H

National
Qualifications
2022

# 2022 Mathematics

# Higher

# Paper 2

# Finalised Marking Instructions

**General marking principles for Higher Mathematics**

*Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.*

*For each question, the marking instructions are generally in two sections:*

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

*In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.*

**(a)** Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

**(b)** If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

**(c)** One mark is available for each •. There are no half marks.

**(d)** If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

**(e)** Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

**(f)** Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

**(g)** If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

**(h)** If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = 1 \text{ or } 3$$

**(i) Horizontal/vertical marking**

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

| | $\bullet^5$ | $\bullet^6$ |
|---|---|---|
| $\bullet^5$ | $x = 2$ | $x = -4$ |
| $\bullet^6$ | $y = 5$ | $y = -7$ |

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$
$\bullet^6$ $y = 5$ and $y = -7$

Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

**(j)** In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\qquad$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50 $\qquad$ $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

**(k)** Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

**(l)** Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $(x^3 + 2x^2 + 3x + 2)(2x + 1)$ written as

  $(x^3 + 2x^2 + 3x + 2) \times 2x + 1$

  $= 2x^4 + 5x^3 + 8x^2 + 7x + 2$

  gains full credit

- repeated error within a question, but not between questions or papers

**(m)** In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

**(n)** You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

**(o)** You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

**(p)** If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

| | |
|---|---|
| Strategy 1 attempt 1 is worth 3 marks. | Strategy 2 attempt 1 is worth 1 mark. |
| Strategy 1 attempt 2 is worth 4 marks. | Strategy 2 attempt 2 is worth 5 marks. |
| From the attempts using strategy 1, the resultant mark would be 3. | From the attempts using strategy 2, the resultant mark would be 1. |

In this case, award 3 marks.

## Marking Instructions for each question

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 1. | (a) | | •¹ determine gradient of AB<br>•² determine gradient of altitude<br>•³ find equation | •¹ $-1$<br>•² $1$<br>•³ $y = x - 4$ | 3 |

**Notes:**

1. •³ is only available to candidates who find and use a perpendicular gradient.
2. At •³, accept any arrangement of a candidate's equation where constant terms have been simplified.

**Commonly Observed Responses:**

Candidate A - BEWARE

Correct gradient from incorrect substitution

$m_{\text{AB}} = \dfrac{2-(-1)}{-4-(-1)} = -1$ •¹ ✗

$m_{\perp} = 1$ •² $\boxed{✓1}$

$y = x - 4$ •³ $\boxed{✓1}$

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 1. | (b) | | •4 determine midpoint of AC<br>•5 determine gradient of median<br>•6 find equation | •4 $(3,1)$<br>•5 5<br>•6 $y=5x-14$ | 3 |

**Notes:**

3. •5 is only available to candidates who use a midpoint to find a gradient.
4. •6 is only available as a consequence of using a 'midpoint' of AC and the point B.
5. At •6, accept any arrangement of a candidate's equation where constant terms have been simplified.
6. •6 is not available as a consequence of using a perpendicular gradient.

**Commonly Observed Responses:**

| Candidate A - Perpendicular bisector of AC | | Candidate B - Altitude through B | |
|---|---|---|---|
| $\text{Midpoint}_{AC}\ (3,1)$ | •1 ✓ | $m_{AC}=\frac{1}{2}$ | •1 ^ |
| $m_{AC}=\frac{1}{2}\Rightarrow m_{\perp}=-2$ | •2 ✗ | $m_{\perp}=-2$ | •2 ✗ |
| $y+2x=7$ | •3 $\boxed{✓\ 2}$ | $y+2x=0$ | •3 $\boxed{✓\ 2}$ |
| For other perpendicular bisectors award 0/3 | | | |

| Candidate C - Median through A | | Candidate D - Median through C | |
|---|---|---|---|
| $\text{Midpoint}_{BC}\left(\frac{9}{2},-\frac{1}{2}\right)$ | •1 ✗ | $\text{Midpoint}_{AB}\left(\frac{1}{2},-\frac{5}{2}\right)$ | •1 ✗ |
| $m_{AM}=\frac{1}{11}$ | •2 $\boxed{✓\ 1}$ | $m_{CM}=\frac{11}{13}$ | •2 $\boxed{✓\ 1}$ |
| $11y=x-10$ | •3 $\boxed{✓\ 2}$ | $13y=11x-38$ | •3 $\boxed{✓\ 2}$ |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| | (c) | | •7 determine $x$-coordinate<br>•8 determine $y$-coordinate | •7 2.5<br>•8 −1.5 | 2 |

**Notes:**

7. For $\left(\frac{10}{4},-\frac{6}{4}\right)$ award 1/2 (do not penalise repeated lack of simplification - *general marking principle* (*l*) ).

