H

National
Qualifications
2022

2022 Mathematics

Higher

Paper 1 (Non-calculator)

Finalised Marking Instructions

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

$$x^2 + 5x + 7 = 9x + 4$$

$$x - 4x + 3 = 0$$

This is a transcription error and so the mark is not awarded.

$$x = 1$$

This is no longer a solution of a quadratic equation, so the mark is not awarded.

The following example is an exception to the above

$$x^2 + 5x + 7 = 9x + 4$$

$$x - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 1 \text{ or } 3$$

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$ Vertical: $\bullet^5$ $x = 2$ and $y = 5$

$\bullet^6$ $y = 5$ and $y = -7$ $\bullet^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $(x^3+2x^2+3x+2)(2x+1)$ written as

  $(x^3+2x^2+3x+2)\times 2x+1$

  $=2x^4+5x^3+8x^2+7x+2$

  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			$\bullet^1$ state gradient $\bullet^2$ state perpendicular gradient $\bullet^3$ find equation of line	$\bullet^1$ $-\frac{5}{2}$ $\bullet^2$ $\frac{2}{5}$ $\bullet^3$ $5y = 2x + 32$	**3**

Notes:

1. At $\bullet^1$, ignore any errors in processing the constant term.
2. At $\bullet^1$ and $\bullet^2$, ignore the appearance of 'x'.
3. $\bullet^3$ is only available as a consequence of using a perpendicular gradient.
4. At $\bullet^3$, accept any arrangement of a candidate's equation where constant terms have been simplified.

Commonly Observed Responses:

Candidate A		Candidate B	
A perpendicular gradient has been clearly stated $5x + 2y = 7$		No communication for perpendicular gradient $5x + 2y = 7$ $y = -\frac{5}{2}x + \frac{7}{2}$	
$m_{\perp} = \frac{2}{5}$	$\bullet^1$ ✓ $\bullet^2$ ✓	$m = \frac{2}{5}$	$\bullet^1$ ^ $\bullet^2$ $\boxed{\checkmark\ 1}$
$5y = 2x + 32$	$\bullet^3$ ✓	$5y = 2x + 32$	$\bullet^3$ $\boxed{\checkmark\ 1}$
Candidate C			
$m = 5$	$\bullet^1$ ✗		
$m_{\perp} = -\frac{1}{5}$	$\bullet^2$ $\boxed{\checkmark\ 1}$		
$x + 5y = 29$	$\bullet^3$ $\boxed{\checkmark\ 1}$		

Question			Generic scheme	Illustrative scheme	Max mark
2.			•[1] apply $m\log_n x = \log_n x^m$	•[1] $\log_3 6^2$	3
			•[2] apply $\log_n x - \log_n y = \log_n \frac{x}{y}$	•[2] $\log_3 \frac{6^2}{4}$	
			•[3] evaluate	•[3] 2	

Notes:

1. Do not penalise the omission of the base of the logarithm at •[1] or •[2].
2. Correct answer with no working, award 0/3.

Commonly Observed Responses:

Candidate A - introducing a variable		Candidate B	
$\log_3 9$	•[1] ✓ •[2] ✓	$2\log_3\left(\frac{6}{4}\right)$	•[2] ✗
$3^x = 9$		$\log_3\left(\frac{6}{4}\right)^2$	•[1] $\boxed{✓ 1}$ •[3] ^
$x = 2$	•[3] ✓		

Question			Generic scheme	Illustrative scheme	Max mark
3.			**Method 1**	**Method 1**	3
			•[1] equate composite function to x	•[1] $h\left(h^{-1}(x)\right)=x$	
			•[2] write $h\left(h^{-1}(x)\right)$ in terms of $h^{-1}(x)$	•[2] $4+\frac{1}{3}h^{-1}(x)=x$	
			•[3] state inverse function	•[3] $h^{-1}(x)=3(x-4)$	
			Method 2	**Method 2**	
			•[1] write as $y=h(x)$ and start to rearrange	•[1] $y=h(x)\Rightarrow x=h^{-1}(y)$ $y-4=\frac{1}{3}x$ or $3y=12+x$	
			•[2] express x in terms of y	•[2] $x=3(y-4)$	
			•[3] state inverse function	•[3] $h^{-1}(y)=3(y-4)$ $\Rightarrow h^{-1}(x)=3(x-4)$	

