Paper 2

Question	Generic Scheme	Illustrative Scheme	Max Mark
1(a)			
$\bullet^1$ calculate gradient of AB $\bullet^2$ use property of perpendicular lines $\bullet^3$ substitute into general equation of a line $\bullet^4$ demonstrate result		$\bullet^1$ $m_{AB}=-3$ $\bullet^2$ $m_{alt}=\frac{1}{3}$ $\bullet^3$ $y-3=\frac{1}{3}(x-13)$ $\bullet^4$ $\ldots\Rightarrow x-3y=4$	**4**

Notes:

1. $\bullet^3$ is only available as a consequence of trying to find and use a perpendicular gradient.
2. $\bullet^4$ is only available if there is/are appropriate intermediate lines of working between $\bullet^3$ and $\bullet^4$.
3. The ONLY acceptable variations for the final equation for the line in $\bullet^4$ are $4=x-3y$, $-3y+x=4$, $4=-3y+x$.

Commonly Observed Responses:

Candidate A

$m_{AB}=\frac{-1-(-5)}{-5-7}=\frac{4}{-12}=-\frac{1}{3}$

$m_{alt}=3$

$y-3=3(x-13)$

$\bullet^4$ is not available

$\bullet^1$ ×

$\bullet^2$ ✓1

$\bullet^3$ ✓1

$\bullet^4$ ×

Candidate B

For $\bullet^4$

$y-3=\frac{1}{3}x-\frac{13}{3}$

$y=\frac{1}{3}x-\frac{4}{3}$

$3y=x-4$ - not acceptable

$3y-x=-4$ - not acceptable

$x-3y=4$ ✓

Question	Generic Scheme	Illustrative Scheme	Max Mark
1(b)			
• [5] calculate midpoint of AC • [6] calculate gradient of median • [7] determine equation of median		• [5] $\mathrm{M}_{\mathrm{AC}} = (4,5)$ • [6] $m_{BM} = 2$ • [7] $y = 2x - 3$	**3**
Notes:			
4. • [6] and • [7] are not available to candidates who do not use a midpoint. 5. • [7] is only available as a consequence of using a non-perpendicular gradient and a midpoint. 6. Candidates who find either the median through A or the median through C or a side of the triangle gain 1 mark out of 3. 7. At • [7] accept $y-(-5)=2(x-(-1))$, $y-5=2(x-4)$, $y-2x+3=0$ or any other rearrangement of the equation.			
Commonly Observed Responses:			
Median through A $\mathrm{M}_{BC} = (6,-1)$ $m_{AM} = \frac{-8}{11}$ $y+1=\frac{-8}{11}(x-6)$ or $y-7=\frac{-8}{11}(x+5)$ **Award 1/3**		**Median through C** $\mathrm{M}_{AB} = (-3,1)$ $m_{CM} = \frac{1}{8}$ $y-3=\frac{1}{8}(x-13)$ or $y-1=\frac{1}{8}(x+3)$ **Award 1/3**	
1(c)			
• [8] calculate x or y coordinate • [9] calculate remaining coordinate of the point of intersection		• [8] $x=1$ or $y=-1$ • [9] $y=-1$ or $x=1$	**2**
Notes:			
8. If the candidate's 'median' is either a vertical or horizontal line then award 1 out of 2 if both coordinates are correct, otherwise award 0.			
Commonly Observed Responses:			
For candidates who find the altitude through B in part (b) $x=-\frac{1}{5}$ • [8] ✓1 $y=-\frac{7}{5}$ • [9] ✓1		**Candidate A** (b) $y-5=2(x-4)$ • [7] ✓ $y=2x-13$ -error (c) $x-3y=4$ $y=2x-13$ Leading to $x=7$ and $y=1$ • [8] × • [9] ✓1	

Question	Generic Scheme	Illustrative Scheme	Max Mark
2 (a)			
$\bullet^1$ interpret notation $\bullet^2$ state a correct expression		$\bullet^1$ $f((1+x)(3-x)+2)$ stated or implied by $\bullet^2$ $\bullet^2$ $10+(1+x)(3-x)+2$ stated or implied by $\bullet^3$	**2**

Notes:

1. $\bullet^1$ is not available for $g(f(x))=g(10+x)$ but $\bullet^2$ may be awarded for $(1+10+x)(3-(10+x))+2$.

