# [C100/SQP321]

Mathematics Higher Paper 1 Specimen Question Paper (for examinations from Diet 2008 onwards)

# NATIONAL QUALIFICATIONS

#### **Read carefully**

Calculators may <u>NOT</u> be used in this paper.

#### Section A – Questions 1–20 (40 marks)

Instructions for completion of **Section A** are given on page two. For this section of the examination you must use an **HB pencil**.

#### Section B (30 marks)

- 1 Full credit will be given only where the solution contains appropriate working.
- 2 Answers obtained by readings from scale drawings will not receive any credit.



# **Read carefully**

- 1 Check that the answer sheet provided is for Mathematics Higher (Section A).
- 2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.
- Check that the answer sheet you have been given has your name, date of birth, SCN (Scottish Candidate Number) and Centre Name printed on it.
   Do not change any of these details.
- 4 If any of this information is wrong, tell the Invigilator immediately.
- 5 If this information is correct, **print** your name and seat number in the boxes provided.
- 6 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below).
- 7 There is **only one correct** answer to each question.
- 8 Rough working should **not** be done on your answer sheet.
- 9 At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.

## Sample Question

A curve has equation  $y = x^3 - 4x$ .

What is the gradient at the point where x = 2?

A 8
B 1
C 0
D -4

The correct answer is **A**—8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).



#### Changing an answer

If you decide to change your answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.

#### FORMULAE LIST

# **Circle:**

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre (-g, -f) and radius  $\sqrt{g^2 + f^2 - c}$ . The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre (a, b) and radius r.

**Scalar Product:**  $a.b = |a| |b| \cos \theta$ , where  $\theta$  is the angle between a and b

or 
$$\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$$
 where  $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

Trigonometric formulae:

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

f(x)	f'(x)
sin ax	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

f(x)	$\int f(x) dx$
sin ax	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

#### SECTION A

#### ALL questions should be attempted.

- 1. If  $y = \frac{x^3 x}{x^2}$ , what is  $\frac{dy}{dx}$ ? A  $\frac{3x^2 - 1}{2x}$ B  $1 + \frac{1}{x^2}$ C  $\frac{3}{2}x - \frac{1}{2}$ D  $x^3 - x - x^{-2}$
- 2. Functions f and g are given by f(x) = 2x 3 and  $g(x) = x^2$ .
  - Find an expression for g(f(x)).
  - A  $g(f(x)) = 4x^2 12x + 9$
  - B  $g(f(x)) = x^2 + 2x 3$
  - $C \quad g(f(x)) = 4x 9$
  - $D \quad g(f(x)) = 2x^3 3x^2$
  - 3. Find  $\int \frac{1}{\sqrt[3]{x}} dx.$ A  $-\frac{3}{2}x^{-\frac{1}{2}} + c$ B  $x^{-3} + c$ C  $\frac{3}{2}x^{\frac{2}{3}} + c$ D  $-2x^{-2} + c$

- **4.** A and B have coordinates (2, 3, -2) and (-1, -4, 0) respectively. What is the distance between A and B?
  - A $\sqrt{6}$ B $\sqrt{17}$ C $\sqrt{62}$
  - D  $\sqrt{148}$
- 5. A sequence is defined by the recurrence relation

 $u_{n+1} = 3u_n - 4, u_0 = -1.$ 

What is the value of  $u_2$ ?

- A –25
- B –10
- C -4
- D –1

6. The diagram shows a sketch of y = f(x). Which of the diagrams below shows a sketch of y = -3 - f(x)?





- 7. Which of the following describes the stationary point on the curve with equation  $y = 3(x-4)^2 5$ ?
  - A minimum at (4, 5)
  - B maximum at (4, 5)
  - C minimum at (4, -5)
  - D maximum at (4, -5)

8. The diagram shows a right-angled triangle with sides of 1,  $2\sqrt{2}$  and 3.

What is the value of  $\sin 2x^{\circ}$ ?



9. *a* and *b* are angles as shown in the diagram. What is the value of sin(a - b)?



 $2\sqrt{2}$ 

1

A 
$$-\frac{7}{\sqrt{85}}$$
  
B  $\frac{2}{\sqrt{85}}$   
C  $\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{17}}$   
D  $\frac{1}{\sqrt{5}} - \frac{1}{\sqrt{17}}$ 

- 10. A circle has equation  $x^2 + y^2 + 8x 6y 12 = 0$ . What is the radius of this circle?
  - A  $\sqrt{2}$
  - B  $\sqrt{19}$
  - C  $\sqrt{37}$
  - D  $\sqrt{88}$
- 11. The points P(1, 3, 7), Q(5, 13, 13) and R(s, 33, 25) are collinear as shown in the diagram.