Notes:

1. In Method 1, accept $4+\frac{1}{3}h^{-1}(x)=x$ for •[1] and •[2].
2. In Method 2, accept '$y-4=\frac{1}{3}x$' without reference to $y=h(x)\Rightarrow x=h^{-1}(y)$ at •[1].
3. In Method 2, accept $h^{-1}(x)=3(x-4)$ without reference to $h^{-1}(y)$ at •[3].
4. In Method 2, beware of candidates with working where each line is not mathematically equivalent. See Candidates A and B for example.
5. At •[3] stage, accept h^{-1} written in terms of any dummy variable eg $h^{-1}(y)=3(y-4)$.
6. $y=3(x-4)$ does not gain •[3].
7. $h^{-1}(x)=3(x-4)$ with no working gains 3/3.

Question	Generic scheme	Illustrative scheme	Max mark
3.	(continued)		

Commonly Observed Responses:

Candidate A $h(x)=4+\frac{1}{3}x$ $y=4+\frac{1}{3}x$ $x=3(y-4)$ $\bullet^1$ ✓ $\bullet^2$ ✓ $y=3(x-4)$ $\bullet^3$ ✗ $h^{-1}(x)=3(x-4)$	**Candidate B** $h(x)=4+\frac{1}{3}x$ $y=4+\frac{1}{3}x$ $x=4+\frac{1}{3}y$ $\bullet^1$ ✗ $y=3(x-4)$ $\bullet^2$ $\boxed{✓ 1}$ $h^{-1}(x)=3(x-4)$ $\bullet^3$ $\boxed{✓ 1}$
Candidate C - BEWARE $h'=\ldots$ $\bullet^3$ ✗	**Candidate D** $x\rightarrow x\div 3\rightarrow x\div 3+4=h(x)$ $\div 3\rightarrow +4$ $\therefore -4\rightarrow \times 3$ $\bullet^1$ ✓ $3(x-4)$ $\bullet^2$ ✓ $h^{-1}(x)=3(x-4)$ $\bullet^3$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
4.			•¹ express first term in differentiable form •² differentiate first term •³ differentiate second term	•¹ $y = x^{\frac{3}{2}} \ldots$ stated or implied by •² •² $\frac{3}{2}x^{\frac{1}{2}} \ldots$ •³ $\ldots + 2x^{-2}$	3

Notes:

1. •² is only available for differentiating a term with a fractional index.
2. Where candidates attempt to integrate throughout, only •¹ is available.

Commonly Observed Responses:

Candidate A - differentiating over two lines		
$y = x^{\frac{3}{2}} + 2x^{-2}$	•¹ ✓	
$y = \frac{3}{2}x^{\frac{1}{2}} + 2x^{-2}$	•² ✓ •³ ✗	

Question			Generic scheme	Illustrative scheme	Max mark
5.			•¹ use $m=\tan\theta$ •² evaluate exact value •³ determine equation	•¹ $m=\tan\frac{\pi}{6}$ or $m=\tan 30°$ •² $\frac{1}{\sqrt{3}}$ •³ eg $y\sqrt{3}=x+2$ or $y=\frac{1}{\sqrt{3}}x+\frac{2}{\sqrt{3}}$	3

Notes:

1. Do not award •¹ for $m=\tan^{-1}\frac{\pi}{6}$. However •² and •³ are still available. Where candidates state $m=\tan^{-1}\frac{\pi}{3}$ only •³ is available.
2. Where candidates make no reference to a trigonometric ratio or use an incorrect trigonometric ratio, •¹ and •² are unavailable.
3. •³ is only available as a consequence of attempting to use a tan ratio. See Candidate F
4. Accept $y=\frac{1}{\sqrt{3}}(x+2)$ for •³, but do not accept $y-0=\frac{1}{\sqrt{3}}(x+2)$.

