National
Qualifications

X8477611

Mathematics
Paper 1 (Non-Calculator)

Marking Instructions

Please note that these marking instructions have not been standardised based on candidate responses. You may therefore need to agree within your centre how to consistently mark an item if a candidate response is not covered by the marking instructions.

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each ○. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = 1 \text{ or } 3$$

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	O5	O6
O5	$x = 2$	$x = -4$
O6	$y = 5$	$y = -7$

Horizontal: O5 $x = 2$ and $x \neq -4$ O6 $y = 5$ and $y \neq -7$

Vertical: O5 $x = 2$ and $y = 5$ O6 $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50 $\frac{4\frac{}{}5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

$(x^3 + 2x^2 + 3x + 2)(2x + 1)$ written as

$(x^3 + 2x^2 + 3x + 2) \times 2x + 1$

$= 2x^4 + 5x^3 + 8x^2 + 7x + 2$

gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Section 1

Question			Generic scheme	Illustrative scheme	Max mark
1.			•[1] use the discriminant •[2] apply condition and simplify •[3] determine the value of k	•[1] $3^2-4\times k\times(-4)$ •[2] $9+16k=0$ or $9=-16k$ •[3] $-\frac{9}{16}$	**3**

Question			Generic scheme	Illustrative scheme	Max mark
2.			$\bullet^1$ start to differentiate $\bullet^2$ complete differentiation $\bullet^3$ evaluate	$\bullet^1$ $5\left(x^2+1\right)^4\ldots$ $\bullet^2$ $\ldots\times 2x$ $\bullet^3$ 160	**3**

Question	Generic scheme	Illustrative scheme	Max mark
3.	**Method 1**	**Method 1**	**3**
	•[1] equate composite function to x	•[1] $f\left(f^{-1}(x)\right)=x$	
	•[2] write $f\left(f^{-1}(x)\right)$ in terms of $f^{-1}(x)$	•[2] $\dfrac{f^{-1}(x)+3}{2}=x$	
	•[3] state inverse function	•[3] $f^{-1}(x)=2x-3$	
	Method 2	**Method 2**	
	•[1] write as $y=f(x)$ and start to rearrange	•[1] $y=f(x)\Rightarrow x=f^{-1}(y)$ $2y=x+3$	
	•[2] express x in terms of y	•[2] $x=2y-3$	
	•[3] state inverse function	•[3] $f^{-1}(y)=2y-3$ $\Rightarrow f^{-1}(x)=2x-3$	

Question			Generic scheme	Illustrative scheme	Max mark
4.			$\bullet^1$ find gradient of first line $\bullet^2$ find gradient of second line $\bullet^3$ interpret results and state conclusion	$\bullet^1$ $-\frac{3}{2}$ $\bullet^2$ $\frac{2}{3}$ $\bullet^3$ $-\frac{3}{2}\times\frac{2}{3}=-1\Rightarrow$ lines are perpendicular.	**3**

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)	(i)	•[1] calculate $\sin p$	•[1] $\frac{3}{\sqrt{10}}$	**1**
		(ii)	•[2] calculate adjacent side •[3] calculate $\cos q$	•[2] $\sqrt{6}$ •[3] $\sqrt{\frac{3}{5}}$ or $\frac{\sqrt{3}}{\sqrt{5}}$	**2**
	(b)		•[4] use addition formula •[5] calculate remaining trig ratios and substitute into formula •[6] calculate $\cos(p+q)$	•[4] $\cos p\cos q-\sin p\sin q$ stated or implied by •[5] •[5] $\frac{1}{\sqrt{10}}\times\frac{\sqrt{6}}{\sqrt{10}}-\frac{3}{\sqrt{10}}\times\frac{2}{\sqrt{10}}$ •[6] $\frac{\sqrt{6}-6}{10}$	**3**

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		•[1] interpret notation •[2] state expression for $f\left(g\left(x\right)\right)$	•[1] $f\left(x^2-2x\right)$ •[2] $2\left(x^2-2x\right)+5$	**2**
	(b)		•[3] state expression for $g\left(f\left(x\right)\right)$	•[3] $\left(2x+5\right)^2-2\left(2x+5\right)$	**1**
	(c)		•[4] express $g\left(f\left(x\right)\right)-f\left(g\left(x\right)\right)$ in standard quadratic form •[5] identify common factor •[6] complete the square •[7] process for c and write in required form	•[4] $2x^2+20x+10$ •[5] $2\left(x^2+10x\ldots\right.$ stated or implied by •[6] •[6] $2\left(x+5\right)^2\ldots$ •[7] $2\left(x+5\right)^2-40$	**4**

Question			Generic scheme	Illustrative scheme	Max mark
7.			•[1] start to integrate •[2] complete integration	•[1] $6\sin\left(3x+\frac{\pi}{4}\right)\ldots$ •[2] $\ldots\times\frac{1}{3}+c$	**2**

Question			Generic scheme	Illustrative scheme	Max mark
8.			•[1] use $m = \tan\theta$	•[1] $m = \tan\frac{2\pi}{3}$ stated or implied by •[2]	**3**
			•[2] evaluate exact value	•[2] $-\sqrt{3}$	
			•[3] determine equation	•[3] $y = -\sqrt{3}x + 4\sqrt{3}$	