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 2. | | | •1 use discriminant<br>•2 apply condition and simplify<br>•3 state range | •1 $(-8)^2-4(2)(4-p)$<br>•2 $32+8p>0$ or $8p>-32$<br>•3 $p>-4$ | 3 |

**Notes:**

1. At •1, treat the inconsistent use of brackets eg $(-8)^2-4\times2\times4-p$ or $-8^2-4(2)(4-p)$ as bad form only if the candidate deals with the unbracketed terms correctly in the next line of working.
2. If candidates have the condition 'discriminant $=0$', then •2 and •3 are unavailable. However, see Candidate E.
3. If candidates have the condition 'discriminant $<0$', 'discriminant $\leq 0$' or 'discriminant $\geq 0$' then •2 is lost but •3 is available.

**Commonly Observed Responses:**

| | |
|---|---|
| **Candidate A - bad form**<br>$(-8)^2-4\times2\times4-p>0$<br>$32+8p>0$ •1 ✓ •2 ✓<br>$p>-4$ •3 ✓ | **Candidate B - no coefficient of $p$**<br>$(-8)^2-4\times2\times4-p>0$<br>$32-p>0$ •1 ✗ •2 $\boxed{✓ 2}$<br>$p<32$ •3 $\boxed{✓ 2}$ |
| **Candidate C - bad form**<br>$-8^2-4\times2\times(4-p)>0$<br>$32+8p>0$ •1 ✓ •2 ✓<br>$p>-4$ •3 ✓ | **Candidate D - not bad form**<br>$-8^2-4\times2\times(4-p)>0$<br>$-96+8p>0$ •1 ✗ •2 $\boxed{✓ 2}$<br>$p>12$ •3 $\boxed{✓ 1}$ |
| **Candidate E - condition stated initially**<br>Real and distinct roots $b^2-4ac>0$<br>$(-8)^2-4(2)(4-p)=0$ •1 ✓<br>$32+8p=0$<br>$p=-4$<br>so $p>-4$ •2 ✓ •3 ✓ | **Candidate F**<br>$8^2-4(2)(4-p)>0$ •1 ✗<br>$32+8p>0$ •2 $\boxed{✓ 1}$<br>$p>-4$ •3 $\boxed{✓ 1}$<br>However, $64-4(2)(4-p)>0$ as the first line of working may be awarded •1 |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 3. | (a) | | •¹ use compound angle formula | •¹ $k\sin x\cos a + k\cos x\sin a$ stated explicitly | 4 |
| | | | •² compare coefficients | •² $k\cos a = 4$ and $k\sin a = 5$ stated explicitly | |
| | | | •³ process for $k$ | •³ $k = \sqrt{41}$ | |
| | | | •⁴ process for $a$ and express in required form | •⁴ $\sqrt{41}\sin(x + 0.896\ldots)$ | |

**Notes:**

1. Accept $k(\sin x\cos a + \cos x\sin a)$ at •¹.
2. Treat $k\sin x\cos a + \cos x\sin a$ as bad form only if the equations at the •² stage both contain $k$.
3. $\sqrt{41}\sin x\cos a + \sqrt{41}\cos x\sin a$ or $\sqrt{41}(\sin x\cos a + \cos x\sin a)$ are acceptable for •¹ and •³.
4. •² is not available for $k\cos x = 4$ and $k\sin x = 5$, however •⁴ may still be gained. See Candidate E.
5. •³ is only available for a single value of $k$, $k > 0$.
6. •⁴ is not available for a value of $a$ given in degrees.
7. Accept values of $a$ which round to 0.9.
8. Candidates may use any form of the wave function for •¹, •² and •³. However, •⁴ is only available if the wave is interpreted in the form $k\sin(x + a)$.
9. Evidence for •⁴ may not appear until part (b) and must appear by the •⁵ stage.

**Commonly Observed Responses:**

| Candidate A | Candidate B | Candidate C |
|---|---|---|
| •¹ ^ | $k\sin x\cos a + k\cos x\sin a$ •¹ ✓ | $\sin x\cos a + \cos x\sin a$ •¹ ✘ |
| $\sqrt{41}\cos a = 4$<br>$\sqrt{41}\sin a = 5$ •² ✓ •³ ✓ | $\cos a = 4$<br>$\sin a = 5$ •² ✘ | $\cos a = 4$<br>$\sin a = 5$ •² $\boxed{✓\ 2}$ |
| | | $k = \sqrt{41}$ •³ ✓ |
| $\tan a = \frac{5}{4}$<br>$a = 0.896\ldots$ | $\tan a = \frac{5}{4}$<br>$a = 0.896\ldots$<br>Not consistent with equations at •². | $\tan a = \frac{5}{4}$<br>$a = 0.896\ldots$ |
| $\sqrt{41}\sin(x + 0.896\ldots)$ •⁴ ✓ | $\sqrt{41}\sin(x + 0.896\ldots)$ •³ ✓ •⁴ ✘ | $\sqrt{41}\sin(x + 0.896\ldots)$ •⁴ ✘ |

| Question | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|
| 3. | (a) | (continued) | | |