Commonly Observed Responses:

Candidate A

(a)
$f(g(x))='g(f(x))'$
$=(1+10+x)(3-(10+x))+2$ $\bullet^1$ ×, $\bullet^2$ ✓1

(b)
$=-75-18x-x^2$ or $-x^2-18x-75$ $\bullet^3$ ✓1
$=-(x^2+18x$ $\bullet^4$ ✓1
$=-(x+9)^2$ $\bullet^5$ ✓1
$=-(x+9)^2+6$

(c)
$-(x+9)^2+6=0$ $\bullet^6$ ✓1
$x=-9+\sqrt{6}$ or $-9-\sqrt{6}$ $\bullet^7$ ✓1

Candidate B

$f(g(x))$ $\bullet^1$ ∧
$=10((1+x)-(3-x))+2$ $\bullet^2$ ×

Candidate C

$f(g(x))$ $\bullet^1$ ∧
$=10((1+x)(3-x)+2)$ $\bullet^2$ ×

2 (b)			
$\bullet^3$ write $f(g(x))$ in quadratic form **Method 1** $\bullet^4$ identify common factor $\bullet^5$ complete the square **Method 2** $\bullet^4$ expand completed square form and equate coefficients $\bullet^5$ process for q and r and write in required form		$\bullet^3$ $15+2x-x^2$ or $-x^2+2x+15$ **Method 1** $\bullet^4$ $-1(x^2-2x$ stated or implied by $\bullet^5$ $\bullet^5$ $-1(x-1)^2+16$ **Method 2** $\bullet^4$ $px^2+2pqx+pq^2+r$ and $p=-1$, $\bullet^5$ $q=-1$ and $r=16$ Note if $p=1$ $\bullet^5$ is not available	**3**

Notes:

2. Accept $16-(x-1)^2$ or $-\left[(x-1)^2-16\right]$ at $\bullet^5$.

Commonly Observed Responses:

Candidate A

$-(x^2-2x-15)$ $\bullet^4$ ✓

$-(x^2-2x+1-1-15)$

$-(x-1)^2-16$ $\bullet^5$ ×

Candidate B

$15+2x-x^2$ $\bullet^3$ ✓

$x^2-2x-15$ $\bullet^4$ ×

$px^2+2pqx+pq^2+r$ and $p=1$

$q=-1 \quad r=-16$ $\bullet^5$ ✓2 eased

Candidate C

$-x^2+2x+15$ $\bullet^3$ ✓

$-(x+1)^2\ldots$ $\bullet^4$ ×

$-(x+1)^2+14$ $\bullet^5$ ×

Candidate D

$15+2x-x^2$ $\bullet^3$ ✓

$x^2-2x-15$ $\bullet^4$ ×

$(x-1)^2-16$ $\bullet^5$ ✓2 eased

Eased, unitary coefficient of x^2 (lower level skill)

Candidate E

$15+2x-x^2$ $\bullet^3$ ✓

$x^2-2x-15$ $\bullet^4$ ✓

$(x-1)^2-16$

so $15+2x-x^2=-(x-1)^2+16$ $\bullet^5$ ✓

Candidate F

$-x^2+2x+15$ $\bullet^3$ ✓

$-(x+1)^2\ldots$ $\bullet^4$ ×

$-(x+1)^2+16$ $\bullet^5$ ✓1

2(c)

$\bullet^6$ identify critical condition	$\bullet^6$ $-1(x-1)^2+16=0$ or $f((g(x))=0$	
$\bullet^7$ identify critical values	$\bullet^7$ 5 and -3	**2**

Notes:

3. Any communication indicating that the denominator cannot be zero gains $\bullet^6$.

4. Accept $x=5$ and $x=-3$ or $x\neq 5$ and $x\neq -3$ at $\bullet^7$.

5. If $x=5$ and $x=-3$ appear without working award 1/2.

Commonly Observed Responses:

Candidate A

$\dfrac{1}{-(x-1)^2+16}$ $\bullet^6$ ✓

$x\neq 5$ $\bullet^7$ ^

Candidate B

$\dfrac{1}{f(g(x))}$

$f(g(x))>0$ $\bullet^6$ ×

$x=-3,\ x=5$ $\bullet^7$ ✓

$-3<x \quad x<5$

3(a)

$\bullet^1$ determine the value of the required term	$\bullet^1$ $22\frac{3}{4}$ or $\frac{91}{4}$ or $22\cdot 75$	**1**

Notes:

1. Do not penalise the inclusion of incorrect units.
2. Accept rounded and unsimplified answers following evidence of correct substitution.