Commonly Observed Responses:

Candidate A		Candidate B	
$m=\tan\frac{\pi}{3}$	•¹ ✗	$m=\frac{1}{\sqrt{3}}$ (with or without a diagram)	•¹ ^ •² ✓ 2
$m=\sqrt{3}$	•² ✓ 1	$y=\frac{1}{\sqrt{3}}x+\frac{2}{\sqrt{3}}$	
$y=\sqrt{3}x+2\sqrt{3}$	•³ ✓ 1		•³ ✓ 1
Candidate C		**Candidate D**	
$m=\tan\theta$ (with or without a diagram)	•¹ ^	$m=\tan\theta$ (with or without a diagram)	•¹ ^
$m=\frac{1}{\sqrt{3}}$	•² ✓ 1	$m=\sqrt{3}$	•² ✗
		$y=\sqrt{3}x+2\sqrt{3}$	•³ ✓ 1
Candidate E		**Candidate F**	
$m=\tan\theta=\frac{\pi}{6}$	•¹ ✗	$m=\tan\frac{\pi}{3}$	•¹ ✗
$m=\frac{1}{\sqrt{3}}$	•² ✓ 1	$m=60$	•² ✗
		$y=60(x+2)$	•³ ✗

Question			Generic Scheme	Illustrative Scheme	Max Mark
6.			•¹ start to integrate	•¹ $\dfrac{(10-3x)^{\frac{1}{2}}}{\frac{1}{2}}\ldots$	4
			•² complete integration	•² $\ldots\times-\dfrac{1}{3}$	
			•³ process limits	•³ $-\dfrac{2}{3}(10-3(2))^{\frac{1}{2}}-\left(-\dfrac{2}{3}(10-3(-5))^{\frac{1}{2}}\right)$	
			•⁴ evaluate integral	•⁴ 2	

Notes:

1. For candidates who **differentiate throughout** or make **no attempt to integrate**, award 0/4.
2. If candidates start to integrate individual terms within the bracket or attempt to expand a bracket or use another invalid approach no further marks are available.
3. Do not penalise the inclusion of '$+c$' or the continued appearance of the integral sign after •¹.
4. •³ is only available for substitution into an expression which is equivalent to the integrand obtained at •².
5. The integral obtained must contain a non-integer power for •⁴ to be available.
6. •⁴ is only available to candidates who deal with the coefficient of x at the •² stage. See Candidate A.

Commonly Observed Responses:

Candidate A		Candidate B - NOT differentiating throughout	
$\dfrac{(10-3x)^{\frac{1}{2}}}{\frac{1}{2}}$	•¹ ✓ •² ^	$-\dfrac{1}{2}(10-3x)^{-\frac{3}{2}}\times-\dfrac{1}{3}$	•¹ ✗ •² ✓
$2(10-3(2))^{\frac{1}{2}}-2(10-3(-5))^{\frac{1}{2}}$	•³ ✓1	$\dfrac{1}{6}(10-3(2))^{-\frac{3}{2}}-\dfrac{1}{6}(10-3(-5))^{-\frac{3}{2}}$	•³ ✓1
-6	•⁴ ✓2 Note 6	$\dfrac{39}{2000}$	•⁴ ✓1
Candidate C		**Candidate D - integrating over two lines**	
$\dfrac{(10-3x)^{\frac{1}{2}}}{\frac{1}{2}}\times-3$	•¹ ✓ •² ✗	$\dfrac{(10-3x)^{\frac{1}{2}}}{\frac{1}{2}}$	
		$\dfrac{(10-3x)^{\frac{1}{2}}}{\frac{1}{2}}\times-\dfrac{1}{3}$	•¹ ✓ •² ^
$-6(10-3(2))^{\frac{1}{2}}-\left(-6(10-3(-5))^{\frac{1}{2}}\right)$	•³ ✓1	$-\dfrac{2}{3}(10-3(2))^{\frac{1}{2}}-\left(-\dfrac{2}{3}(10-3(-5))^{\frac{1}{2}}\right)$	•³ ✓1
18	•⁴ ✓1	2	•⁴ ✓1