**Commonly Observed Responses:**

| Candidate D - errors at $\bullet^2$ | Candidate E - use of $x$ at $\bullet^2$ | Candidate F |
|---|---|---|
| $k\sin x\cos a + k\cos x\sin a$ $\bullet^1$ ✓ | $k\sin x\cos a + k\cos x\sin a$ $\bullet^1$ ✓ | $k\sin A\cos B + k\cos A\sin B$ $\bullet^1$ ✗ |
| $k\cos a = 5$ <br> $k\sin a = 4$ $\bullet^2$ ✗ | $k\cos x = 4$ <br> $k\sin x = 5$ $\bullet^2$ ✗ | $k\cos A = 4$ <br> $k\sin A = 5$ $\bullet^2$ ✗ |
| $\tan a = \frac{4}{5}$ <br> $a = 0.674\ldots$ | $\tan x = \frac{5}{4}$ <br> $x = 0.896\ldots$ | $\tan A = \frac{5}{4}$ <br> $A = 0.896\ldots$ |
| $\sqrt{41}\sin(x+0.674\ldots)$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓ 1}$ | $\sqrt{41}\sin(x+0.896\ldots)$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓ 1}$ | $\sqrt{41}\sin(x+0.896\ldots)$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓ 1}$ |

| Question | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|
| | (b) | $\bullet^5$ link to (a) | $\bullet^5$ $\sqrt{41}\sin(x+0.896\ldots) = 5.5$ | 3 |
| | | $\bullet^6$ solve for $(x+a)$ | $\bullet^6$ $\overset{\bullet^6}{1.033\ldots}$, $\overset{\bullet^7}{2.108\ldots}$ | |
| | | $\bullet^7$ solve for $x$ | $\bullet^7$ $0.137\ldots$, $1.212\ldots$ | |

**Notes:**

10. In part (b), where candidates work in degrees throughout, the maximum mark available is 2/3.
11. $\bullet^7$ is only available for two solutions within the stated range. Ignore 'solutions' outwith the range.
12. At $\bullet^7$ accept values of $x$ which round to 0.1 or 1.2

**Commonly Observed Responses:**

| Candidate G - converting to radians | Candidate H - working in degrees and truncation |
|---|---|
| ⋮ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓ | ⋮ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓ |
| $\sqrt{41}\sin(x+51.3\ldots)$ $\bullet^4$ ✗ | $\sqrt{41}\sin(x+51.3)$ $\bullet^4$ ✗ |
| $\sqrt{41}\sin(x+51.3\ldots) = 5.5$ $\bullet^5$ $\boxed{✓ 1}$ | $\sqrt{41}\sin(x+51.3) = 5.5$ $\bullet^5$ $\boxed{✓ 1}$ |
| $x+51.3\ldots = 59.1\ldots, 120.8\ldots$ <br> $x = 7.8\ldots, 69.4\ldots$ $\bullet^6$ $\boxed{✓ 1}$ | $x+51.3 = 59.1, 120.9$ <br> $x = 7.8, 69.6$ $\bullet^6$ $\boxed{✓ 1}$ $\bullet^7$ ^ |
| $x = \frac{7.9\pi}{180}, \frac{69.5\pi}{180}$ $\bullet^7$ $\boxed{✓ 1}$ | |

| Candidate I - working in degrees | Candidate J - working in degrees |
|---|---|
| ⋮ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓ | ⋮ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓ |
| $\sqrt{41}\sin(x+51.3\ldots)$ $\bullet^4$ ✗ | $\sqrt{41}\sin(x+51.3\ldots)$ $\bullet^4$ ✗ |
| $\sqrt{41}\sin(x+51.3\ldots) = 5.5$ $\bullet^5$ $\boxed{✓ 1}$ | $\sqrt{41}\sin(x+51.3\ldots) = 5.5$ $\bullet^5$ $\boxed{✓ 1}$ |
| $x+51.3\ldots = 59.1\ldots$ <br> $x = 7.8\ldots$ $\bullet^6$ ^ $\bullet^7$ ^ | $x+51.3\ldots = 59.1\ldots, 120.8\ldots$ $\bullet^6$ ^ $\bullet^7$ ^ |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 4. | (a) | | $\bullet^1$ state appropriate integral<br><br>$\bullet^2$ integrate<br><br>$\bullet^3$ substitute limits<br><br>$\bullet^4$ evaluate area | $\bullet^1$ $\int_{-1}^{2}\left(x^3-5x^2+2x+8\right)dx$<br><br>$\bullet^2$ $\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x$<br><br>$\bullet^3$ $\left(\frac{1}{4}(2)^4-\frac{5}{3}(2)^3+(2)^2+8(2)\right)$ $-\left(\frac{1}{4}(-1)^4-\frac{5}{3}(-1)^3+(-1)^2+8(-1)\right)$<br><br>$\bullet^4$ $\frac{63}{4}$ or 15.75 | 4 |

**Notes:**

1. Limits and '$dx$' must appear at the $\bullet^1$ stage for $\bullet^1$ to be awarded.
2. Where a candidate differentiates one or more terms at $\bullet^2$, then $\bullet^3$ and $\bullet^4$ are not available.
3. Candidates who substitute limits without integrating, do not gain $\bullet^3$ or $\bullet^4$.
4. Do not penalise the inclusion of '$+c$'.
5. Do not penalise the continued appearance of the integral sign after $\bullet^1$.
6. $\bullet^4$ is not available where solutions include statements such as $-\frac{63}{4}=\frac{63}{4}$. See Candidate C.