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
3 (b)			
Method 1 **(Considering both limits)** •^{2} know how to calculate limit •^{3} know how to calculate limit •^{4} calculate limit •^{5} calculate limit •^{6} interpret limits and state conclusion **Method 2** **(Frog first then numerical for toad)** •^{2} know how to calculate limit •^{3} calculate limit •^{4} determine the value of the highest term less than 50 •^{5} determine the value of the lowest term greater than 50 •^{6} interpret information and state conclusion **Method 3** **(Numerical method for toad only)** •^{2} continues numerical strategy •^{3} exact value •^{4} determine the value of the highest term less than 50 •^{5} determine the value of the lowest term greater than 50 •^{6} interpret information and state conclusion **Method 4** **(Limit method for toad only)** •^{2} & •^{3} know how to calculate limit •^{4} & •^{5} calculate limit • •^{6} interpret limit and state conclusion		**Method 1** •^{2} $\frac{32}{1-\frac{1}{3}}$ or $L=\frac{1}{3}L+32$ •^{3} $\frac{13}{1-\frac{3}{4}}$ or $L=\frac{3}{4}L+13$ •^{4} 48 •^{5} 52 •^{6} $52>50$ $\therefore$ toad will escape **Method 2** •^{2} $\frac{32}{1-\frac{1}{3}}$ or $L=\frac{1}{3}L+32$ •^{3} 48 •^{4} $49\cdot803...$ •^{5} $50\cdot352...$ •^{6} $50\cdot352>50$ $\therefore$ toad will escape **Method 3** •^{2} numerical strategy •^{3} $30\cdot0625$ •^{4} $49\cdot803...$ •^{5} $50\cdot352...$ •^{6} $50\cdot352>50$ $\therefore$ toad will escape **Method 4** •^{2} & •^{3} $\frac{13}{1-\frac{3}{4}}$ or $L=\frac{3}{4}L+13$ •^{4} & •^{5} 52 •^{6} $52>50$ $\therefore$ toad will escape	**5**

Notes:

3. $\bullet^6$ is unavailable for candidates who do not consider the toad in their conclusion.
4. For candidates who only consider the frog numerically award 1/5 for the strategy.

Commonly Observed Responses:

Error with frogs limit - Frog Only

$L_F = \dfrac{34}{1-\frac{1}{3}}$ $\bullet^2$ ×

$\bullet^3$ ×

$\bullet^4$ ✓1

$L_F = 51$ $\bullet^5$ ✓1

$51 > 50$ $\bullet^6$ ✓1

$\therefore$ frog will escape.

Using Method 3 - Toad Only

$\bullet^2$ ✓

$\bullet^3$ ✓

$\bullet^4$ missing ^

$\bullet^5$ $50 \cdot 352...$ ✓

$\bullet^6$ $50.352 > 50$

so the toad escapes. ✓

Using Method 3- Toad Only

$\bullet^2$ ✓

$\bullet^3$ ✓

$\bullet^4$ missing ^

$\bullet^5$ $50 \cdot 1$..rounding error ×

$\bullet^6$ $50.1 > 50$ ✓1

so the toad escapes.

Using Method 3 - Toad Only

$\bullet^2$ ✓

$\bullet^3$ ✓

$\bullet^4$ $49 \cdot 7$..rounding error ×

$\bullet^5$ $50 \cdot 1...$ ✓1

$\bullet^6$ $50.1 > 50$ ✓1

so the toad escapes.

Toad Conclusions

Limit = 52

This is greater than the height of the well and so the toad will escape – award $\bullet^6$.

However

Limit =52 and so the toad escapes - $\bullet^6$ ^.

Iterations

$f_1 = 32$	$t_1 = 13$
$f_2 = 42 \cdot 667$	$t_2 = 22 \cdot 75$
$f_3 = 46 \cdot 222$	$t_3 = 30 \cdot 0625$
$f_4 = 47 \cdot 407$	$t_4 = 35 \cdot 547$
$f_5 = 47 \cdot 802$	$t_5 = 39 \cdot 660$
$f_6 = 47 \cdot 934$	$t_6 = 42 \cdot 745$
$f_7 = 47 \cdot 978$	$t_7 = 45 \cdot 059$
$f_8 = 47 \cdot 993$	$t_8 = 46 \cdot 794$
$f_9 = 47 \cdot 998$	$t_9 = 48 \cdot 096$
	$t_{10} = 49 \cdot 072$
	$t_{11} = 49 \cdot 804$
	$t_{12} = 50 \cdot 353$

Question	Generic Scheme	Illustrative Scheme	Max Mark
4 (a)			
$\bullet^1$ know to equate $f(x)$ and $g(x)$ $\bullet^2$ solve for x		$\bullet^1$ $\frac{1}{4}x^2-\frac{1}{2}x+3=\frac{1}{4}x^2-\frac{3}{2}x+5$ $\bullet^2$ $x=2$	**2**

Notes:

1. $\bullet^1$ and $\bullet^2$ are not available to candidates who: (i) equate zeros, (ii) give answer only without working, (iii) arrive at $x=2$ with erroneous working.

Commonly Observed Responses:

Candidate A

$y=\frac{1}{4}x^2-\frac{1}{2}x+3$

$y=\frac{1}{4}x^2-\frac{3}{2}x+5$ $\bullet^1$ ✓

subtract to get

$0=x-2$

$x=2$ $\bullet^2$ ✓

Candidate B

$\frac{1}{4}x^2-\frac{1}{2}x=-3$

$\frac{1}{4}x^2-\frac{3}{2}x=-5$ $\bullet^1$ ×

$x=2$ $\bullet^2$ ×

In this case the candidate has equated zeros

Candidate C

$f(x)=\frac{1}{4}x^2-\frac{1}{2}x+3$ $\quad$ $g(x)=\frac{1}{4}x^2-\frac{3}{2}x+5$

$f'(x)=\frac{1}{2}x-\frac{1}{2}$ $\quad$ $g'(x)=\frac{1}{2}x-\frac{3}{2}$

. $\quad$.