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.	(a)	(i)	•1 determine $\sin r$	•1 $\dfrac{1}{\sqrt{10}}$	**1**
		(ii)	•2 determine $\sin q$	•2 $\dfrac{3}{\sqrt{13}}$	**1**

Notes:

1. In (a)(ii), where candidates do not simplify the perfect square see Candidates A and B.

Commonly Observed Responses:

Candidate A	**Candidate B** - simplification in part (b)
$\sin q = \dfrac{\sqrt{9}}{\sqrt{13}}$ •2 $\boxed{✓ 2}$	(a)(ii) $\sin q = \dfrac{\sqrt{9}}{\sqrt{13}}$ •2 ✓ (b) $\sin(q-r) = \dfrac{7}{\dots}$ Roots have been simplified in (b)

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.	(b)		•[3] select appropriate formula and express in terms of p and q •[4] substitute into addition formula •[5] evaluate $\sin(q-r)$	•[3] $\sin q\cos r - \cos q\sin r$ stated or implied by •[4] •[4] $\frac{3}{\sqrt{13}}\times\frac{3}{\sqrt{10}}-\frac{2}{\sqrt{13}}\times\frac{1}{\sqrt{10}}$ •[5] $\frac{7}{\sqrt{130}}$	**3**

Notes:

2. Award •[3] for candidates who write $\sin\left(\frac{3}{\sqrt{13}}\right)\times\cos\left(\frac{3}{\sqrt{10}}\right)-\sin\left(\frac{2}{\sqrt{13}}\right)\times\cos\left(\frac{1}{\sqrt{10}}\right)$. •[4] and •[5] are unavailable.
3. For any attempt to use $\sin(q-r)=\sin q-\sin r$, •[4] and •[5] are unavailable.
4. At •[5], the answer must be given as a single fraction. Accept $\frac{7}{\sqrt{13}\sqrt{10}}$, $\frac{7\sqrt{10}}{10\sqrt{13}}$ and $\frac{7\sqrt{13}}{13\sqrt{10}}$.
5. Do not penalise trigonometric ratios which are less than –1 or greater than 1.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
8.			**Method 1**	**Method 1**	4
			•¹ apply $\log_6 x + \log_6 y = \log_6 xy$	•¹ $\log_6\left(x(x+5)\right) = \ldots$	
			•² write in exponential form	•² $x(x+5) = 6^2$	
			•³ express in standard quadratic form	•³ $x^2 + 5x - 36 = 0$	
			•⁴ solve quadratic and state solution of log equation	•⁴ -9, 4 and $x > 0 \Rightarrow x = 4$	
			Method 2	**Method 2**	
			•¹ apply $\log_6 x + \log_6 y = \log_6 xy$	•¹ $\log_6\left(x(x+5)\right) = \ldots$	
			•² apply $m\log_6 x = \log_6 x^m$	•² $\ldots = \log_6 6^2$	
			•³ express in standard quadratic form	•³ $x^2 + 5x - 36 = 0$	
			•⁴ solve quadratic and state solution of log equation	•⁴ -9, 4 and $x > 0 \Rightarrow x = 4$	

Notes:

1. Accept $\log_6 x(x+5) = \ldots$ for •¹ .
2. •² is not available for $x(x+5) = 2^6$; however candidates may still gain •³ and •⁴.
3. •³ and •⁴ are only available if the quadratic reached at •³ is obtained by applying the rules in •¹ and •².
4. •⁴ is only available for solving a polynomial of degree two or higher.
5. At •⁴, accept any indication that -9 has been discarded. For example, scoring out $x = -9$ or underlining $x = 4$.

