**[C100/SQP248]**

# Higher Mathematics

Time: 1 hour 30 minutes

NATIONAL QUALIFICATIONS

Units 1, 2 and 3
Paper 2
Specimen Question Paper **(Revised)**
for use in and after 2004

**Read Carefully**

1 **Calculators may be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

SCOTTISH QUALIFICATIONS AUTHORITY

**FORMULAE LIST**

**Circle:**

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre $(a, b)$ and radius $r$.

**Scalar Product:** $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where $\theta$ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

**Trigonometric formulae:**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

**Table of standard derivatives:**

| $f(x)$ | $f'(x)$ |
|---|---|
| $\sin ax$ | $a\cos ax$ |
| $\cos ax$ | $-a\sin ax$ |

**Table of standard integrals:**

| $f(x)$ | $\int f(x)dx$ |
|---|---|
| $\sin ax$ | $-\frac{1}{a}\cos ax + C$ |
| $\cos ax$ | $\frac{1}{a}\sin ax + C$ |

**All questions should be attempted.** *Marks*

**1.** ABCD is a parallelogram. A, B and C have coordinates (2, 3), (4, 7) and (8, 11). Find the equation of DC. **(4)**

**2.** Trees are sprayed weekly with the pesticide, "Killpest", whose manufacturers claim it will destroy 60% of all pests. Between the weekly sprayings, it is estimated that 300 new pests invade the trees.

A new pesticide, "Pestkill", comes onto the market. The manufacturers claim that it will destroy 80% of existing pests but it is estimated that 360 new pests per week will invade the trees.

Which pesticide will be more effective in the long term? **(6)**

**3.** (*a*) Show that the function $f(x) = 2x^2 + 8x - 3$ can be written in the form $f(x) = a(x + b)^2 + c$ where $a$, $b$ and $c$ are constants. **(3)**

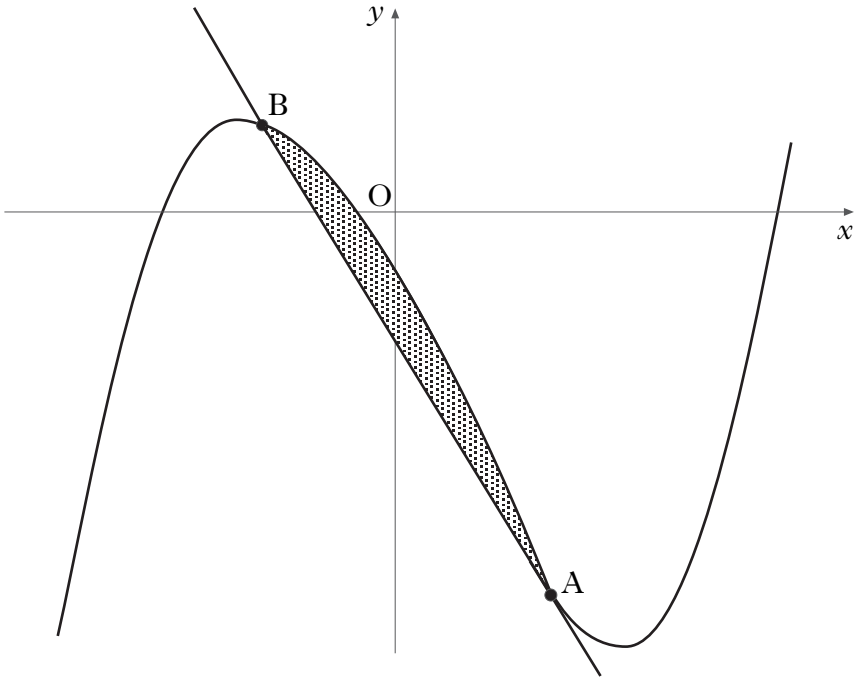
(*b*) Hence, or otherwise, find the coordinates of the turning point of the function $f$. **(1)**

**4.** In the diagram below, a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, −8).

(*a*) Find the equation of the tangent at A. **(4)**

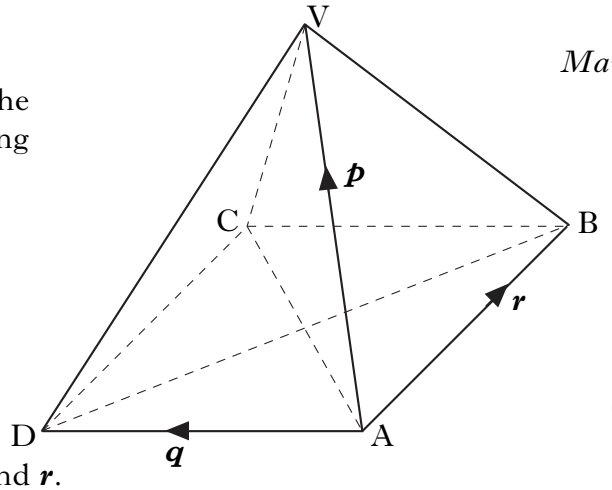
(*b*) Hence find the coordinates of B. **(5)**

(*c*) Find the area of the shaded part which represents the land bounded by the river and the road. **(5)**



*Marks*

**5.** VABCD is a square-based pyramid. The length of AD is 3 units and each sloping face is an equilateral triangle.




$\overrightarrow{AV} = \boldsymbol{p}$, $\overrightarrow{AD} = \boldsymbol{q}$ and $\overrightarrow{AB} = \boldsymbol{r}$.

(*a*) (i) Evaluate $\boldsymbol{p}.\boldsymbol{q}$.

(ii) Hence evaluate $\boldsymbol{p}.(\boldsymbol{q} + \boldsymbol{r})$. **(4)**

(*b*) (i) Express $\overrightarrow{CV}$ in terms of $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$.

