NATIONAL QUALIFICATIONS

Mathematics
Practice Paper B
Paper 2
Assessing Units 1, 2 & 3

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g\ ,-f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a\ ,b)$ and radius r.

Scalar Product: $a \cdot b = |a||b|\cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$
$$\sin 2A = 2\sin A\cos A$$

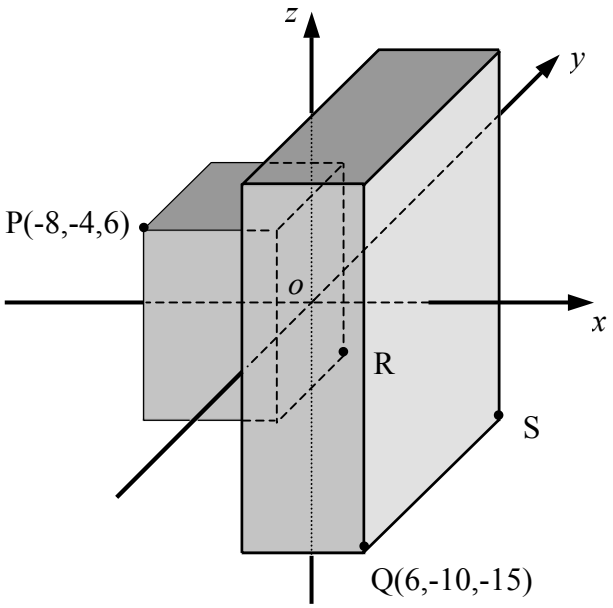
$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

All questions should be attempted

1. A metal casting is in the shape of two cuboids.

The casting is positioned relative to a set of rectangular axes as shown below.
Both cuboids have been centred along the x-axis.



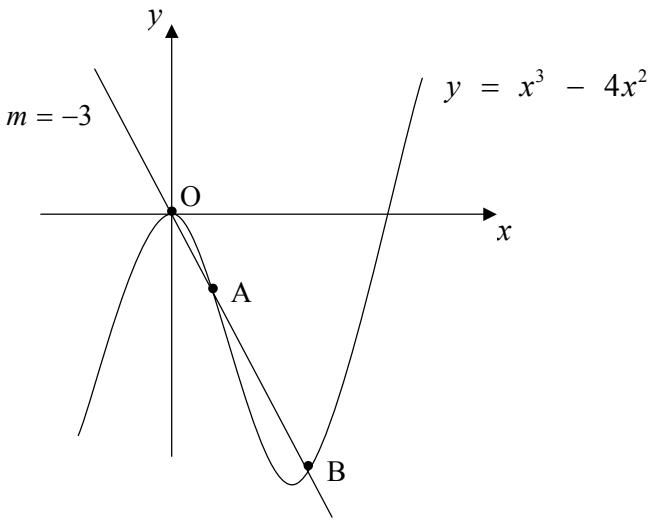
(*a*) Given that corners P and Q have coordinates $(-8,-4,6)$ and $(6,-10,-15)$ respectively , write down the coordinates of corners R and S. **(2)**

(*b*) Hence show that the corners P , R and S are collinear. **(2)**

(*c*) Find $\overrightarrow{RP}$ and $\overrightarrow{RQ}$ **(1)**

(*d*) Calculate the size of $\angle PRQ$. **(5)**

2. The line which passes through the origin with gradient -3 intersects the curve with equation $y = x^3 - 4x^2$ at two further points A and B, as shown in the diagram below.



(*a*) Establish the coordinates of A and B. **(5)**

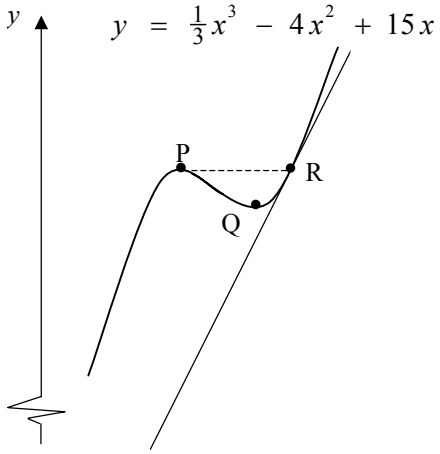
(*b*) Hence show that OA is half the length of AB. **(3)**

3. A curve has as its equation $y = \frac{1}{3}x^3 - 4x^2 + 15x$.

Part of the graph of this curve is shown in the diagram opposite.

The diagram is not drawn to scale.

The tangent at the point R on the curve is also shown



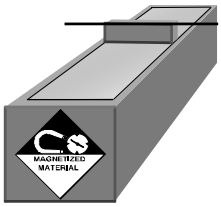
(*a*) Find the coordinates of the stationary points. **(6)**

(*b*) Establish the coordinates of the point R given that PR is parallel to the x – axis and that the x – coordinate of R is a **whole** number. Hence find the equation of the tangent at R. **(5)**

(*c*) This tangent meets the curve at a second point, state the coordinates of this second point. **(1)**

4. The diagram opposite shows a small bar magnet which is part of an electrical control circuit.

When first placed in the circuit the magnetic strength of the magnet is rated at 100 *mfu* (magnetic flux units).