Commonly Observed Responses:

Candidate A		Candidate B	
$\log_6\left(x(x+5)\right) = 2$	•¹ ✓	$\log_6\left(x(x+5)\right) = 2$	•¹ ✓
$x(x+5) = 12$	•² ✗	$x(x+5) = 64$	•² ✗
$x^2 + 5x - 12 = 0$	•³ $\boxed{✓\ 1}$	$x^2 + 5x - 64 = 0$	•³ $\boxed{✓\ 1}$
$\frac{-5 \pm \sqrt{73}}{2}$ and $x > 0 \Rightarrow x = \frac{-5 + \sqrt{73}}{2}$	•⁴ $\boxed{✓\ 1}$	$\frac{-5 \pm \sqrt{281}}{2}$ and $x > 0 \Rightarrow x = \frac{-5 + \sqrt{281}}{2}$	•⁴ $\boxed{✓\ 1}$

Question			Generic Scheme	Illustrative Scheme	Max Mar
9.			•[1] substitute for $\cos 2x°$ into equation	•[1] $2\cos^2 x° - 1\ldots$	5
			•[2] express in standard quadratic form	•[2] $2\cos^2 x° - 5\cos x° + 2 = 0$	
			•[3] factorise	•[3] $(2\cos x° - 1)(\cos x° - 2) = 0$	
				•[4] •[5]	
			•[4] solve for $\cos x°$	•[4] $\cos x° = \frac{1}{2}$ $\cos x° = 2$	
			•[5] solve for x	•[5] $x = 60,\ 300$ 'no solutions'	

Notes:

1. •[1] is not available for simply stating $\cos 2x° = 2\cos^2 x° - 1$ with no further working.
2. In the event of $\cos^2 x° - \sin^2 x°$ or $1 - 2\sin^2 x°$ being substituted for $\cos 2x°$, •[1] cannot be awarded until the equation reduces to a quadratic in $\cos x°$.
3. Substituting $2\cos^2 \mathrm{A} - 1$ or $2\cos^2 \alpha - 1$ for $\cos 2x°$ at the •[1] stage should be treated as bad form provided the equation is written in terms of x at •[2] stage. Otherwise, •[1] is not available.
4. Do not penalise the omission of degree signs.
5. '$= 0$' must appear by •[3] stage for •[2] to be awarded. However, for candidates using the quadratic formula to solve the equation, '$= 0$' must appear at •[2] stage for •[2] to be awarded.
6. $\cos x° = \dfrac{5 \pm \sqrt{9}}{4}$ gains •[3].
7. Candidates may express the equation obtained at •[2] in the form $2c^2 - 5c + 2 = 0$ or $2x^2 - 5x + 2 = 0$. In these cases, award •[3] for $(2c - 1)(c - 2) = 0$ or $(2x - 1)(x - 2) = 0$.
   However, •[4] is only available if $\cos x°$ appears explicitly at this stage. See Candidate A.
8. The equation $2 + 2\cos^2 x° - 5\cos x° = 0$ does not gain •[2] unless •[3] has been awarded.
9. •[4] and •[5] are only available as a consequence of trying to solve a quadratic equation. See Candidate B. However, •[5] is not available if the quadratic equation has repeated roots.
10. •[3], •[4] and •[5] are not available for any attempt to solve a quadratic equation written in the form $ax^2 + bx = c$. See Candidate C.
11. •[5] is only available for 2 valid solutions within the stated range. Ignore 'solutions' outwith the range. However, see Candidate E.
12. Accept $\cancel{\cos x° = 2}$ for •[5]. See Candidate A.