**Commonly Observed Responses:**

**Candidate A**

$\int_{-1}^{2}\left(x^3-5x^2+2x+8\right)$ $\bullet^1$ ✗

$=\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x$ $\bullet^2$ ✓

$\bullet^3$ ^

$=\frac{63}{4}$ $\bullet^4$ $\boxed{✓ 1}$

**Candidate B - evidence of substitution using a calculator**

$\int\left(x^3-5x^2+2x+8\right)dx$ $\bullet^1$ ✗

$=\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x$ $\bullet^2$ ✓

$=\frac{32}{3}-\left(-\frac{61}{12}\right)$ $\bullet^3$ ✓

$=\frac{63}{4}$ $\bullet^4$ ✓

**Candidate C - communication for $\bullet^4$**

$\int_{2}^{-1}\left(x^3-5x^2+2x+8\right)dx$ $\bullet^1$ ✓

... $\bullet^2$ ✓ $\bullet^3$ ✓

$=-\frac{63}{4}$, hence area is $\frac{63}{4}$. $\bullet^4$ ✓

However $-\frac{63}{4}=\frac{63}{4}$ square units does not gain $\bullet^4$

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 4. | (b) | | **Method 1** | **Method 1** | 3 |
| | | | •5 state appropriate integral | •5 $\int_2^4 \left(x^3-5x^2+2x+8\right)dx$ | |
| | | | •6 evaluate integral | •6 $-\frac{16}{3}$ | |
| | | | •7 interpret result and evaluate total area | •7 $\frac{253}{12}$ or 21.083... | |
| | | | **Method 2** | **Method 2** | |
| | | | •5 state appropriate integral | •5 $\int_2^4 \left(0-\left(x^3-5x^2+2x+8\right)\right)dx$ | |
| | | | •6 substitute limits | •6 $-\left(\frac{1}{4}(4)^4-\frac{5}{3}(4)^3+(4)^2+8(4)\right)-\left(-\left(\frac{1}{4}(2)^4-\frac{5}{3}(2)^3+(2)^2+8(2)\right)\right)$ | |
| | | | •7 evaluate total area | •7 $\frac{253}{12}$ or 21.083... | |

**Notes:**

7. For candidates who only consider $\int_{-1}^{4} \ldots dx$ or any other invalid integral, award 0/3.
8. In part (b), at •5 do not penalise the omission of '$dx$'.
9. In Method 1, •5 may be awarded for $\left[\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x\right]_2^4$

   or $\left(\frac{1}{4}(4)^4-\frac{5}{3}(4)^3+(4)^2+8(4)\right)-\left(\frac{1}{4}(2)^4-\frac{5}{3}(2)^3+(2)^2+8(2)\right)$.
10. In Method 2, •5 may be awarded for $\left[\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x\right]_4^2$ or •5 and •6 may be awarded for

    $\left(\frac{1}{4}(2)^4-\frac{5}{3}(2)^3+(2)^2+8(2)\right)-\left(\frac{1}{4}(4)^4-\frac{5}{3}(4)^3+(4)^2+8(4)\right)$ .
11. •7 is not available to candidates where solutions include statements such as $-\frac{16}{3}=\frac{16}{3}$ square units. See Candidate D.
12. In Method 1, where a candidate's integral leads to a positive value, •7 is not available.
13. Where a candidate has differentiated in both parts of the question see Candidate E.

| Question | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|
| 4. | (b) | (continued) | | |

**Commonly Observed Responses:**

**Candidate D - communication for •7**

$\int_2^4 (x^3 - 5x^2 + 2x + 8)\,dx = -\frac{16}{3}$ •5 ✓ •6 ✓

$\frac{63}{4} + \frac{16}{3} = \frac{253}{12}$ •7 ✓

However, •7 is not available where statements such as "$-\frac{16}{3} = \frac{16}{3}$ square units" or "ignore negative" appear.