$x=1$ $\quad$ $x=3$ $\bullet^1$ ✓ $\bullet^2$ ✓

$\therefore x=2$

Question	Generic Scheme	Illustrative Scheme	Max Mark
4 (b)			
• [3] know to integrate • [4] interpret limits • [5] use 'upper – lower' • [6] integrate • [7] substitute limits • [8] evaluate area between $f(x)$ and $h(x)$ • [9] state total area		• [3] $\int$ • [4] $\int_0^2$ • [5] $\int_0^2 (\frac{1}{4}x^2 - \frac{1}{2}x + 3) - (\frac{3}{8}x^2 - \frac{9}{4}x + 3)\,dx$ • [6] $-\frac{1}{24}x^3 + \frac{7}{8}x^2$ accept unsimplified integral • [7] $\left(-\frac{1}{24}\times 2^3 + \frac{7}{8}\times 2^2\right) - 0$ • [8] $\frac{19}{6}$ • [9] $\frac{19}{3}$	**7**

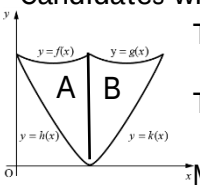
Notes:

2. If limits $x = 0$ and $x = 2$ appear ex nihilo award •[4].
4. If a candidate differentiates at •[6] then •[6], •[7] and •[8] are not available. However, •[9] is still available.
5. Candidates who substitute at •[7], without attempting to integrate at •[6], cannot gain •[6], •[7] or •[8]. However, •[9] is still available.
6. Evidence for •[8] may be implied by •[9].
7. •[9] is a strategy mark and should be awarded for correctly multiplying their solution at •[8], or for any other valid strategy applied to previous working.
8. For •[5] both a term containing a variable and the constant term must be dealt with correctly.
9. In cases where •[5] is not awarded, •[6] may be gained for integrating an expression of equivalent difficulty ie a polynomial of at least degree two. •[6] is unavailable for the integration of a linear expression.
10. •[8] must be as a consequence of substituting into a term where the power of x is not equalto 1 or 0.

Commonly Observed Responses:

Candidate A – Valid Strategy

Candidates who use the strategy:



Total Area = Area A + Area B

Then mark as follows:

Mark Area A for $\bullet^3$ to $\bullet^8$ then mark Area B for $\bullet^3$ to $\bullet^8$ and award the higher of the two.
$\bullet^9$ is available for correctly adding two equal areas.

Candidate B – Invalid Strategy

For example, candidates who integrate each of the four functions separately within an invalid strategy

$\bullet^3$ ✓

Gain $\bullet^4$ if limits correct on

$$\int f(x) \text{ and } \int h(x)$$

or

$$\int g(x) \text{ and } \int k(x)$$

$\bullet^5$ is unavailable

Gain $\bullet^6$ for calculating either

$$\int f(x) \text{ or } \int g(x)$$

and

$$\int h(x) \text{ or } \int k(x)$$

Gain $\bullet^7$ for correctly substituting at least twice
Gain $\bullet^8$ for evaluating at least two integrals correctly
$\bullet^9$ is unavailable

Candidate C

$\int_0^2 (\frac{1}{4}x^2 - \frac{1}{2}x + 3 - \frac{3}{8}x^2 - \frac{9}{4}x + 3)\ dx$

$\int_0^2 (-\frac{1}{8}x^2 - \frac{11}{4}x)\ dx$ $\bullet^5$ ✓

$\frac{-1}{24}x^3 - \frac{11}{8}x^2$ $\bullet^6$ ×

Candidate D

$\int_0^2 (\frac{1}{4}x^2 - \frac{1}{2}x + 3 - \frac{3}{8}x^2 - \frac{9}{4}x + 3)\ dx$

$\int_0^2 (-\frac{1}{8}x^2 - \frac{11}{4}x + 6)\ dx$ $\bullet^5$ ×

$-\frac{1}{24}x^3 - \frac{11}{8}x^2 + 6x$ $\bullet^6$ ✓1

Candidate E

$\int ... = -\frac{1}{3}$ cannot be negative so $= \frac{1}{3}$ $\bullet^8$ ×

however, $= -\frac{1}{3}$ so Area $= \frac{1}{3}$ $\bullet^8$ ✓

Candidate F

$\int_0^2 (\frac{1}{4}x^2 - \frac{1}{2}x + 3 - \frac{3}{8}x^2 - \frac{9}{4}x + 3)\ dx$

