[C100/SQP328]

Mathematics
Higher
Paper 1

Specimen Question Paper
Example 2 based on 2004 Examination Paper
(for examinations from Diet 2008 onwards)

NATIONAL QUALIFICATIONS

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1–20 (40 marks)

Instructions for completion of **Section A** are given on page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks)

1 Full credit will be given only where the solution contains appropriate working.

2 Answers obtained by readings from scale drawings will not receive any credit.

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher (Section A)**.

2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.

3 Check that the answer sheet you have been given has **your name**, **date of birth**, **SCN** (Scottish Candidate Number) and **Centre Name** printed on it.

Do not change any of these details.

4 If any of this information is wrong, tell the Invigilator immediately.

5 If this information is correct, **print** your name and seat number in the boxes provided.

6 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below).

7 There is **only one correct** answer to each question.

8 Rough working should **not** be done on your answer sheet.

9 At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.

Sample Question

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where $x = 2$?

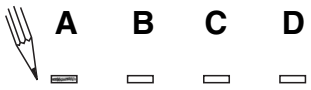
A 8

B 1

C 0

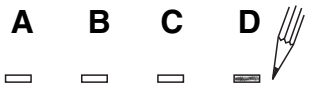
D −4

The correct answer is **A—**8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).



Changing an answer

If you decide to change your answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

SECTION A

ALL questions should be attempted.

1. The line through P(7, p) and Q(4, –5) has a gradient of 3.
What is the value of p?

A –14

B 4

C 6

D 8

2. A sequence is defined by the recurrence relation $u_{n+1} = u_n + 5,\ u_0 = -3$.
What is the value of u_2?

A 3
B 5
C 7
D 9

3. What is the gradient of the line perpendicular to the line with equation $3y = -2x + 1$?

A –3

B 1

C $\frac{3}{2}$

D 5

4. $f(x) = x^3 - x^2 - 5x - 3$.
What is the remainder when $f(x)$ is divided by $(x + 3)$?

A –24
B –3
C 36
D 48

5. If $x^2 - 16x + 27$ is written in the form $(x + p)^2 + q$, find the value of q.

A -37

B 11

C 27

D 43

6. What is the derivative of $(8 - 2x^2)^{\frac{2}{3}}$?

A $-\frac{8}{3}x(8 - 2x^2)^{-\frac{1}{3}}$

B $(8 - 4x)^{\frac{2}{3}}$

C $\frac{2}{3}(8 - 4x)^{-\frac{1}{3}}$

D $\frac{3}{5}(8 - 2x^2)^{\frac{5}{3}}$

7. On dividing $f(x)$ by $(x - 1)$, the remainder is zero and the quotient is $x^2 - 4x - 5$.
Find $f(x)$ in its fully factorised form.

A $(x - 1)(x - 1)(x + 5)$

B $(x + 1)(x - 5)$

C $(x - 1)(x - 1)$

D $(x - 1)(x + 1)(x - 5)$

8. A sequence is generated by the recurrence relation $u_{n+1} = 0{\cdot}4u_n + 3$.
What is the limit of this sequence as $n \to \infty$?

A $\frac{1}{5}$

B $\frac{15}{7}$

C 5

D $\frac{15}{2}$

9. Find all the values of x in the interval $0 < x < 2\pi$ for which $\tan x = -\sqrt{3}$.

A $\frac{5\pi}{6}, \frac{11\pi}{6}$

B $\frac{2\pi}{3}, \frac{4\pi}{3}$

C $\frac{2\pi}{3}, \frac{5\pi}{3}$

D $\frac{5\pi}{3}, \frac{7\pi}{3}$

10. P = (–3, 4, 7), Q = (–1, 8, 3) and R = (0, 10, 1).

Find the ratio in which Q divides PR.

A 2 : 1

B 3 : –1

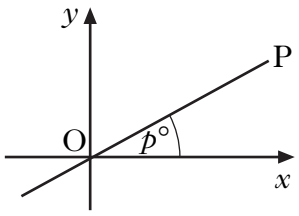
C 1 : 2

D 3 : 1

11. The diagram shows the line OP with equation $2y = x$.

The angle between OP and the positive direction of the x-axis is $p°$.

Find an expression for angle p.



A $\tan^{-1}\frac{1}{2}$

B $\tan^{-1}1$

C $\tan^{-1}2$

D $-\tan^{-1}\frac{1}{2}$

12. Which one of the following is true for the function g where $g'(x) = x^2 + 2x + 1$?

A g is never increasing.

B g is decreasing then increasing.

C g is increasing then decreasing.

D g is never decreasing.

13. Simplify $\log_2(x + 1) - 2\log_2 3$.

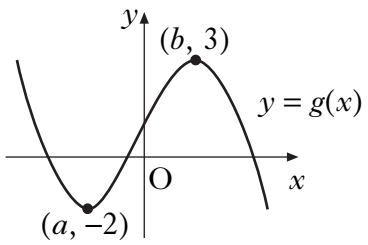
A $\log_2\left(\frac{x+1}{9}\right)$

B $\log_2(x - 8)$

C $\log_2(x - 2)$

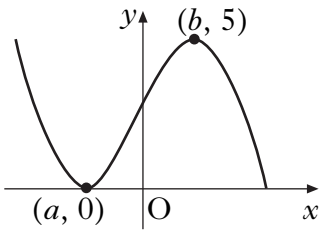
D $\log_2 6(x + 1)$

14. The diagram shows the graph of $y = g(x)$.

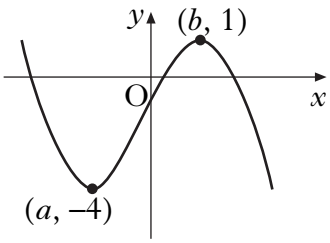


Which diagram below shows the graph of $y = 3 - g(x)$?