Question		Generic Scheme	Illustrative Scheme	Max Mark
9.	(continued)			

Commonly Observed Responses:

Candidate A

$2\cos^2 x^\circ - 1 = 5\cos x^\circ - 3$	$\bullet^1$ ✓
$2c^2 - 5c + 2 = 0$	$\bullet^2$ ✓
$(2c-1)(c-2) = 0$	$\bullet^3$ ✓
$c = \frac{1}{2},\ c = 2$	$\bullet^4$ ✗
$x = 60,\ 300 \quad \cancel{\cos x^\circ = 2}$	$\bullet^5$ $\boxed{\checkmark\ 1}$

Candidate B - not solving a quadratic

$2\cos^2 x^\circ - 1 = 5\cos x^\circ - 3$	$\bullet^1$ ✓
$2\cos^2 x^\circ - 5\cos x^\circ + 2 = 0$	$\bullet^2$ ✓
$-3\cos x^\circ + 2 = 0$	$\bullet^3$ ✗
$\cos x^\circ = \frac{2}{3}$	$\bullet^4$ $\boxed{\checkmark\ 2}$ $\bullet^5$ ^

Candidate C - not in standard quadratic form

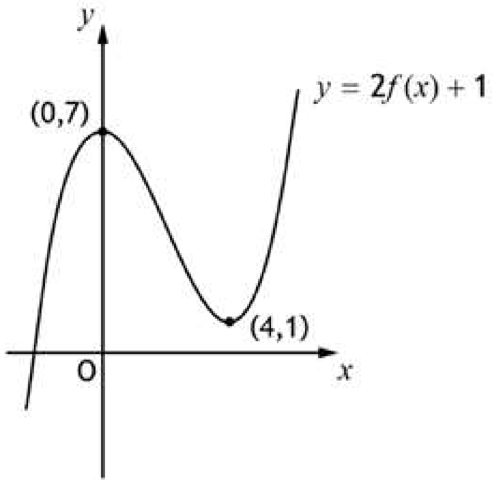
$2\cos^2 x^\circ - 1 = 5\cos x^\circ - 3$	$\bullet^1$ ✓
$2\cos^2 x^\circ - 5\cos x^\circ = -2$	$\bullet^2$ $\boxed{\checkmark\ 2}$
$\cos x^\circ(2\cos x^\circ - 5) = -2$	$\bullet^3$ $\boxed{\checkmark\ 2}$
$\cos x^\circ = -2,\ 2\cos x^\circ - 5 = -2$	
$\Rightarrow \cos x = \frac{3}{2}$	$\bullet^4$ ✗
No solutions	$\bullet^5$ ✗

Candidate D - reading $\cos 2x^\circ$ as $\cos^2 x^\circ$

$\cos^2 x^\circ = 5\cos x^\circ - 3$	$\bullet^1$ ✗
$\cos^2 x^\circ - 5\cos x^\circ + 3 = 0$	$\bullet^2$ $\boxed{\checkmark\ 1}$
$\cos x^\circ = \frac{5 \pm \sqrt{13}}{2}$	$\bullet^3$ $\boxed{\checkmark\ 1}$
	$\bullet^4$ ^ $\bullet^5$ ^

Candidate E

⋮	$\bullet^1$ ✓ $\bullet^2$ ✓
$(\cos x^\circ - 1)(\cos x^\circ - 2) = 0$	$\bullet^3$ ✗
$\cos x^\circ = 1, \qquad \cos x^\circ = 2$	$\bullet^4$ $\boxed{\checkmark\ 1}$
$x = 0 \qquad$ No solutions	$\bullet^5$ $\boxed{\checkmark\ 1}$

Question			Generic Scheme	Illustrative Scheme	Max Mark
10.	(a)		•¹ vertical scaling by a factor of 2 identifiable from graph •² vertical translation of ‘+1’ units identifiable from graph •³ transformations applied in correct order	•¹ •² •³ 	**3**

Notes:

1. •¹, •² and •³ are only available for a ‘cubic’ with a maximum and minimum turning point.
2. Ignore intersections (or lack of intersections) with the original graph.