(ii) Hence show that angle CVA is 90°. **(4)**

**6.** $f(x) = 2\cos x° + 3\sin x°$.

(*a*) Express $f(x)$ in the form $k\cos(x - \alpha)°$ where $k > 0$ and $0 \leq \alpha < 360$. **(4)**

(*b*) Hence solve $f(x) = 0{\cdot}5$ for $0 \leq x < 360$. **(3)**

(*c*) Find the $x$-coordinate of the point nearest to the origin where the graph of $f(x) = 2\cos x° + 3\sin x°$ cuts the $x$-axis for $0 \leq x < 360$. **(2)**

**7.** (*a*) Show that $2\cos 2x° - \cos^2 x° = 1 - 3\sin^2 x°$. **(3)**

(*b*) **Hence**

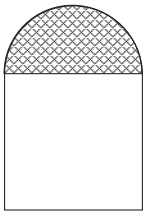
(i) write the equation $2\cos 2x° - \cos^2 x° = 2\sin x°$ in terms of $\sin x°$

(ii) solve this equation in the interval $0 \leq x < 90$. **(4)**

**8.** The roots of the equation $(x - 1)(x + k) = -4$ are equal.
Find the values of $k$. **(5)**

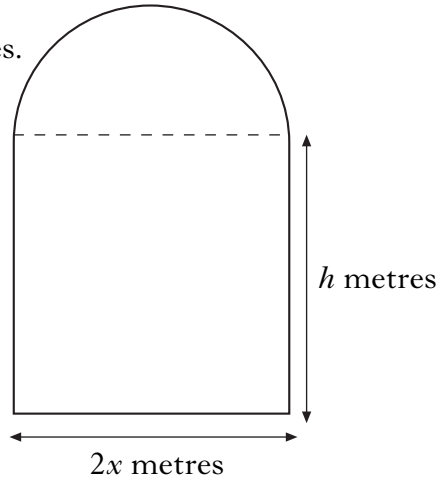
*Marks*

**9.** A window in the shape of a rectangle surmounted by a semicircle is being designed to let in the maximum amount of light.



The glass to be used for the semicircular part is stained glass which lets in one unit of light per square metre; the rectangular part uses clear glass which lets in 2 units of light per square metre.

The rectangle measures $2x$ metres by $h$ metres.




(*a*) (i) If the perimeter of the whole window is 10 metres, express $h$ in terms of $x$. **(2)**

(ii) Hence show that the amount of light, $L$, let in by the window is given by $L = 20x - 4x^2 - \frac{3}{2}\pi x^2$. **(2)**

(*b*) Find the values of $x$ and $h$ that must be used to allow this design to let in the maximum amount of light. **(6)**

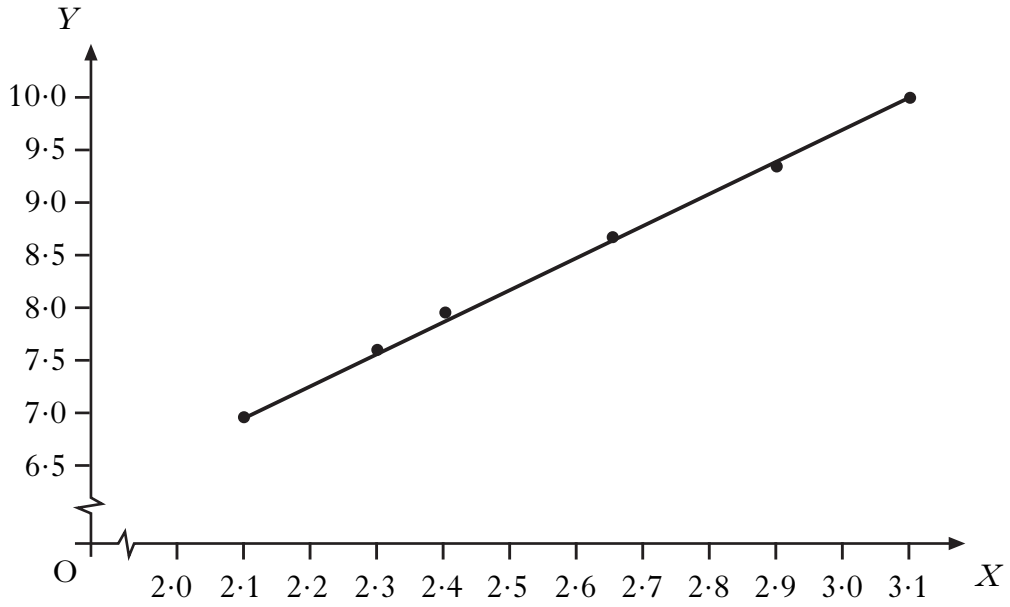
*Marks*

**10.** Six spherical sponges were dipped in water and weighed to see how much water each could absorb. The diameter ($x$ millimetres) and the gain in weight ($y$ grams) were measured and recorded for each sponge. It is thought that $x$ and $y$ are connected by a relationship of the form $y = ax^b$.

By taking logarithms of the values of $x$ and $y$, the table below was constructed.

| | | | | | | |
|---|---|---|---|---|---|---|
| $X\ (= \log_e x)$ | 2·10 | 2·31 | 2·40 | 2·65 | 2·90 | 3·10 |
| $Y\ (= \log_e y)$ | 7·00 | 7·60 | 7·92 | 8·70 | 9·38 | 10·00 |

A graph was drawn and is shown below.




Find the equation of the line in the form $Y = mX + c$. **(3)**

[*END OF SPECIMEN QUESTION PAPER*]