$\int_0^2 (-\frac{1}{8}x^2 + \frac{7}{4}x)\ dx$ $\bullet^5$ ✓

$-\frac{1}{24}x^3 + \frac{7}{8}x^2$ $\bullet^6$ ✓

Question	Generic Scheme	Illustrative Scheme	Max Mark
5(a)			
• ^{1} state centre of C_1 • ^{2} state radius of C_1 • ^{3} calculate distance between centres of C_1 and C_2 • ^{4} calculate radius of C_2		• ^{1} $(-3,-5)$ • ^{2} 5 • ^{3} 20 • ^{4} 15	**4**
Notes:			
1. For • ^{4} to be awarded radius of C_2 must be greater than the radius of C_1. 2. Beware of candidates who arrive at the correct solution by finding the point of contact by an invalid strategy. 3. • ^{4} is for $\text{Distance}_{c1c2} - r_{c1}$ but only if the answer obtained is greater than r_{c1}.			
Commonly Observed Responses:			

Question	Generic Scheme	Illustrative Scheme	Max Mark
5 (b)			
$\bullet^5$ find ratio in which centre of C_3 divides line joining centres of C_1 and C_2 $\bullet^6$ determine centre of C_3 $\bullet^7$ calculate radius of C_3 $\bullet^8$ state equation of C_3		$\bullet^5$ $3:1$ $\bullet^6$ $(6,7)$ $\bullet^7$ $r=20$ (answer must be consistent with distance between centres) $\bullet^8$ $(x-6)^2+(y-7)^2=400$	**4**

Notes:

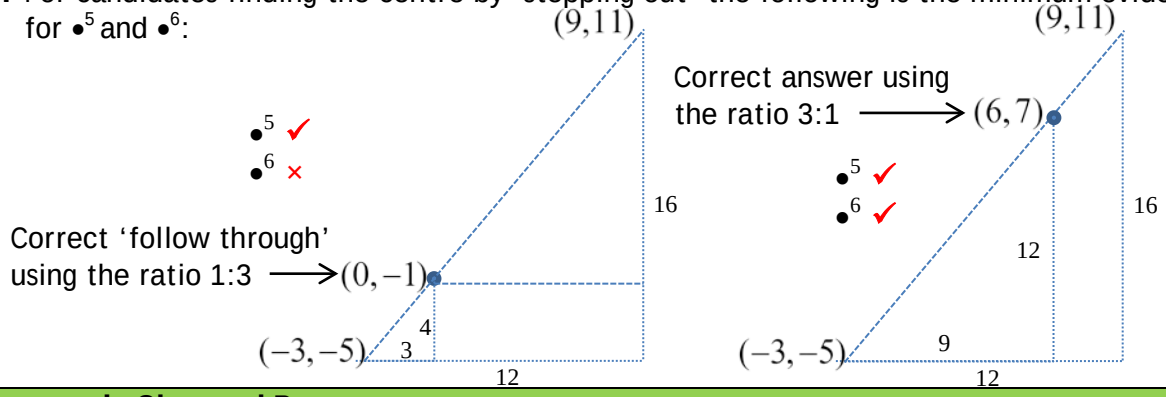
4. For $\bullet^5$ accept ratios $\pm3:\pm1,\ \pm1:\pm3,\ \mp3:\pm1,\ \mp1:\pm3$ (or the appearance of $\frac{3}{4}$).

5. $\bullet^7$ is for $r_{c2}+r_{c1}$.

6. Where candidates arrive at an incorrect centre or radius from working then $\bullet^8$ is available. However $\bullet^8$ is not available if either centre or radius appear ex nihilo (see note 5).

7. Do not accept 20^2 for $\bullet^8$.

8. For candidates finding the centre by 'stepping out' the following is the minimum evidence for $\bullet^5$ and $\bullet^6$:



Commonly Observed Responses:

Candidate A

using the mid-point of centres:

centre $C_3=(3,3)$

radius of $C_3=20$

$(x-3)^2+(y-3)^2=400$

$\bullet^5$ ×
$\bullet^6$ ✓2
$\bullet^7$ ✓
$\bullet^8$ ✓1

Candidate B

$C_1=(-3,-5)$ — 1:3 — $C_2(9,11)$ $\quad r=20$

$C_3=\frac{1}{4}\begin{pmatrix}0\\-4\end{pmatrix}$

$C_3=(0,-1)$

$x^2+(y+1)^2=400$

$\bullet^5$ ✓note 4
$\bullet^6$ ✓2
$\bullet^7$ ✓
$\bullet^8$ ✓1

Candidate C - touches C_1 internally only

$\bullet^5$ ×
$\bullet^6$ centre $C_3=(3,3)$ ×
$\bullet^7$ radius of C_3 = radius of $C_2=15$ ✓1
$\bullet^8(x-3)^2+(y-3)^2=225$ ✓1