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•[1] set $y = y$ and arrange in standard form •[2] factorise and state x-coordinate of A	•[1] $6x^2 - 18x = 0$ •[2] $x = 3$	**2**
	(b)		•[3] know to integrate and interpret limits •[4] use 'upper − lower' •[5] integrate •[6] substitute limits •[7] evaluate	•[3] $\int_0^3 \ldots \, dx$ •[4] $\int_0^3 \left(\left(x^3 - 7x^2 + 12x + 3 \right) - \left(x^3 - x^2 - 6x + 3 \right) \right) dx$ •[5] $-\frac{6}{3}x^3 + \frac{18}{2}x^2$ •[6] $\left(-2 \times 3^3 + 9 \times 3^2 \right) - 0$ •[7] 27 (units^2)	**5**

Question			Generic scheme	Illustrative scheme	Max mark
10.			•[1] know to use (synthetic) division or substitution	•[1] eg $\begin{array}{r\|rrrr} \ldots & 6 & -13 & 0 & 4 \\ \hline & 6 & & & \end{array}$ **OR** $6\times(\ldots)^3-13\times(\ldots)^2+4$	**4**
			•[2] complete process using a root	•[2] eg $\begin{array}{r\|rrrr} 2 & 6 & -13 & 0 & 4 \\ & & 12 & -2 & -4 \\ \hline & 6 & -1 & -2 & 0 \end{array}$ **OR** $6\times(2)^3-13\times(2)^2+4=0$	
			•[3] identify quadratic factor	•[3] $6x^2-x-2$	
			•[4] express in fully factorised form	•[4] $(x-2)(3x-2)(2x+1)$	

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		•[1] state maximum value	•[1] 7	**1**
	(b)	(i)	•[2] state maximum value	•[2] 13	**1**
		(ii)	•[3] state value of x	•[3] 10	**1**

Section 2

Part A

Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)	(i)	•1 identify components of $\overrightarrow{AB}$ •2 find $\left\|\overrightarrow{AB}\right\|$	•1 eg $\begin{pmatrix} 3 \\ -6 \\ 6 \end{pmatrix}$ or $\sqrt{3^2+(-6)^2+6^2}$ •2 9	**2**
		(ii)	•3 state ratio	•3 $3:2$	**1**
	(b)		•4 determine coordinates	•4 $(9,-8,5)$	**1**

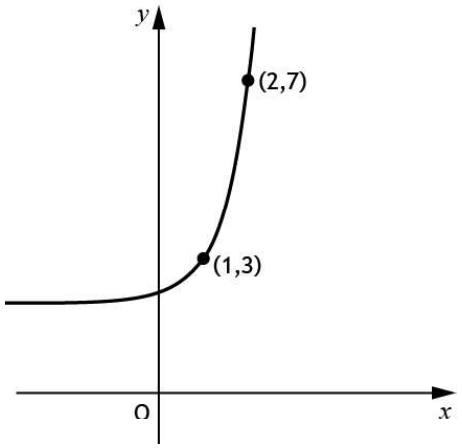
Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		•[1] begin to find u_6 •[2] determine u_5	•[1] $20 = \frac{2}{3}u_6 + 8$ •[2] $u_5 = 15$	**2**
	(b)		•[3] know how to find limit •[4] evaluate limit	•[3] $L = \frac{2}{3}L + 8$ or $L = \frac{8}{1-\frac{2}{3}}$ •[4] 24	**2**

Question			Generic scheme	Illustrative scheme	Max mark
14.			•[1] expand •[2] calculate $\mathbf{u}.\mathbf{u}$ or $\mathbf{u}.\mathbf{v}$ •[3] calculate $\mathbf{u}.\mathbf{v}$ or $\mathbf{u}.\mathbf{u}$ and complete calculation	•[1] $\mathbf{u}.\mathbf{u}+\mathbf{u}.\mathbf{v}$ •[2] 16 or -10 •[3] -10 or 16 leading to 6	**3**

Part B

Question			Generic scheme	Illustrative scheme	Max mark
15.			•[1] determine radius of circle •[2] find coordinates of P •[3] state equation of circle	•[1] 2 stated or implied by •[3] •[2] $(8,7)$ stated or implied by •[3] •[3] $(x-8)^2+(y-7)^2=4$	**3**

Question			Generic scheme	Illustrative scheme	Max mark
16.			•[1] use laws of logarithms	•[1] $\log_2 3^2$ stated or implied by •[3]	**4**
			•[2] use laws of logarithms	•[2] $\log_2 (12 \times 6)$ stated or implied by •[3]	
			•[3] use laws of logarithms	•[3] $\log_2 \frac{12 \times 6}{3^2}$	
			•[4] evaluate expression	•[4] 3	

Question			Generic scheme	Illustrative scheme	Max mark
17.	(a)		•1 in first quadrant, graph reflected in $y=x$ passing through $(1,3)$ and $(2,7)$ •2 in second quadrant, graph approaching $y=2$ from above	•1, •2  	**2**
	(b)		**Method 1** •3 interpret information and start to solve •4 solve and state coordinates **Method 2** •3 determine $f^{-1}(x)$ •4 evaluate $f^{-1}(0)$ and state coordinates	**Method 1** •3 $\log_5(x-2)+1=0$ leading to $\log_5(x-2)=-1$ •4 $\left(0,\frac{11}{5}\right)$ **Method 2** •3 $5^{(x-1)}+2$ •4 $\left(0,\frac{11}{5}\right)$	**2**

[END OF MARKING INSTRUCTIONS]