Commonly Observed Responses:

Where the image of (4,0) is not (4,1), that point must be annotated (or drawn to within tolerance). In the following table, the images of the given points must be stationary points for the marks to be awarded.

Image of (0,3)	Image of (4,0)	Award...	
(0,8)	(4,2)	2/3	Transformation in wrong order
(0,4) (0,4) (0,4)	(8,1) (4,1) (2,1)	1/3 1/3 1/3	Only vertical translation correct
(0,5)	(4,−1)	2/3	Evidence of vertical scaling and transformation in correct order
(0,6) (0,7) (1,6) (−1,6)	(4,0) any incorrect point (5,0) (3,0)	1/3 1/3 1/3 1/3	Evidence of vertical scaling
(0,−2) (0,4)	(4,1) (−4,1)	1/3 1/3	Evidence of vertical translation
(0,5) (0,2)	any other point any other point	0/3 0/3	Insufficient evidence of scaling/translation

Question			Generic Scheme	Illustrative Scheme	Max Mark
10.	(b)		•4 state coordinates of stationary points	•4 $(0,3)$ and $(8,0)$	**1**

Notes:

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
11.	**Method 1**	**Method 1**	3
	•¹ identify common factor	•¹ $2(x^2+6x\ldots$ stated or implied by •²	
	•² complete the square	•² $2(x+3)^2\ldots$	
	•³ process for r and write in required form	•³ $2(x+3)^2+5$	
	Method 2	**Method 2**	
	•¹ expand completed square form	•¹ $px^2+2pqx+pq^2+r$	
	•² equate coefficients	•² $p=2$, $2pq=12$, $pq^2+r=23$	
	•³ process for q and r and write in required form	•³ $2(x+3)^2+5$	

Notes:

1. $2(x+3)^2+5$ with no working gains •¹ and •² only. However, see Candidate E.

Commonly Observed Responses:

Candidate A $2(x^2+6)+23$ $2\left((x+3)^2-9\right)+23$ •¹ ✓ •² ✓ $2(x+3)^2+5$ •³ ✓ See the exception to marking principle (h)	**Candidate B** $px^2+2pqx+pq^2+r$ •¹ ✓ $p=2$, $2pq=12$, $pq^2+r=23$ •² ✓ $q=3$, $r=5$ •³ ^ •³ is lost as answer is not in completed square form
Candidate C $2(x^2+12x)+23$ •¹ ✗ $2\left((x+6)^2-36\right)+23$ •² $\boxed{\checkmark\ 1}$ $2(x+6)^2-49$ •³ $\boxed{\checkmark\ 1}$	**Candidate D** $2\left((x+6)^2-36\right)+23$ •¹ ✗ •² ✗ $2(x+6)^2-49$ •³ $\boxed{\checkmark\ 1}$
Candidate E $2(x+3)^2+5$ •¹ ✓ •² ✓ Check: $=2(x^2+6x+9)+5$ $=2x^2+12x+18+5$ $=2x^2+12x+23$ •³ ✓	

Question			Generic Scheme	Illustrative Scheme	Max Mark
12.			•¹ start to differentiate •² complete differentiation •³ evaluate derivative	•¹ $4\cos\left(3x-\frac{\pi}{3}\right)\ldots$ •² $\ldots\times 3$ •³ $6\sqrt{3}$	3

Notes:

1. Where candidates make no attempt to differentiate or use another invalid approach, •² and •³ are not available.
2. At the •¹ and •² stage, candidates who work in degrees cannot gain •¹. However •² and •³ are still available.
3. At the •³ stage, do not penalise candidates who work in degrees or in radians and degrees.
4. Ignore the appearance of $+c$ at any stage.