Candidate D - touches C_2 internally only

$\bullet^5$ ×
$\bullet^6$ centre $C_3=(3,3)$ ×
$\bullet^7$ radius of C_3 = radius of $C_1=5$ ✓1
$\bullet^8(x-3)^2+(y-3)^2=25$ ✓1

Candidate E - centre C_3 collinear with C_1, C_2

$\bullet^5$ ×
$\bullet^6$ e.g. centre $C_3=(21,27)$ ×
$\bullet^7$ radius of $C_3=45$ (touch C_1 internally only) ✓1
$\bullet^8(x-21)^2+(y-27)^2=2025$ ✓1

Question	Generic Scheme	Illustrative Scheme	Max Mark
6 (a)			
• [1] Expands • [2] Evaluate $\mathbf{p.q}$ • [3] Completes evaluation		• [1] $\mathbf{p.q}+\mathbf{p.r}$ • [2] $4\frac{1}{2}$ • [3] $\ldots+0=4\frac{1}{2}$	**3**
Notes:			
1. For $\mathbf{p}.(\mathbf{q}+\mathbf{r})=\mathbf{pq}+\mathbf{pr}$ with no other working •[1] is not available.			
Commonly Observed Responses:			
6 (b)			
• [4] correct expression		• [4] $\mathbf{-q}+\mathbf{p}+\mathbf{r}$ or equivalent	**1**
6 (c)			
• [5] correct substitution • [6] start evaluation • [7] find expression for $\lvert\mathbf{r}\rvert$		• [5] $\mathbf{-q.q}+\mathbf{q.p}+\mathbf{q.r}$ • [6] $-9+\ldots.+3\lvert\mathbf{r}\rvert\cos 30^{o}=9\sqrt{3}-\frac{9}{2}$ • [7] $\lvert\mathbf{r}\rvert=\frac{3\sqrt{3}}{\cos 30}$	**3**
Notes:			
2. Award •[5] for $\mathbf{-q}^{2}+\mathbf{q.p}+\mathbf{q.r}$			
Commonly Observed Responses:			
Candidate A $\mathbf{-q.q}+\mathbf{q.p}+\mathbf{q.r}=9\sqrt{3}-\frac{9}{2}$ •[5] ✓ $-9+\frac{9}{2}+3\lvert\mathbf{r}\rvert\cos 150^{o}=9\sqrt{3}-\frac{9}{2}$ •[6] × $\lvert\mathbf{r}\rvert=\frac{3\sqrt{3}}{\cos 150}$ •[7] ✓1		**Candidate B** $\mathbf{-q.q}+\mathbf{q.p}+\mathbf{q.r}=9\sqrt{3}-\frac{9}{2}$ •[5] ✓ $-9+\frac{9}{2}+3\lvert\mathbf{r}\rvert\cos 30^{o}=9\sqrt{3}-\frac{9}{2}$ •[6] ✓ $\lvert\mathbf{r}\rvert=6$ •[7] ✓	

Question	Generic Scheme	Illustrative Scheme	Max Mark
7 (a)			
• [1] integrate a term • [2] complete integration with constant		• [1] $\frac{3}{2}\sin 2x$ OR x • [2] $x+c$ OR $\frac{3}{2}\sin 2x+c$	**2**
Notes:			
Commonly Observed Responses:			
7 (b)			
• [3] substitute for cos 2x • [4] substitute for 1 and complete		• [3] $3\left(\cos^2 x-\sin^2 x\right)\ldots$ or $\ldots(\sin^2 x+\cos^2 x)$ • [4] $\ldots\left(\sin^2 x+\cos^2 x\right)=4\cos^2 x-2\sin^2 x$	**2**
Notes:			
1. Any valid substitution for $\cos 2x$ is acceptable for •[3]. 2. Candidates who show that $4\cos^2 x-2\sin^2 x=3\cos 2x+1$ may gain both marks. 3. Candidates who quote the formula for $\cos 2x$ in terms of A but do not use in the context of the question cannot gain •[3].			
Commonly Observed Responses:			
Candidate A $3\cos 2x+1=(2\cos^2 x-1)+(2\cos^2 x-1)+(1-2\sin^2 x)+1$ $=4\cos^2 x-2\sin^2 x$		•[3] ✓ •[4] ✓	
Candidate B $4\cos^2 x-2\sin^2 x=2(\cos 2x+1)-(1-\cos 2x)$ $=3\cos 2x+1$		•[3] ✓ •[4] ✓	
7 (c)			
• [5] interpret link • [6] state result		• [5] $-\frac{1}{2}\int\ldots$ • [6] $-\frac{3}{4}\sin 2x-\frac{1}{2}x+c$	**2**
Notes:			
Commonly Observed Responses:			
Candidate A $\int \sin^2 x-2\cos^2 x\,dx$ $=\int(3\cos 2x+1)\,dx$ •[5] × $\frac{3}{2}\sin 2x+x+c$ •[6] ×			

Question	Generic Scheme	Illustrative Scheme	Max Mark
8 (a) (i)			
$\bullet^1$ calculate T when $x = 20$		$\bullet^1$ 10·4 or 104	**1**
8 (a) (ii)			
$\bullet^2$ calculate T when $x = 0$		$\bullet^2$ 11 or 110	**1**

Notes:

1. Accept correct answers with no units.
2. Accept $5\sqrt{436}$ or $10\sqrt{109}$ or equivalent for $T(20)$.
3. For correct substitution alone, with no calculation $\bullet^1$ and $\bullet^2$ are not available.
4. For candidates who calculate T when $x = 0$ at $\bullet^1$ then $\bullet^2$ is available as follow through for calculating T when $x = 20$ in part(ii).

