H

National
Qualifications
2016

2016 Mathematics

Higher Paper 2

Finalised Marking Instructions

Specific Marking Instructions for each question

Question			Generic Scheme	Illustrative Scheme	Max Mark
1.	(a)	i	●[1] state the midpoint M	●[1] $(2,4)$	**1**
		ii	●[2] calculate gradient of median ●[3] determine equation of median	●[2] 4 ●[3] $y=4x-4$	**2**

Notes:

1. ●[3] is not available as a consequence of using a perpendicular gradient.
2. Accept any rearrangement of $y=4x-4$ for ●[3].
3. On this occasion, accept $y-4=4(x-2)$ or $y-(-4)=4(x-0)$; however, in future candidates should expect that the final equation will only be accepted when it involves a single constant term.
4. ●[3] is only available as a consequence of using points M and P, or any other point which lies on PM, for example the midpoint $(1,0)$.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		●[1] calculate gradient of PR ●[2] use property of perpendicular lines ●[3] determine equation of line	●[1] 1 ●[2] -1 stated or implied by ●[6] ●[3] $y=-x+6$	**3**

Notes:

5. ●[6] is only available as a consequence of using M and a perpendicular gradient.
6. Candidates who use a gradient perpendicular to QR cannot gain ●[4] but ●[5] and ●[6] are still available. See Candidate A.
7. Beware of candidates who use the coordinates of P and Q to arrive at $m=-1$. See Candidate B.
8. On this occasion, accept $y-4=-1(x-2)$; however, in future candidates should expect that the final equation will only be accepted when it involves a single constant term.

Question			Generic Scheme	Illustrative Scheme	Max Mark
Commonly Observed Responses:					
Candidate A $m_{QR}=\frac{1}{4}$ $\bullet^4$ ✗ $m_{perp}=-4$ $\bullet^5$ ✓1 $y=-4x+12$ $\bullet^6$ ✓1				**Candidate B - BEWARE** $m_{PQ}=\frac{2-(-4)}{-6-0}$ $\bullet^4$ ^ $=-1$ $\bullet^5$ ✗ $y-4=-1(x-2)$ $\bullet^6$ ✓2 $y=-x+6$ Note: $\bullet^4$ $\bullet^5$ and $\bullet^6$ may still be available for any candidate that demonstrates that PQ is also perpendicular to PR.	
	(c)		$\bullet^1$ find the midpoint of PR $\bullet^2$ substitute x-coordinate into equation of L. $\bullet^3$ verify y-coordinate and communicate conclusion	**Method 1** $\bullet^1$ $(5,1)$ $\bullet^2$ $y=-5+6$ $(1=-x+6)$ $\bullet^3$ $y=1(x=5)$ $\therefore$ L passes through the midpoint of PR	3
			$\bullet^7$ find the midpoint of PR $\bullet^8$ substitute x and y coordinates into the equation of L $\bullet^9$ verify the point satisfies the equation and communicate conclusion	**Method 2** $\bullet^7$ $x+y=6$ sub $(5,1)$ $\bullet^8$ $5+1=6$ $\bullet^9$ $\therefore$ point $(5,1)$ satisfies equation.	
			$\bullet^7$ find the midpoint of PR $\bullet^8$ find equation of PR $\bullet^9$ use simultaneous equations and communicate conclusion	**Method 3** $\bullet^7$ $(5,1)$ $\bullet^8$ $y=x-4$ $\bullet^9$ $y=1,\ x=5$ $\therefore$ L passes through the midpoint of PR	

Question			Generic Scheme	Illustrative Scheme	Max Mark
				Method 4	
			•[7] find the midpoint of PR	•[7] $(5,1)$	
			•[8] find equation of perpendicular bisector of PR	•[8] $y-1=-1(x-5) \rightarrow y=-x+6$	
			•[9] communicate conclusion	•[9] The equation of the perpendicular bisector is the same as L therefore L passes through the midpoint of PR.	

Notes:

9. A relevant statement is required for •[9] to be awarded.
10. Erroneous working accompanied by a statement such as "L does not pass through the midpoint." does NOT gain •[9].
11. Beware of candidates substituting $(1,5)$ instead of $(5,1)$
12. On this occasion, for Method 3, at •[8] accept $y-1=1(x-5)$; however, in future candidates should expect that the final equation will only be accepted when it involves a single constant term.

Commonly Observed Responses:

Candidate C

$(5,1)$ mid - point •[7] ✓

$y+x=6$

Sub$(5,1)$ •[8] ✗

$5+1=6$

$\therefore$ point $(5,1)$ satisfies equation. •[9] ✗

Candidate has substituted 5 for y and 1 for x.