**Candidate E - differentiation in (a) and (b)**

(a) $\int_{-1}^{2} (x^3 - 5x^2 + 2x + 8)\,dx$ •1 ✓

$= 3x^2 - 10x + 2$ •2 ✘

$= (3(2)^2 - 10(2) + 2) - (3(-1)^2 - 10(-1) + 2)$ •3 ✘

$= -21$

Area $= 21$ •4 ✘

(b) $(3(4)^2 - 10(4) + 2) - (3(2)^2 - 10(2) + 2) = 16$ •5 ✓ •6 $\boxed{✓ 1}$

Total Area $= 5$ •7 $\boxed{✓ 2}$ see note 12

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 5. | (a) | (i) | •¹ interpret notation<br><br>•² state expression for $f\left(g\left(x\right)\right)$ | •¹ $f\left(3x+5\right)$ or $\left(g\left(x\right)\right)^2-2$<br><br>•² $\left(3x+5\right)^2-2$ | 2 |
| | | (ii) | •³ state expression for $g\left(f\left(x\right)\right)$ | •³ $3\left(x^2-2\right)+5$ | 1 |

**Notes:**

1. For $f\left(g\left(x\right)\right)=\left(3x+5\right)^2-2$ without working, award both •¹ and •².

**Commonly Observed Responses:**

**Candidate A**

(a)(i) $f\left(g\left(x\right)\right)=3\left(x^2-2\right)+5$ •¹ ✘ •² $\boxed{✓ 1}$

(a)(ii) $g\left(f\left(x\right)\right)=\left(3x+5\right)^2-2$ •³ $\boxed{✓ 1}$

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 5. | (b) | | •[4] interpret information and expand<br>•[5] express inequality in standard quadratic form<br>•[6] determine zeros of quadratic equation<br>•[7] state range with justification | •[4] $9x^2+30x+25-2<3x^2-6+5$<br>•[5] $6x^2+30x+24<0$<br>•[6] $-4, -1$<br>•[7] $-4<x<-1$ with eg sketch or table of signs | 4 |

**Notes:**

2. Candidates who do not work with an inequation from the outset lose •[4], •[5] and •[7]. However, •[6] is still available. See Candidate D.
3. Accept the appearance of $-4,-1$ within inequalities for •[6].
4. At •[7] accept "$x>-4$ and $x<-1$" or "$x>-4$, $x<-1$" together with the required justification.

**Commonly Observed Responses:**

| Candidate B | | Candidate C | |
|---|---|---|---|
| $9x^2+30x+25-2<3x^2-6+5$ | •[4] ✓ | $9x^2+30x+25-2<3x^2-6+5$ | •[4] ✓ |
| $6x^2+30x+24<0$ | •[5] ✓ | $6x^2+30x+24=0$ | •[5] ✗ |
| $6x^2+30x+24=0$ | | $x=-1,\ x=-4$ | •[6] ✓ |
| $x=-1,\ x=-4$ | •[6] ✓ | $-4<x<-1$ with sketch | •[7] ✗ |
| $-4<x<-1$ with sketch | •[7] ✓ | | |

| Candidate D | |
|---|---|
| $9x^2+30x+25-2=3x^2-6+5$ | •[4] ✗ |
| $6x^2+30x+24=0$ | •[5] ✗ |
| $x=-1,\ x=-4$ | •[6] ✓ |
| For $f(g(x))<g(f(x))$ | |
| $-4<x<-1$ with sketch | •[7] ✗ |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 6. | | | •1 write in integrable form | •1 $1-3x^{-2}$ | 5 |
| | | | •2 integrate one term | •2 $x$ or $\ldots-\frac{3x^{-1}}{-1}$ | |
| | | | •3 complete integration | •3 $\ldots-\frac{3x^{-1}}{-1}+c$ or $x\ldots+c$ | |
| | | | •4 interpret information given and substitute for $x$ and $y$ | •4 $6=3+3(3)^{-1}+c$ | |
| | | | •5 state expression for $y$ | •5 $y=x+3x^{-1}+2$ | |

**Notes:**

1. For candidates who make no attempt to integrate only •1 is available.
2. For candidates who omit $+c$ only •1 and •2 are available.
3. For candidates who differentiate either term, •3, •4, and •5 are not available.

