\theta = \dfrac{-44}{\sqrt{85136}}$
	•[5] calculates angle	•[5] angle PRQ = $98 \cdot 7^\circ$
2a	**ans:** A(1,-3) and B(3,-9) **5 marks**	
	•[1] know to use system of equations •[2] for equation of line •[3] for equating *y*'s •[4] for solving cubic •[5] finds coordinates of A and B	•[1] evidence •[2] $y = -3x$ •[3] $-3x = x^3 - 4x^2$ •[4] $x(x-1)(x-3) = 0 \ \therefore \ x = 1$ *or* $x = 3$ •[5] A(1,-3) and B(3,-9)

	Give 1 mark for each •	Illustration(s) for awarding each mark
2b	**ans:** proof **3 marks**	
	•[1] chooses suitable strategy	•[1] proof
	•[2] calculates lengths of OA and AB	•[2] OA = $\sqrt{10}$ and AB = $2\sqrt{10}$
	•[3] final statement and keeping exact values	•[3] evidence
3a	**ans:** P(3,18) and Q($5,16\frac{2}{3}$) **6 marks**	
	•[1] attempting to differentiate	•[1] $x^2 - 8x + 15$
	•[2] stating $\frac{dy}{dx} = 0$	•[2] $x^2 - 8x + 15 = 0$
	•[3] factorise	•[3] $(x-3)(x-5) = 0$
	•[4] find x coordinates	•[4] $x = 3$ or $x = 5$
	•[5] finds coordinates of P	•[5] P(3,18)
	•[6] finds coordinates of Q	•[6] Q($5,16\frac{2}{3}$)
b	**ans:** $y = 3x$ **5 marks**	
	•[1] find y coordinate of R	•[1] $y = 18$
	•[2] trial and error to find x	•[2] that $x > 5$ until $x = 6$
	•[3] knows $\frac{dy}{dx} = m$	•[3] evidence
	•[4] evaluating derivative at $x = 6$	•[4] $m = 3$
	•[5] find equation of line	•[5] $y = 3x$
c	**ans: (0,0)** **1 mark**	
	•[1] finds second point	•[1] (0,0)
4a.	**ans:** $84 \cdot 935$ **3 marks**	
	•[1] for correct multiplier	•[1] $0 \cdot 96$
	•[2] for setting up calculation	•[2] $= (0 \cdot 96)^4 \times 100$
	•[3] for calculation	•[3] $= 84 \cdot 935$
b	**ans:** $y = 3x$ **4 marks**	
	•[1] for setting up recurrance relation	•[1] $U_{n+1} = (0 \cdot 96)^4 . U_n + 12$
	•[2] for setting out at least 5 lines of calc	•[2] evidence
	•[3] for looking at lower value before add 12	•[3] evidence
	•[4] for solution	•[4] $\approx 76 \cdot 64$ $\therefore$ strength < 77 &not acceptable

	Give 1 mark for each •	Illustration(s) for awarding each mark
4c	**ans:** Magnetic saturation $106 \cdot 2$ *mfu* **3 marks**	
	$\bullet^1$ for knowing limit formula or equivalent	$\bullet^1$ evidence
	$\bullet^2$ substituting into formula	$\bullet^2$ $L = \dfrac{16}{1 - (0 \cdot 96)^4}$
	$\bullet^3$ calculating limit	$\bullet^3$ Magnetic saturation at $106 \cdot 2$ *mfu*
5a	**ans:** A(3,9) and B(6,0) **2 marks**	
	$\bullet^1$ finding coordinates of A	$\bullet^1$ A(3,9)
	$\bullet^2$ finding coordinates of B	$\bullet^2$ B(6,0)
b	**ans:** proof **3 marks**	
	$\bullet^1$ establ. a perfect square	$\bullet^1$ evidence
	$\bullet^2$ expand	$\bullet^2$ $y = x^2 - 12x + 36$
	$\bullet^3$ check point lies on curve	$\bullet^3$ evidence
c	**ans: 36 square units** **6 marks**	
	$\bullet^1$ set up integral	$\bullet^1$ evidence
	$\bullet^2$ correct limits	$\bullet^2$ $A = \int_3^6 \left(18x - 2x^2 - 36\right)$
	$\bullet^3$ integrate correctly	$\bullet^3$ $A = \left[9x^2 - \dfrac{2x^3}{3} - 36x\right]_3^6$
	$\bullet^4$ substituting and knowing to subtract	$\bullet^4$ evidence
	$\bullet^5$ calculating answer	$\bullet^5$ $A = (-36) - (-45) = 9$ square units
	$\bullet^6$ find total area	$\bullet^6$ $= 4 \times 9 = 36$ square units.
6a	**ans:** P(12,0) and Q(0,6) **6 marks**	
	$\bullet^1$ for finding centre	$\bullet^1$ C(6,-2)
	$\bullet^2$ for finding gradient to tangent	$\bullet^2$ m = 2
	$\bullet^3$ for finding gradient of tangent	$\bullet^3$ $m_{tan} = -\frac{1}{2}$
	$\bullet^4$ for using equation of line with point and *m*	$\bullet^4$ $y - b = m(x - a)$
	$\bullet^5$ for finding equation of line	$\bullet^5$ $x + 2y = 12$ (or equiv.)
	$\bullet^6$ finds points P and Q	$\bullet^6$ P(12,0) and Q(0,6).

	Give 1 mark for each •	Illustration(s) for awarding each mark
6b	**ans: proof** **3 marks** $\bullet^1$ for realising a rectangle is present $\bullet^2$ for correct use of x $\bullet^3$ for correct use of y	 $\bullet^1$ evidence $\bullet^2$ evidence $\bullet^3$ evidence
6c	**ans: proof** **3 marks** $\bullet^1$ find midpoint of chord $\bullet^2$ for proving M lies on TQ	 $\bullet^1$ M(6,0) $\bullet^2$ evidence

Total 60 marks