Commonly Observed Responses:

a) **(i)** 10·4 $\bullet^1$ ✓ **See note 1**

(ii) 110 $\bullet^2$ ✓

b) leading to 9·8 seconds $\bullet^{10}$ × **See note 7**

Question	Generic Scheme	Illustrative Scheme	Max Mark
8 (b)			
	• [3] write function in differential form	• [3] $5(36+x^2)^{\frac{1}{2}}+\ldots$	
	• [4] start differentiation of first term	• [4] $5\times\frac{1}{2}(\ \ )^{-\frac{1}{2}}\ldots$	
	• [5] complete differentiation of first term	• [5] $\ldots\ldots\ldots\times 2x\ldots\ldots$	
	• [6] complete differentiation and set candidate's derivative $=0$	• [6] $5x(36+x^2)^{-\frac{1}{2}}-4=0$	
	• [7] start to solve	• [7] $5x=4(36+x^2)^{\frac{1}{2}}$ or $\frac{5x}{(36+x^2)^{\frac{1}{2}}}=4$	
	• [8] know to square both sides	• [8] $25x^2=16(36+x^2)$ or $\frac{25x^2}{(36+x^2)}=16$	
	• [9] find value of x	• [9] $x=8$	
	• [10] calculate minimum time	• [10] T = 9·8 or 98 no units required	**8**

Notes:

5. Incorrect expansion of $(\ldots)^{\frac{1}{2}}$ at stage •[3] only •[6] and •[10] are available as follow through.
6. Incorrect expansion of $(\ldots)^{-\frac{1}{2}}$ at stage •[7] only •[10] is available as follow through.
7. Where candidates have omitted units, then •[10] is only available if the implied units are consistent throughout their solution.
8. •[10] is only available as a follow through for a value which is less than the values obtained for the two extremes.

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
9.			
$\bullet^1$ use compound angle formula $\bullet^2$ compare coefficients $\bullet^3$ process for *k* $\bullet^4$ process for *a* $\bullet^5$ equates expression for *h* to 100 $\bullet^6$ write in standard format and attempt to solve $\bullet^7$ solve equation for $1{\cdot}5t$ $\bullet^8$ process solutions for *t*		$\bullet^1$ $k\sin 1{\cdot}5t\cos a - k\cos 1{\cdot}5t\sin a$ $\bullet^2$ $k\cos a = 36$, $k\sin a = 15$ **stated explicitly** $\bullet^3$ $k = 39$ $\bullet^4$ $a = 0{\cdot}39479\ldots\text{rad}$ or $22{\cdot}6^\circ$ $\bullet^5$ $39\sin(1{\cdot}5t - 0{\cdot}39479\ldots) + 65 = 100$ $\bullet^6$ $\sin(1{\cdot}5t - 0{\cdot}39479\ldots) = \frac{35}{39}$ $\Rightarrow 1{\cdot}5t - 0{\cdot}39479\ldots = \sin^{-1}\left(\frac{35}{39}\right)$ $\bullet^7$ $1{\cdot}5t = 1{\cdot}508$ and $2{\cdot}422$ $\bullet^8$ $t = 1{\cdot}006$ and $1{\cdot}615$	**8**

Notes:

1. Treat $k\sin 1{\cdot}5t\cos a - \cos 1{\cdot}5t\sin a$ as bad form only if the equations at the $\bullet^2$ stage both contain *k*.
2. $39\sin 1{\cdot}5t\cos a - 39\cos 1{\cdot}5t\sin a$ or $39(\sin 1{\cdot}5t\cos a - \cos 1{\cdot}5t\sin a)$ is acceptable for $\bullet^1$ and $\bullet^3$.
3. Accept $k\cos a = 36$ and $-k\sin a = -15$ for $\bullet^2$.
4. $\bullet^2$ is not available for $k\cos 1{\cdot}5t = 36$ and $k\sin 1{\cdot}5t = 15$, however, $\bullet^4$ is still available.
5. $\bullet^3$ is only available for a single value of k, $k > 0$.
6. $\bullet^4$ is only available for a single value of a.
7. The angle at $\bullet^4$ must be consistent with the equations at $\bullet^2$ even when this leads to an angle outwith the required range.
8. Candidates who identify and use any form of the wave equation may gain $\bullet^1$, $\bullet^2$ and $\bullet^3$, however, $\bullet^4$ is only available if the value of a is interpreted for the form $k\sin(1{\cdot}5t - a)$.
9. Candidates who work consistently in degrees cannot gain $\bullet^8$.
10. Do not penalise additional solutions at $\bullet^8$.
11. On this occasion accept any answers which round to $1{\cdot}0$ and $1{\cdot}6$ (2 significant figures required).