**Commonly Observed Responses:**

| | |
|---|---|
| **Candidate A - incomplete substitution**<br>$y=x+3x^{-1}+c$ •1 ✓ •2 ✓ •3 ✓<br>$y=3+3(3)^{-1}+c$<br>$c=-4$ •4 ^<br>$y=x+3x^{-1}-4$ •5 $\boxed{✓ 1}$ | **Candidate B - partial integration**<br>$y=1+3x^{-1}+c$ •1 ✓ •2 ✓ •3 ✗<br>$6=1+3(3)^{-1}+c$ •4 $\boxed{✓ 1}$<br>$c=4$<br>$y=x+3x^{-1}+4$ •5 $\boxed{✓ 1}$ |
| **Candidate C - inconsistent working**<br>$\frac{dy}{dx}=1-\frac{3}{x^2}$<br>$x-3x^{-2}$ •1 ✗<br>$y=x-\frac{3x^{-1}}{-1}+c$ •2 $\boxed{✓ 1}$ •3 $\boxed{✓ 1}$ | **Candidate D - inconsistent working**<br>$\frac{dy}{dx}=1-\frac{3}{x^2}$<br>$x-3x^{-2}$ •1 ✗<br>$y=\frac{x^2}{2}-\frac{3x^{-1}}{-1}+c$ •2 $\boxed{✓ 1}$ •3 $\boxed{✓ 1}$ |
| **Candidate E**<br>**integration not complete at •3 stage**<br>$\frac{dy}{dx}=1-3x^{-2}$ •1 ✓<br>$y=x-\frac{3x^{-1}}{-1}$ •2 ✓ •3 ✗<br>$y=x+3x^{-1}+c$ | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 7. | | | **Method 1** | **Method 1** | 5 |
| | | | •[1] state equation of line | •[1] $\log_5 y = -2\log_5 x + 3$ | |
| | | | •[2] introduce logs | •[2] $\log_5 y = -2\log_5 x + 3\log_5 5$ | |
| | | | •[3] use laws of logs | •[3] $\log_5 y = \log_5 x^{-2} + \log_5 5^3$ | |
| | | | •[4] use laws of logs | •[4] $\log_5 y = \log_5 5^3 x^{-2}$ | |
| | | | •[5] state $k$ and $n$ | •[5] $k = 125,\ n = -2$ | |
| | | | **Method 2** | **Method 2** | |
| | | | •[1] state equation of line | •[1] $\log_5 y = -2\log_5 x + 3$ | |
| | | | •[2] use laws of logs | •[2] $\log_5 y = \log_5 x^{-2} + 3$ | |
| | | | •[3] use laws of logs | •[3] $\log_5 \dfrac{y}{x^{-2}} = 3$ | |
| | | | •[4] use laws of logs | •[4] $\dfrac{y}{x^{-2}} = 5^3$ | |
| | | | •[5] state $k$ and $n$ | •[5] $k = 125, n = -2$ | |
| | | | **Method 3** | **Method 3**<br>**The equations at •[1], •[2], and •[3] must be stated explicitly.** | |
| | | | •[1] introduce logs to $y = kx^n$ | •[1] $\log_5 y = \log_5 kx^n$ | |
| | | | •[2] use laws of logs | •[2] $\log_5 y = n\log_5 x + \log_5 k$ | |
| | | | •[3] interpret intercept | •[3] $\log_5 k = 3$ | |
| | | | •[4] use laws of logs | •[4] $k = 125$ | |
| | | | •[5] interpret gradient | •[5] $n = -2$ | |

| Question | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|
| 7. | (continued) | | | |
| | | **Method 4** | **Method 4** | |
| | | •¹ interpret point on log graph | •¹ $\log_5 x = 0$ and $\log_5 y = 3$ | |
| | | •² convert from log to exponential form | •² $x = 1, y = 5^3$ | |
| | | •³ interpret point and convert | •³ $\log_5 x = 2$ **and** $\log_5 y = -1$<br>$x = 5^2$ and $y = 5^{-1}$ | |
| | | •⁴ substitute into $y = kx^n$ and evaluate $k$ | •⁴ $5^3 = k(1)^n \Rightarrow k = 125$ | |
| | | •⁵ substitute other point into $y = kx^n$ and evaluate $n$ | •⁵ $5^{-1} = 5^3 \times 5^{2n}$<br>$\Rightarrow 3 + 2n = -1$<br>$\Rightarrow n = -2$ | |

**Notes:**

1. In any method, marks may only be awarded within a valid strategy using $y = kx^n$.
2. Markers must identify the method which best matches the candidates approach; markers must not mix and match between methods.
3. Penalise the omission of base 5 at most once in any method.
4. In Method 4, candidates may use $(2,-1)$ for •¹ and •² and $(0,3)$ for •³.
5. Do not accept $k = 5^3$.
6. In Method 3, do not accept $m = -2$ or gradient $= -2$ for •⁵.
7. Accept $y = 125x^{-2}$ for •⁵.

**Commonly Observed Responses:**

| | |
|---|---|
| | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 8. | (a) | | $\bullet^1$ determine expression for area of pond<br><br>$\bullet^2$ obtain expression for $y$<br><br>$\bullet^3$ demonstrate result | $\bullet^1$ $(x-3)(y-2)$ stated or implied by $\bullet^3$<br><br>$\bullet^2$ $y=\dfrac{150}{x}$<br><br>$\bullet^3$ $A(x)=(x-3)\left(\dfrac{150}{x}-2\right)$<br>eg $A(x)=\dfrac{150x}{x}-\dfrac{450}{x}-2x+6$<br>$A(x)=156-2x-\dfrac{450}{x}$ | 3 |

**Notes:**

1. Accept any legitimate variations for the area of the pond in $\bullet^1$, eg $A=150-2(x-3)-2(y)(1.5)$.
2. Do not penalise the omission of brackets at $\bullet^1$. See Candidate A.
3. The substitution for $y$ at $\bullet^3$ must be clearly shown for $\bullet^3$ to be available.