Question			Generic Scheme	Illustrative Scheme	Max Mark
2.			•[1] use the discriminant	•[1] $(-2)^2-4(1)(3-p)$	**3**
			•[2] simplify and apply the condition for no real roots	•[2] $-8+4p<0$	
			•[3] state range	•[3] $p<2$	

Notes:

1. At the •[1] stage, treat $(-2)^2-4(1)3-p$ and $-2^2-4(1)(3-p)$ as bad form only if the candidate deals with the 'bad form' **term** correctly in the inequation at •[2].
2. If candidates have the condition 'discriminant = 0', then •[2] and •[3] are unavailable.
3. If candidates have the condition 'discriminant $>$ 0', 'discriminant $\geq$ 0' or 'discriminant $\leq$ 0' then •[2] is lost, but •[3] is available **provided the discriminant has been simplified correctly at** •[2].
4. If a candidate works with an equation, then •[2] and •[3] are unavailable. However, see Candidate D.

Commonly Observed Responses:

Candidate A

$(-2)^2-4(1)3-p$	•[1]	✓
$-8+4p<0$	•[2]	✓
$p<2$	•[3]	✓

Candidate B

$(-2)^2-4(1)(3-p)$	•[1]	✓
$-8-4p<0$	•[2]	✗
$p>-2$	•[3]	✓1

Candidate C

$(-2)^2-4(1)3-p$	•[1]	✗	
$-8-p<0$	•[2]	✓2	eased
$p>-8$	•[3]	✓2	eased

Candidate D - Special Case

$b^2-4ac<0$		
$(-2)^2-4(1)(3-p)=0$	•[1]	✓
$-8+4p=0$		
	•[2]	✓
$p=2$		
$p<2$	•[3]	✓2

•2 is awarded since the condition (first line), its application (final line) and the simplification of the discriminant all appear.

Candidate E

$-2^2-4(1)(3-p)$	•[1]	✓
$-8+4p<0$	•[2]	✓
$p<2$	•[3]	✓

Candidate F

$-2^2-4(1)(3-p)$	•[1]	✗
$-16+4p<0$	•[2]	✓2
$p<4$	•[3]	✓1

Candidate G

$-2^2-4(1)(3-p)=0$	•[1]	✓
$-8+4p=0$	•[2]	✗
$p=2$	•[3]	✗

Question			Generic Scheme	Illustrative Scheme	Max Mark
3.	(a)	i	•1 know to substitute $x=-1$ •2 complete evaluation, interpret result and state conclusion	**Method 1** •1 $2(-1)^3-9\times(-1)^2+3\times(-1)+14$ •2 $=0$ $\therefore$ $(x+1)$ is a factor	**2**
			•1 know to use $x=-1$ in synthetic division •2 complete division, interpret result and state conclusion	**Method 2** •1 $\begin{array}{r\|rrrr} -1 & 2 & -9 & 3 & 14 \\ & & -2 & & \\ \hline & 2 & -11 & & \end{array}$ •2 $\begin{array}{r\|rrrr} -1 & 2 & -9 & 3 & 14 \\ & & -2 & 11 & -14 \\ \hline & 2 & -11 & 14 & 0 \end{array}$ remainder = 0 $\therefore$ $(x+1)$ is a factor	
			•1 start long division and find leading term in quotient •2 complete division, interpret result and state conclusion	**Method 3** •1 $\begin{array}{r l} & 2x^2 \\ (x+1) & \overline{)\,2x^3-9x^2+3x+14} \end{array}$ •2 $\begin{array}{r l} & 2x^2-11x+14 \\ (x+1) & \overline{)\,2x^3-9x^2+3x+14} \\ & \underline{2x^3+2x^2} \\ & \quad -11x^2+3x \\ & \quad \underline{-11x^2-11x} \\ & \qquad\quad 14x+14 \\ & \qquad\quad \underline{14x+14} \\ & \qquad\qquad\quad 0 \end{array}$ remainder = 0 $\therefore$ $(x+1)$ is a factor	

Question			Generic Scheme	Illustrative Scheme	Max Mark

Notes:

1. Communication at $\bullet^2$ must be consistent with working at that stage ie a candidate's working must arrive legitimately at 0 before $\bullet^2$ can be awarded.
2. Accept any of the following for $\bullet^2$:
   - '$f(-1)=0$ so $(x+1)$ is a factor'
   - 'since remainder = 0, it is a factor'
   - the 0 from any method linked to the word 'factor' by eg 'so', 'hence', '$\therefore$', '$\rightarrow$', '$\Rightarrow$'
3. Do not accept any of the following for $\bullet^2$:
   - double underlining the zero or boxing the zero without comment
   - '$x=1$ is a factor', '$(x-1)$ is a factor', '$(x-1)$ is a root', '$x=-1$ is a root', '$(x+1)$ is a root'
   - the word 'factor' only with no link

Commonly Observed Responses:

		ii	$\bullet^3$ state quadratic factor $\bullet^4$ find remaining linear factors or substitute into quadratic formula $\bullet^5$ state solution	$\bullet^3$ $2x^2-11x+14$ $\bullet^4$ $....(2x-7)(x-2)$ or $\dfrac{11\pm\sqrt{(-11)^2-4\times2\times14}}{2\times2}$ $\bullet^5$ $x=-1,\ 2,\ 3{\cdot}5$	**3**

Notes:

4. On this occasion, the appearance of "$=0$" is not required for $\bullet^5$ to be awarded.
5. Be aware that the solution, $x=-1,\ 2,\ 3{\cdot}5$, may not appear until part (b).

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)	(i)	•[6] state coordinates	•[6] $(-1,0)$ and $(2,0)$	1

Notes:

6. '-1 and 2' does not gain •[6]
7. $x=-1, y=0$ **and** $x=2, y=0$ gains •[6]

Commonly Observed Responses:

		(ii)	•[7] know to integrate with respect to x	•[7] $\int(2x^3-9x^2+3x+14)\,dx$	4
			•[8] integrate	•[8] $\frac{2x^4}{4}-\frac{9x^3}{3}+\frac{3x^2}{2}+14x$	
			•[9] interpret limits and substitute	•[9] $\left(\frac{2\times2^4}{4}-\frac{9\times2^3}{3}+\frac{3\times2^2}{2}+14\times2\right)-\left(\frac{2\times(-1)^4}{4}-\frac{9\times(-1)^3}{3}+\frac{3\times(-1)^2}{2}+14\times(-1)\right)$	
			•[10] evaluate integral	•[10] 27	

Candidate A

$\int(2x^3-9x^2+3x+14)\,dx$ •[7] ✓

$\frac{2x^4}{4}-\frac{9x^3}{3}+\frac{3x^2}{2}+14x$ •[8] ✓

27 •[9] ^ •[10] ✓1

Candidate B

$\int(2x^3-9x^2+3x+14)\,dx$ •[7] ✓

$\frac{2x^4}{4}-\frac{9x^3}{3}+\frac{3x^2}{2}+14x$ •[8] ✓

$\left(\frac{2\times(-1)^4}{4}-\frac{9\times(-1)^3}{3}+\frac{3\times(-1)^2}{2}+14\times(-1)\right)-\left(\frac{2\times2^4}{4}-\frac{9\times2^3}{3}+\frac{3\times2^2}{2}+14\times2\right)$ •[9] ✗

-27, hence area is 27 •[10] ✓1

However, $-27=27$ does not gain •[10].

Question			Generic Scheme	Illustrative Scheme	Max Mark
			Candidate C $\int \ldots = -27$ cannot be negative so $= 27$ $\bullet^{10}$ ✗ However, $\int \ldots = -27$ so Area $= 27$ $\bullet^{10}$ ✓		

Notes:

8. $\bullet^{7}$ is not available to candidates who omit ' dx '
9. Do not penalise the absence of brackets at the $\bullet^{7}$ stage
10. Where a candidate differentiates one or more terms at $\bullet^{8}$, then $\bullet^{8}$, $\bullet^{9}$ and $\bullet^{10}$ are not available.
11. Candidates who substitute limits without integrating do not gain $\bullet^{8}$, $\bullet^{9}$ or $\bullet^{10}$.
12. For candidates who make an error in (a), $\bullet^{9}$ is available only if the lower limit is negative and the upper limit is positive.
13. Do not penalise the inclusion of '$+c$'.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
4.	(a)		●[1] centre of C_1 ●[2] radius of C_1 ●[3] centre of C_2 ●[4] radius of C_2	●[1] $(-5,6)$ ●[2] 3 ●[3] $(3,0)$ ●[4] 5	**4**
Notes:					
Commonly Observed Responses:					
	(b)		●[5] calculate the distance between the centres ●[6] calculate the sum of the radii ●[7] interpret significance of calculations	●[1] 10 ●[2] 8 ●[3] $8<10$ $\therefore$ the circles do not intersect	**3**
Notes:					
1. For ●[7] to be awarded a comparison must appear. 2. Candidates who write '$r_1+r_2<\text{D}$', or similar, must have identified the value of r_1+r_2 and the value of D somewhere in their solution for ●[7] to be awarded. 3. Where earlier errors lead to the candidate dealing with non-integer values, do not penalise inaccuracies in rounding unless they lead to an inconsistent conclusion.					
Commonly Observed Responses:					