Commonly Observed Responses:

Candidate A	Candidate B	Candidate C
Differentiating over two lines $f'(x)=4\cos\left(3x-\frac{\pi}{3}\right)$ •¹ ✓ $f'(x)=12\cos\left(3x-\frac{\pi}{3}\right)$ •² ^ $6\sqrt{3}$ •³ ✓1	$4\cos\left(3x-\frac{\pi}{3}\right)\times\frac{1}{3}$ •¹ ✓ •² ✗ $\frac{2\sqrt{3}}{3}$ •³ ✓1	$4\cos\left(3x-\frac{\pi}{3}\right)$ •¹ ✓ •² ^ $2\sqrt{3}$ •³ ✓1

Candidate D	Candidate E	Candidate F
$\pm 12\sin\left(3x-\frac{\pi}{3}\right)$ •¹ ✗ •² ✗ ± 6 •³ ✓1	$\pm 4\sin\left(3x-\frac{\pi}{3}\right)\ldots$ •¹ ✗ $\ldots\times 3$ •² ✓1 ± 6 •³ ✓1	$-12\cos\left(3x-\frac{\pi}{3}\right)$ •¹ ✗ •² ✓ $-6\sqrt{3}$ •³ ✓1

Question			Generic Scheme	Illustrative Scheme	Max Mark
13.	(a)	(i)	•[1] use -2 in synthetic division or evaluation of the cubic	•[1] $\begin{array}{r\|rrrr} -2 & 1 & -2 & -20 & -24 \\ & & & & \\ \hline & 1 & & & \end{array}$ or $(-2)^3-2(-2)^2-20(-2)-24$	**2**
			•[2] complete division/evaluation and interpret result	•[2] $\begin{array}{r\|rrrr} -2 & 1 & -2 & -20 & -24 \\ & & -2 & 8 & 24 \\ \hline & 1 & -4 & -12 & \boxed{0} \end{array}$ Remainder $=0 \therefore (x+2)$ is a factor or $f(-2)=0 \therefore (x+2)$ is a factor	
		(ii)	•[3] state quadratic factor	•[3] $x^2-4x-12$	**3**
			•[4] find remaining factors or apply the quadratic formula	•[4] $(x+2)$ and $(x-6)$ or $\dfrac{4\pm\sqrt{(-4)^2-4(1)(-12)}}{2(1)}$	
			•[5] state solution	•[5] $-2, 6$	

Notes:

1. Communication at •[2] must be consistent with working at that stage - a candidate's working must arrive legitimately at 0 before •[2] can be awarded.
2. Accept any of the following for •[2]:
   - '$f(-2)=0$ so $(x+2)$ is a factor'
   - 'since remainder $=0$, it is a factor'
   - the '0' from any method linked to the word 'factor' by 'so', 'hence', $\therefore, \rightarrow, \Rightarrow$ etc.
3. Do not accept any of the following for •[2]:
   - double underlining the '0' or boxing the '0' without comment
   - '$x=-2$ is a factor', '... is a root'
   - the word 'factor' only, with no link.

Commonly Observed Responses:

	(b)		•[6] state value of k	•[6] 3	**1**

Notes:

1. Accept $y=f(x-3)$ or $f(x-3)$ for •[6].

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
14.	(a)	(i)	•[1] state coordinates of centre •[2] state radius	•[1] $(7,-5)$ •[2] 10	**2**
		(ii)	•[3] substitute coordinates of P and evaluate •[4] communicate result	•[3] $(-2-7)^2+(7+5)^2=225$ •[4] $225>100 \therefore$ P lies outside	**2**

Notes:

1. Accept $x=7,\ y=-5$ for •[1].
2. Do not accept $g=7,\ f=-5$ or $7,\ -5$ for •[1].

Commonly Observed Responses:

Candidate A		Candidate B	
$d=15$	•[3] ✓	$d=\sqrt{225}$	•[3] ✓
$15>10 \therefore$ P lies outside	•[4] ✓	$\sqrt{225}>10 \therefore$ P lies outside	•[4] ✓

Candidate C			
$d=15$	•[3] ✓		
$r=10$			
$d>r \therefore$ P lies outside	•[4] ✓		

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		•[5] determine first value of r •[6] determine second value of r	•[5] 5 •[6] 25	**2**

Notes:

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]