(*a*) When the circuit is switched on, heat is produced.
This heat disturbs the dipoles within the domains of the magnet producing a demagnetization of the magnet (i.e. a decrease in magnetic strength).

During any six hour period, when the circuit is running, the magnet is known to lose 4% of the magnetic strength it had at the beginning of the period.

Calculate the magnetic strength of the magnet after the circuit has been running continuously for 24 hours. **(3)**

(*b*) At the end of each 24 hour period, the circuit (with the magnet in place) is automatically passed through a very intense electric field. This allows the magnet to regain some of its lost strength.
A single pass through the field and the magnet regains 12 *mfu* of strength.

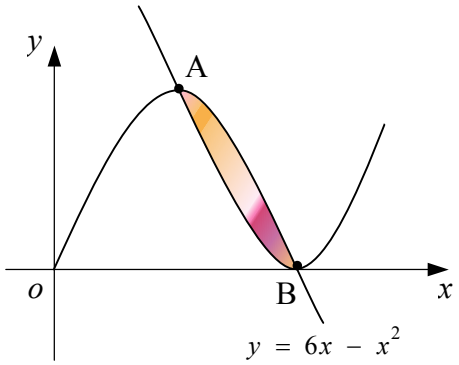
The 24 hour cycle described above is now left to run uniterrupted for a number of weeks.

Given that a magnet which falls below a strength of 77 *mfu* will not function properly in the circuit, comment on whether or not the above conditions are satisfactory ? **(4)**
Your answer must be accompanied with the appropriate working.

(*c*) Although an expensive option the company decide to change their strategy.
They buy new equipment so that at the end of each 24 hour period the circuit is passed through an electric field many times stronger than the original.
The result for the magnet is a gradual increase in magnetic strength until "magnetic saturation" is reached.
(It is known that the strength of a magnet cannot be increased beyond a certain limit.)

Calculate the strength of this particular magnet when "magnetic saturation" occurs given that the new equipment is now adding 16 *mfu* at the end of each 24 hour period. **(3)**

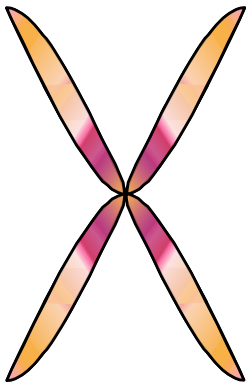
5. The diagram below shows parts of the graphs of two quadratic curves.



(*a*) The curve with its maximum turning point at A has as its equation $y = 6x - x^2$.
Establish the coordinates of A and B. **(2)**

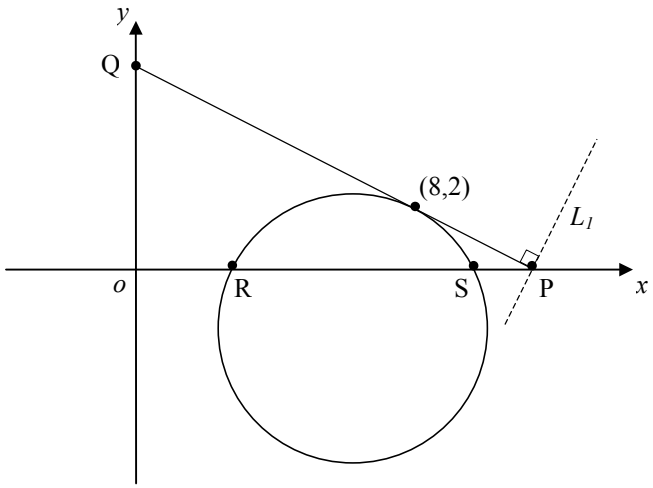
(*b*) The second quadratic curve passes through the point A and has its turning point at B.
Show that this second curve has as its equation $y = x^2 - 12x + 36$. **(3)**

(*c*) The design below has been created by reflecting the area enclosed between the two curves in the horizontal and vertical axes.
Calculate the area of this design in square units.



(6)

6. The diagram below shows the circle with equation $x^2 + y^2 - 12x + 4y + 20 = 0$.
The tangent at the point $(8,2)$ on the circle meets the x – axis at P and the y – axis at Q as shown.
The circle cuts the x – axis at the points R and S.



(*a*) Find the coordinates of the points P and Q. **(6)**

(*b*) Given that the perpendicular to the line PQ through P (line L_1) is also a tangent to the circle, show that its point of tangency, T, is $(10,-4)$. **(3)**

(*c*) Prove that the line TQ passes through the mid-point of the chord RS. **(3)**

[END OF QUESTION PAPER]