Mathematics

Higher

Paper 2

Specimen Question Paper

(for examinations from Diet 2008 onwards)

NATIONAL
QUALIFICATIONS

Read Carefully

1 **Calculators may be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

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FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\quad \mathbf{a}.\mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \cos \theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

$\qquad$ or $\quad \mathbf{a}.\mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:
$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

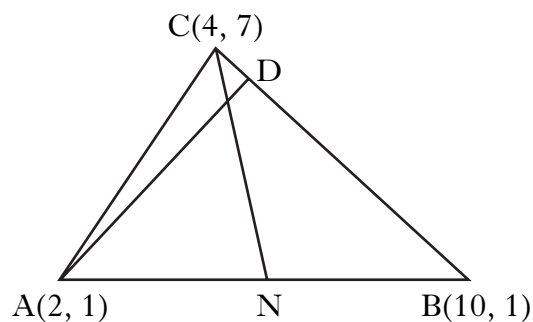| $f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\dfrac{1}{a} \cos ax + C$
$\cos ax$	$\dfrac{1}{a} \sin ax + C$

ALL questions should be attempted.

1. Triangle ABC has coordinates A(2, 1), B(10, 1) and C(4, 7).

    (*a*) Find the equation of the median CN. **3**

    (*b*) Find the equation of the altitude AD. **3**

    (*c*) The median from (*a*) and the altitude from (*b*) intersect at P. Find the coordinates of P. **3**

    (*d*) The point Q lies on AB and has coordinates (8, 1).

    Show that PQ is parallel to BC. **2**
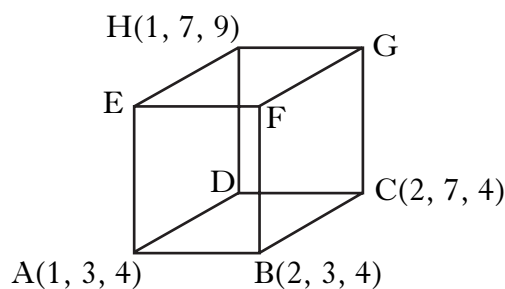
C(4, 7)

D

A(2, 1)   N   B(10, 1)

2. The diagram shows a wire framework in the shape of a cuboid with the edges parallel to the axes.

Relative to these axes, A, B, C and H have coordinates (1, 3, 4), (2, 3, 4), (2, 7, 4) and (1, 7, 9) respectively.

    (*a*) State the lengths of AB, AD and AE. **1**

    (*b*) Write down the components of $\overrightarrow{HB}$ and $\overrightarrow{HC}$ and hence or otherwise calculate the size of angle BHC. **7**

H(1, 7, 9)   G

E   F

D   C(2, 7, 4)

A(1, 3, 4)   B(2, 3, 4)

3. (*a*) Express $5\sin x° - 12\cos x°$ in the form $k\sin(x - a)°$ where $k > 0$ and $0 < a < 360$. **4**

    (*b*) Hence solve the equation $5\sin x° - 12\cos x° = 6 \cdot 5$ in the interval $0 < x < 360$. **3**

4. The diagram shows a parabola with equation $y = 2x^2 - 2x + 3$.
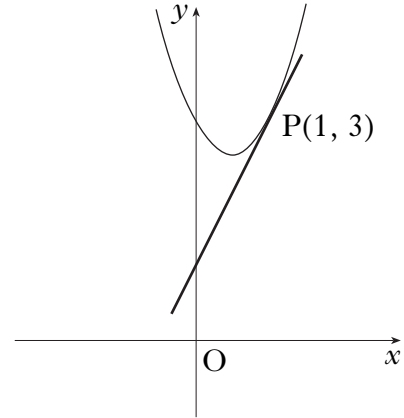
A tangent to the parabola has been drawn at P(1, 3).

(*a*) Find the equation of this tangent. **4**

A circle has equation $x^2 + y^2 + 8y + 11 = 0$.

(*b*) Show that the line from (*a*) is also a tangent to this circle and state the coordinates of the point of contact Q. **6**

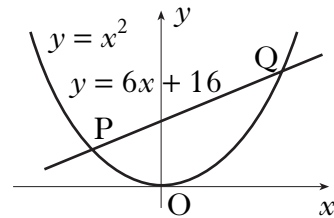(*c*) Determine the ratio in which the *y*-axis cuts the line QP. **3**

5. The diagram shows a curve with equation $y = x^2$ and a straight line with equation $y = 6x + 16$ intersecting the curve at P and Q.

(*a*) Calculate the exact value of the area enclosed by the curve and the straight line. **7**
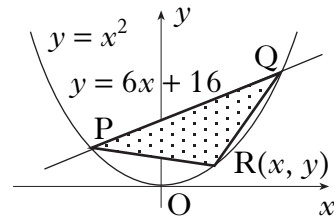
The second diagram shows a third point, R, lying on the curve between P and Q.

(*b*) The area, A, of triangle PQR, is given by $A(x) = -5x^2 + 30x + 80$.

Determine the maximum area of this triangle, and express your answer as a fraction of the area enclosed by the curve and the straight line. **4**

6. Radium decays exponentially and its half-life is 1600 years.

If A_0 represents the amount of radium in a sample to start with and $A(t)$ represents the amount remaining after *t* years, then $A(t) = A_0 e^{-kt}$.

(*a*) Determine the value of *k*, correct to 4 significant figures. **3**

(*b*) Hence find what percentage, to the nearest whole number, of the original amount of radium will be remaining after 3200 years. **2**
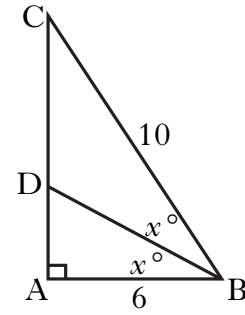
7. Triangle ABC is right-angled at A and BD is the bisector of angle ABC.

AB = 6 units and CB = 10 units.

Determine the exact value of BD, expressing your answer in its simplest form.

5

[*END OF QUESTION PAPER*]