

**[C100/SQP328]**

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Mathematics

Higher

Paper 2

Specimen Question Paper

Example 2 based on 2004 Examination Paper  
(for examinations from Diet 2008 onwards)

NATIONAL  
QUALIFICATIONS

**Read Carefully**

- 1 **Calculators may be used in this paper.**
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.

## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

**Scalar Product:**  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

**Trigonometric formulae:**  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

**Table of standard derivatives:**

| $f(x)$    | $f'(x)$      |
|-----------|--------------|
| $\sin ax$ | $a \cos ax$  |
| $\cos ax$ | $-a \sin ax$ |

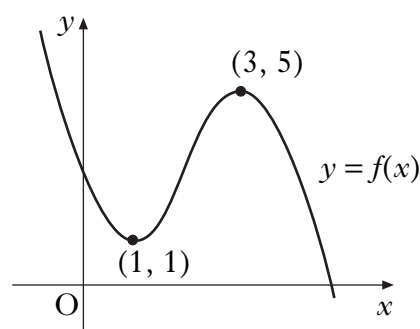
**Table of standard integrals:**

| $f(x)$    | $\int f(x) dx$             |
|-----------|----------------------------|
| $\sin ax$ | $-\frac{1}{a} \cos ax + C$ |
| $\cos ax$ | $\frac{1}{a} \sin ax + C$  |

**ALL questions should be attempted.**

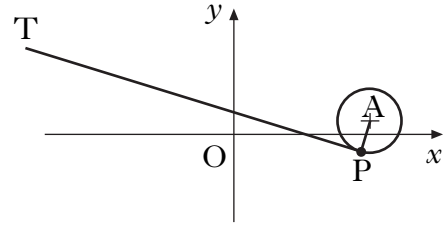
*Marks*

1. Given that  $\vec{QP} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$  and  $\vec{QR} = \begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix}$ , find the size of angle PQR. **5**
2. Prove that the roots of the equation  $2x^2 + px - 3 = 0$  are real for all values of  $p$ . **4**
3. The point  $P(x, y)$  lies on the curve with equation  $y = 6x^2 - x^3$ .
- (a) Find the value of  $x$  for which the gradient of the tangent at P is 12. **5**
- (b) Hence find the equation of the tangent at P. **2**
4. (a) Express  $3\cos x^\circ + 5\sin x^\circ$  in the form  $k\cos(x^\circ - a^\circ)$  where  $k > 0$  and  $0 \leq a \leq 90$ . **4**
- (b) Hence solve the equation  $3\cos x^\circ + 5\sin x^\circ = 4$  for  $0 \leq x \leq 90$ . **3**
5. The graph of the cubic function  $y = f(x)$  is shown in the diagram. There are turning points at  $(1, 1)$  and  $(3, 5)$ .  
Sketch the graph of  $y = f'(x)$ . **3**



6. The circle with centre A has equation  $x^2 + y^2 - 12x - 2y + 32 = 0$ . The line PT is a tangent to this circle at the point P(5, -1).

(a) Show that the equation of this tangent is  $x + 2y = 3$ .

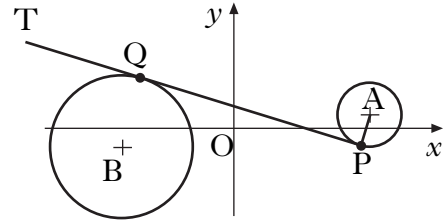


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The circle with centre B has equation  $x^2 + y^2 + 10x + 2y + 6 = 0$ .

(b) Show that PT is also a tangent to this circle.

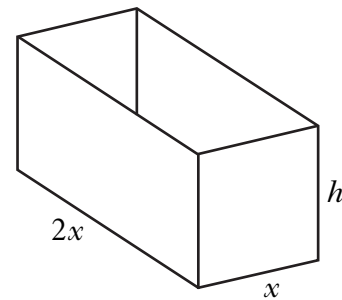
(c) Q is the point of contact. Find the length of PQ.



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7. An open cuboid measures internally  $x$  units by  $2x$  units by  $h$  units and has an inner surface area of 12 units<sup>2</sup>.



(a) Show that the volume,  $V$  units<sup>3</sup>, of the cuboid is given by  $V(x) = \frac{2}{3}x(6 - x^2)$ .

3

(b) Find the exact value of  $x$  for which this volume is a maximum.

5

8. The amount  $A_t$  micrograms of a certain radioactive substance remaining after  $t$  years decreases according to the formula  $A_t = A_0 e^{-0.002t}$ , where  $A_0$  is the amount present initially.

(a) If 600 micrograms are left after 1000 years, how many micrograms were present initially?

3

(b) The half-life of a substance is the time taken for the amount to decrease to half of its initial amount. What is the half-life of this substance?

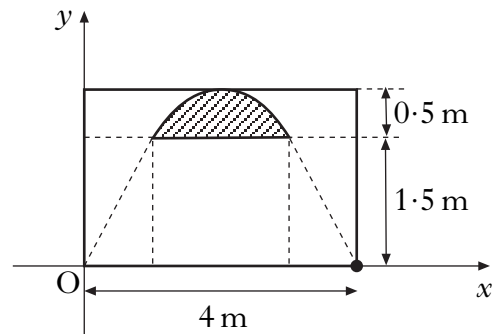
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9. An architectural feature of a building is a wall with arched windows. The curved edge of each window is parabolic.

The second diagram shows one such window. The shaded part represents the glass.

The top edge of the window is part of the parabola with equation  $y = 2x - \frac{1}{2}x^2$ .

Find the area in square metres of the glass in one window.



8

[END OF QUESTION PAPER]