**Commonly Observed Responses:**

| | |
|---|---|
| **Candidate A**<br>$A(x)=x-3\times y-2$ $\quad\bullet^1$ ✓<br>$A(x)=x-3\times\dfrac{150}{x}-2$ $\quad\bullet^2$ ✓<br>$A(x)=156-2x-\dfrac{450}{x}$ $\quad\bullet^3$ ^ | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 8. | (b) | | •4 express $A$ in differentiable form | •4 $156-2x-450x^{-1}$ stated or implied by •5 | 6 |
| | | | •5 differentiate | •5 $-2+450x^{-2}$ | |
| | | | •6 equate expression for derivative to 0 | •6 $-2+450x^{-2}=0$ | |
| | | | •7 solve for $x$ | •7 $x=15$ | |
| | | | •8 verify nature of stationary point | •8 table of signs for derivative $\therefore$ maximum<br>or<br>$A''(x)=-900x^{-3}$ and $A''(15)<0$<br>$\therefore$ maximum | |
| | | | •9 determine maximum area | •9 $A=96\left(\text{m}^2\right)$ | |

**Notes:**

4. For a numerical approach award 0/6.
5. •6 can be awarded for $450x^{-2}=2$.
6. For candidates who integrate any term at the •5 stage, only •6 is available on follow through for setting their 'derivative' to 0.
7. •7, •8, and •9 are only available for working with a derivative which contains an index $\leq -2$.
8. $\sqrt{\frac{450}{2}}$ must be simplified at •7 or •8 for •7 to be awarded.
9. Ignore the appearance of $-15$ at mark •7.
10. •8 is not available to candidates who consider a value of $x \leq 0$ in the neighbourhood of 15.
11. •9 is still available in cases where a candidate's table of signs does not lead legitimately to a maximum at •8.
12. •8 and •9 are not available to candidates who state that the maximum exists at a negative value of $x$.

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 8. | (b) | **(continued)** | | | |

**Notes (continued)**

For the table of signs for a derivative, accept:

| $x$ | $15^-$ | $15$ | $15^+$ |
|---|---|---|---|
| $A'(x)$ | $+$ | $0$ | $-$ |
| Slope or shape | / | — | \ |

| $x$ | $\rightarrow$ | $15$ | $\rightarrow$ |
|---|---|---|---|
| $A'(x)$ | $+$ | $0$ | $-$ |
| Slope or shape | / | — | \ |

Arrow are taken to mean 'in the neighbourhood of'

| $x$ | $a$ | $15$ | $b$ |
|---|---|---|---|
| $A'(x)$ | $+$ | $0$ | $-$ |
| Slope or shape | / | — | \ |

Where $0 < a < 15$ and $b > 15$

For the table of signs for a derivative, **do not** accept:

| $x$ | $\rightarrow$ | $-15$ | $\rightarrow$ | $15$ | $\rightarrow$ |
|---|---|---|---|---|---|
| $A'(x)$ | $-$ | $0$ | $+$ | $0$ | $-$ |
| Slope or shape | \ | — | / | — | \ |

Since the function is discontinuous $-15 \rightarrow 15$ is not acceptable

| $x$ | $a$ | $-15$ | $b$ | $15$ | $c$ |
|---|---|---|---|---|---|
| $A'(x)$ | $-$ | $0$ | $+$ | $0$ | $-$ |
| Slope or shape | \ | — | / | — | \ |

Since the function is discontinuous $-15 < b < 15$ is not acceptable

- For this question do not penalise the omission of '$x$' or the word 'shape'/'slope'.
- Stating values of $A'(x)$ is an acceptable alternative to writing '+' or '–' signs.
- Acceptable variations of $A'(x)$ are: $A'$, $a'(x)$, $\frac{dA}{dx}$, and $-2+450x^{-2}$ .

**Commonly Observed Responses:**

| **Candidate B - differentiating over multiple lines** | | **Candidate C - differentiating over multiple lines** | |
|---|---|---|---|
| | •4 ^ | $A(x) = 156 - 2x - 450x^{-1}$ | •4 ✓ |
| $A'(x) = -2 - 450x^{-1}$ | | $A'(x) = -2 - 450x^{-1}$ | |
| $A'(x) = -2 + 450x^{-2}$ | •5 ✗ | $A'(x) = -2 + 450x^{-2}$ | •5 ✗ |
| $-2 + 450x^{-2} = 0$ | •6 $\boxed{✓ 1}$ | $-2 + 450x^{-2} = 0$ | •6 $\boxed{✓ 1}$ |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 9. | (a) | | •1 substitute for $y$ in equation of circle | •1 $x^2+(3x+7)^2-4x-6(3x+7)-7=0$ | 5 |
| | | | •2 arrange in standard quadratic form | •2 $10x^2+20x=0$ | |
| | | | •3 factorise | •3 $10x(x+2)=0$ | |
| | | | | •4 •5 | |
| | | | •4 state $x$ coordinates | •4 0 $-2$ | |
| | | | •5 state corresponding $y$ coordinates | •5 7 1 | |

**Notes:**

1. •1 is only available if '$=0$' appears by the •3 stage.
2. At •3, the quadratic must lead to two distinct real roots for •4 and •5 to be available.
3. At •3 do not penalise candidates who fail to extract the common factor or who have divided the quadratic equation by 10.
4. If a candidate arrives at an equation which is not a quadratic at •2 stage, then •3, •4 and •5 are not available
5. •3 is available for substituting correctly into the quadratic formula.
6. •4 and •5 may be marked either horizontally or vertically.
7. Ignore incorrect labelling of P and Q.