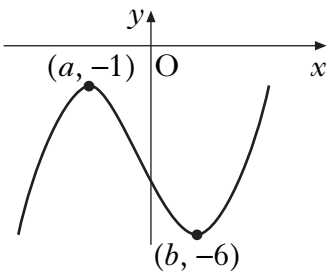
A



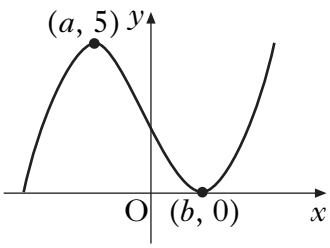
B



C



D



15. Points P and Q have coordinates (1, 3, –1) and (2, 5, 1) and T is the midpoint of PQ. What is the position vector of T?

A $\begin{pmatrix} -\frac{3}{2} \\ -4 \\ 0 \end{pmatrix}$

B $\begin{pmatrix} \frac{3}{2} \\ 4 \\ 0 \end{pmatrix}$

C $\begin{pmatrix} -\frac{1}{2} \\ -1 \\ -1 \end{pmatrix}$

D $\begin{pmatrix} -1 \\ -2 \\ -2 \end{pmatrix}$

16. A = (–3, 4, 7) and B = (–1, 8, 3).
If $\overrightarrow{AD} = 4\overrightarrow{AB}$, what are the coordinates of D?

A (–9, –8, –13)

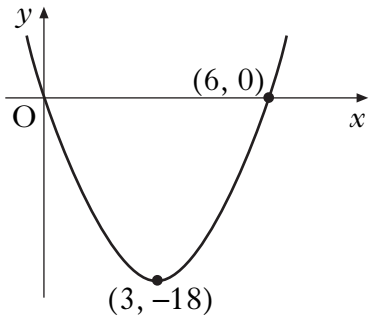
B (5, –4, 1

C (–6, 8, 14)

D (5, 20, –9)

17. The equation of the parabola shown is of the form $y = kx(x - 6)$.

What is the value of k?



A 0

B $\frac{1}{144}$

C 2

D 6

18. Given that $y = 3\cos 5x$, find $\frac{dy}{dx}$.

A $15\cos 5x$

B $-15\sin 5x$

C $-15\cos x$

D $3\cos 5$

19. Find $\int (4x+1)^{\frac{1}{2}}\, dx$.

A $\frac{1}{6}(4x+1)^{\frac{3}{2}} + c$

B $\frac{1}{4}(4x+1) + c$

C $\frac{1}{4}(4x+1)^{\frac{3}{2}} + c$

D $2(4x+1)^{-\frac{3}{2}} + c$

20. Given that $\int (3x+1)^{-\frac{1}{2}}\,dx = \frac{2}{3}(3x+1)^{\frac{1}{2}} + c$, find $\int_0^1 (3x+1)^{-\frac{1}{2}}\,dx$.

A $\frac{2}{3}$

B $\frac{4}{3}$

C 2

D $\sqrt{2}$

[*END OF SECTION A*]

SECTION B

ALL questions should be attempted.

Marks

21. (*a*) Find the stationary points on the curve with equation $y = x^3 + 3x^2 - 9x + 5$ and justify their nature. **7**

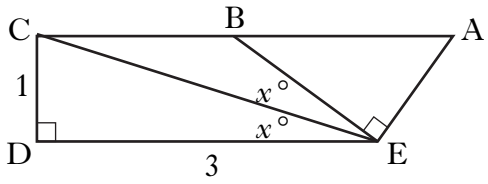
(*b*) The curve passes through the point (–5, 0). Sketch the curve. **2**

22. Solve the equation $\log_x 8 + \log_x 4 = 5$. **4**

23. Solve the equation $\sin 2x - \cos x = 0$ for $0 \leq x \leq 2\pi$. **5**

24. In the diagram,

angle DEC = angle CEB = $x°$ and
angle CDE = angle BEA = 90°.
CD = 1 unit; DE = 3 units.



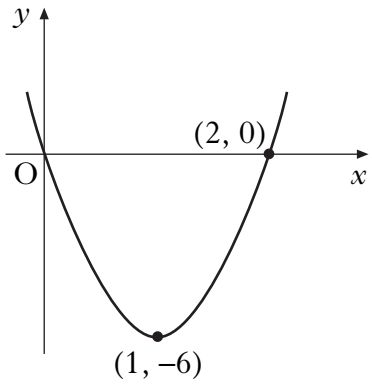
By writing angle DEA in terms of $x°$, find the exact value of cos (DÊA). **7**

25. The diagram shows a parabola with equation

$$y = 6x(x - 2).$$

This parabola is the graph of $y = f'(x)$.

Given that $f(1) = 4$, find the formula for $f(x)$. **5**



[*END OF SECTION B*]

[*END OF QUESTION PAPER*]