What is the value of s?

- A 9
- **B** 10
- C 13
- D 31

12. If  $2x^2 - 12x + 11$  is expressed in the form  $2(x - b)^2 + c$ , what is the value of c?

- A –25
- В —7
- C 11
- D 23
- **13.** The curve y = f(x) is such that  $\frac{dy}{dx} = 3x^2 + 9x + 1$  and the curve passes through the origin. What is the equation of the curve?

R

Q

Ρ

A  $y = x^{3} + \frac{9}{2}x^{2} + x$ B  $y = 6x^{3} + 9x^{2}$ C  $y = 3x^{3} + 9x^{2} + x + 1$ D y = 6x + 9 14. For what value of k does the equation  $x^2 - 3x + k = 0$  have equal roots?

$$A -\frac{9}{4}$$
$$B -\frac{1}{12}$$
$$C 0$$
$$D -\frac{9}{4}$$

**15.** The point P(-1, 2) lies on the circle with equation  $x^2 + y^2 - 6x - 8y + 5 = 0$ . What is the gradient of the tangent at P?

- A -2 $B -\frac{1}{3}$  $C \frac{6}{7}$  $D \frac{1}{2}$
- **16.** What is the value of  $\int_{0}^{\frac{\pi}{6}} 4\cos 2x \, dx$ ?

A 
$$-2$$
  
B  $-\sqrt{\frac{3}{2}}$   
C  $\sqrt{3}$   
D 4

17. The graph shown in the diagram has equation of the form  $y = \sin(px) + q$ .

What are the values of p and q?



а

b

	Þ	q
А	2	1
В	$\frac{1}{2}$	1
С	2	2
D	$\frac{1}{2}$	2

18. The vectors *a*, *b* and *c* are represented by the sides of a right-angled triangle as shown in the diagram.

|a| = 3 and |c| = 5.

Here are two statements about these vectors:

(1) 
$$a.c = 9$$
  
(2)  $a.b = -1$ 

Which of the following is true?

- A neither statement is correct
- B only statement (1) is correct
- C only statement (2) is correct
- D both statements are correct

19. If 
$$\log_3 t = 2 + \log_3 5$$
, what is the value of t?

- A 7
- B 10
- C 25
- D 45

**20.** If  $3^k = e^4$ , find an expression for *k*.

- A  $k = \sqrt[3]{4^e}$ B  $k = \frac{e^4}{3}$
- C  $k = 4 / \log_e 3$
- D  $k = 1 / \log_e 3$

[END OF SECTION A]

#### SECTION B

#### ALL questions should be attempted.

- 21. A firm cleans the factory floor on a daily basis with disinfectant. It has a choice of two products, either "A" or "B". Product A removes 70% of all germs but during the next 24 hours, 300 "new" germs per sq unit are estimated to appear. Product B removes 80% of all germs but during the next 24 hours, 350 "new" germs per sq unit are estimated to appear. For product A, let  $u_n$  represent the number of germs per sq unit on the floor immediately before disinfecting for the nth time. For product B, let  $v_n$  represent the number of germs per sq unit on the floor immediately before disinfecting for the nth time. (a) Write down a recurrence relation for each product to show the number of germs per sq unit present prior to disinfecting. 2 4 (b) Determine which product is more effective in the long term. 22. (a) Find the stationary points on the curve with equation  $y = x^3 - 9x^2 + 24x - 20$  and justify their nature. 7 (b) (i) Show that  $(x-2)^2(x-5) = x^3 - 9x^2 + 24x - 20$ . (ii) Hence sketch the graph of  $y = x^3 - 9x^2 + 24x - 20$ . 4
  - 23. The diagram shows a sketch of functions f and g where  $f(x) = x^3 + 5x^2 36x + 32$  and  $g(x) = -x^2 + x + 2$ .

The two graphs intersect at the points A, B and C.

Determine the *x*-coordinate of each of these three points.



**24.** Find the solution(s) of the equation  $\sin^2 p - \sin p + 1 = \cos^2 p$  for  $\frac{\pi}{2} .$ 

#### 5

8

#### [END OF SECTION B]

[END OF QUESTION PAPER]

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