**Commonly Observed Responses:**

**Candidate A - substituting for $y$**

$\left(\frac{y-7}{3}\right)^2+y^2-4\left(\frac{y-7}{3}\right)-6y-7=0$ •1 ✓

$\frac{10y^2-80y+70}{9}=0$ •2 ✓

$10(y-1)(y-7)=0$ •3 ✓

$y=1$ or $y=7$ •4 ✓

$x=-2$ or $x=0$ •5 ✓

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 9. | (b) | | $\bullet^6$ state centre of circle<br><br>$\bullet^7$ calculate midpoint of PQ<br><br>$\bullet^8$ calculate radius of small circle<br><br>$\bullet^9$ state equation of small circle | $\bullet^6$ $(2, 3)$<br><br>$\bullet^7$ $(-1, 4)$<br><br>$\bullet^8$ $\sqrt{10}$<br><br>$\bullet^9$ $(x-2)^2+(y-3)^2=10$ | 4 |

**Notes:**

8. Evidence for $\bullet^6$ may appear in part (a).
9. Where a candidate uses coordinates for P and Q without supporting working, $\bullet^7$ is not available, however $\bullet^8$ and $\bullet^9$ may be awarded.
10. Where candidates find the equation of the larger circle $\bullet^8$ and $\bullet^9$ are not available.

**Commonly Observed Responses:**

**Candidate B - using substitution**

Equation of smaller circle of form

$(x-2)^2+(y-3)^2=r^2$ $\bullet^6$ ✓

Midpoint PQ $(-1, 4)$ $\bullet^7$ ✓

$(-1-2)^2+(4-3)^2=r^2$

$r^2=10$ $\bullet^8$ ✓

$(x-2)^2+(y-3)^2=10$ $\bullet^9$ ✓

**Candidate C - using tangency**

Equation of smaller circle of form

$(x-2)^2+(y-3)^2=r^2$ $\bullet^6$ ✓

Since $y=3x+7$ is tangent to smaller circle

$10x^2+20x+20-r^2=0$ has equal roots

$\Rightarrow 20^2-4(10)(20-r^2)=0$ $\bullet^7$ ✓

$\Rightarrow r^2=10$ $\bullet^8$ ✓

$(x-2)^2+(y-3)^2=10$ $\bullet^9$ ✓

**Candidate D - using P or Q to mid-point as radius**

⋮

$r=\sqrt{(-2+1)^2+(1-4)^2}=\sqrt{10}$ $\bullet^8$ ✗

or

$r=\sqrt{(0+1)^2+(7-4)^2}=\sqrt{10}$ $\bullet^8$ ✗

$(x-2)^2+(y-3)^2=10$ $\bullet^9$ $\boxed{✓ 2}$

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| 10. | (a) | | •¹ evaluate $P$ for $t = 24.55$ | •¹ 929 | 1 |

**Notes:**

1. Accept any answer which rounds 929.0368007… to at least 2 significant figures.

**Commonly Observed Responses:**

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|---|---|---|---|---|
| | (b) | | •² substitute for $P$ and $D$ | •² $850 = 0.188807(600-210)^k$ | 4 |
| | | | •³ arrange equation in the form $a = b^k$ | •³ $\dfrac{850}{0.188807} = (600-210)^k$ | |
| | | | •⁴ write in logarithmic form | •⁴ eg $\ln\left(\dfrac{850}{0.188807}\right) = \ln(600-210)^k$ or $k = \log_{(600-210)} \dfrac{850}{0.188807}$ | |
| | | | •⁵ solve for $k$ | •⁵ 1.41 | |

**Notes:**

2. •³ may be implied by •⁴.
3. Any base may be used at •⁴ stage.
4. Accept 1.4 at •⁵.
5. The calculation at •⁵ must follow from the valid use of exponentials and logarithms at •³ and •⁴. See Candidate A.
6. For candidates who take an iterative approach to arrive at the value $t = 1.41$ award 1/4. However, if, in the iterations $P$ is calculated for $t = 1.405$ and $t = 1.415$ then award 4/4.

**Commonly Observed Responses:**

| Candidate A - invalid use of exponentials | | Candidate B - transcription error | |
|---|---|---|---|
| $850 = 0.188807(600-210)^k$ | •² ✓ | $850 = \underline{0.18807}(600-210)^k$ | •² ✗ |
| $850 = 73.63473^k$ | •³ ✗ •⁴ ✗ •⁵ ✗ | $4519.59\ldots = 390^k$ | •³ ✓ 1 |
| $\log_{73.63473} 850 = k$ | | $\log_{390} 4519.59\ldots$ | •⁴ ✓ 1 |
| 1.56… | | 1.41… | •⁵ ✓ 1 |

**[END OF MARKING INSTRUCTIONS]**