Commonly Observed Responses:

Response 1: Missing information in working.

Candidate A

$39\cos a = 36$

$-39\sin a = -15$ $\bullet^1$ ^

$\tan a = \frac{15}{36}$ $\bullet^2$ ✓ $\bullet^3$ ✓

$a = 0{\cdot}39479\ldots\text{rad or } 22{\cdot}6^\circ$ $\bullet^4$ ✓

Candidate B

$\cos a = 36$ $\bullet^1$ ^

$\sin a = 15$ $\bullet^2$ ×

$\tan a = \frac{15}{36}$ $\bullet^3$ ^

$a = 0{\cdot}39479\ldots\text{rad or } 22{\cdot}6^\circ$ $\bullet^4$ ×

Does not satisfy equations at $\bullet^2$

Candidate C

$k\sin 1{\cdot}5t\cos a - k\cos 1{\cdot}5t\sin a$

$k\cos a = 36, \quad k\sin a = 15$ $\bullet^1$ ✓

$k = 39 \text{ or } -39$ $\bullet^2$ ✓ $\bullet^3$ × $\bullet^4$ ×

$\tan a = \frac{15}{36}$

$a = 0{\cdot}39479\ldots\text{rad or } 22{\cdot}6^\circ$

or

$a = 3{\cdot}53638\ldots\text{rad or } 202{\cdot}6^\circ$

Response 2: Correct expansion of $k\sin(x + a)^\circ$ and possible errors for $\bullet^2$ and $\bullet^4$

Candidate D

$k\cos a = 36$ $\bullet^2$ ✓

$k\sin a = 15$

$\tan a = \frac{36}{15}$ $\bullet^4$ ×

$a = 1{\cdot}176\ldots\text{rad or } 67{\cdot}4^\circ$

Candidate E

$k\cos a = 15$ $\bullet^2$ ×

$k\sin a = 36$

$\tan a = \frac{36}{15}$ $\bullet^4$ ✓1

$a = 1{\cdot}176\ldots\text{rad or } 67{\cdot}4^\circ$

Candidate F

$k\cos a = 36$ $\bullet^2$ ×

$k\sin a = -15$

$\tan a = \frac{-15}{36}$ $\bullet^4$ ✓1

$a = 5{\cdot}888\ldots\text{rad or } 337{\cdot}4^\circ$

Response 3: Labelling incorrect, $\sin(A - B) = \sin A\cos B - \cos A\sin B$ from formula list.

Candidate G

$k\sin A\cos B - k\cos A\sin B$

$k\cos a = 36$ $\bullet^1$ ×

$k\sin a = 15$ $\bullet^2$ ✓

$\tan a = \frac{15}{36}$ $\bullet^4$ ✓

$a = 0{\cdot}39479\ldots\text{rad or } 22{\cdot}6^\circ$

Candidate H

$k\sin A\cos B - k\cos A\sin B$

$k\cos 1{\cdot}5t = 36$ $\bullet^1$ ×

$k\sin 1{\cdot}5t = 15$ $\bullet^2$ ×

$\tan 1{\cdot}5t = \frac{15}{36}$ $\bullet^4$ ✓1

$1{\cdot}5t = 0{\cdot}39479\ldots\text{rad or } 22{\cdot}6^\circ$

Candidate I

$k\sin A\cos B - k\cos A\sin B$

$k\cos B = 36$ $\bullet^1$ ×

$k\sin B = 15$ $\bullet^2$ ✓1

$\tan B = \frac{15}{36}$ $\bullet^4$ ✓1

$B = 0{\cdot}39479\ldots\text{rad or } 22{\cdot}6^\circ$

Candidate J

$39\sin(1{\cdot}5t - 0{\cdot}395) = 100$ $\bullet^5$ ×

$\sin(1{\cdot}5t - 0{\cdot}395) = \frac{100}{39}$

$1{\cdot}5t - 0{\cdot}395 = \sin^{-1}\frac{100}{39}$ $\bullet^6$ ✓1 $\bullet^7$ × $\bullet^8$ ×

Candidate K

$39\sin(1{\cdot}5t - 0{\cdot}395) = 100$ $\bullet^6$ × $\bullet^7$ × $\bullet^8$ ×

$1{\cdot}5t - 0{\cdot}395 = \sin^{-1}\frac{39}{100}$

[END OF MARKING INSTRUCTIONS]