Question			Generic Scheme	Illustrative Scheme	Max Mark
5.	(a)		•[1] find $\overrightarrow{AB}$	•[1] $\begin{pmatrix} -8 \\ 16 \\ 2 \end{pmatrix}$	**2**
			•[2] find $\overrightarrow{AC}$	•[2] $\begin{pmatrix} -2 \\ -8 \\ 16 \end{pmatrix}$	

Notes:

1. For candidates who find **both** $\overrightarrow{BA}$ **and** $\overrightarrow{CA}$ correctly, only •[2] is available (repeated error).
2. Accept vectors written horizontally.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		**Method 1**	**Method 1**	**4**
			•[1] evaluate $\overrightarrow{AB}.\overrightarrow{AC}$	•[1] $\overrightarrow{AB}.\overrightarrow{AC} = 16 - 128 + 32 = -80$	
			•[2] evaluate $\lvert\overrightarrow{AB}\rvert$ and $\lvert\overrightarrow{AC}\rvert$	•[2] $\lvert\overrightarrow{AB}\rvert = \lvert\overrightarrow{AC}\rvert = 18$	
			•[3] use scalar product	•[3] $\cos BAC = \dfrac{-80}{18 \times 18}$	
			•[4] calculate angle	•[4] $104 \cdot 3°$ or $1 \cdot 82$ radians	
			Method 2	**Method 2**	
			•[3] calculate length of BC	•[3] $BC = \sqrt{808}$	
			•[4] calculate lengths of AB and AC	•[4] $AB = AC = 18$	
			•[5] use cosine rule	•[5] $\cos BAC = \dfrac{18^2 + 18^2 - \sqrt{808}^2}{2 \times 18 \times 18}$	
			•[6] calculate angle	•[6] $104 \cdot 3°$ or $1 \cdot 82$ radians	

Question	Generic Scheme	Illustrative Scheme	Max Mark

Notes:

3. Accept $\sqrt{324}$ at $\bullet^4$ and $\bullet^5$.
4. $\bullet^5$ is not available to candidates who simply state the formula $\cos\theta = \frac{a.b}{|a||b|}$.

   However $\cos\theta = \frac{-80}{18\times18}$ is acceptable. Similarly for Method 2.
5. Accept correct answers rounded to $104°$ or $1{\cdot}8$ radians.
6. Due to $\overrightarrow{AB}$ and $\overrightarrow{AC}$ having equal magnitude, $\bullet^4$ is not available unless both $|\overrightarrow{AB}|$ **and** $|\overrightarrow{AC}|$ have been stated.
7. $\bullet^6$ is only available as a result of using a valid strategy.
8. $\bullet^6$ is only available for a single angle.
9. For a correct answer with no working award 0/4.

Commonly Observed Responses:

Candidate A

$\overrightarrow{BA}.\overrightarrow{AC} = -16 + 128 - 32 = 80$	$\bullet^3$ ✗
$\lvert\overrightarrow{AB}\rvert = \lvert\overrightarrow{AC}\rvert = 18$	$\bullet^4$ ✓1
$\cos\theta = \frac{80}{18\times18}$	$\bullet^5$ ✓1
	$\bullet^6$ ✗
$75{\cdot}7$ or $1{\cdot}32$ radians	

Question			Generic Scheme	Illustrative Scheme	Max Mark
6.	(a)		•¹ state the number	•¹ 200	**1**

Notes:

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		•² interpret context and form equation •³ knowing to use logarithms appropriately. •⁴ simplify •⁵ evaluate t	•² $2 = e^{0\cdot107t}$ •³ $\ln 2 = \ln\left(e^{0\cdot107t}\right)$ •⁴ $\ln 2 = 0\cdot107t$ •⁵ $t = 6\cdot478...$	**4**

Notes:

1. Accept $400 = 200e^{0\cdot107t}$ or equivalent for •²
2. Any base may be used at the •³ stage.
3. •³ may be assumed by •⁴.
4. Accept $t = 6\cdot5$.
5. At •⁵ ignore incorrect units. However, see Candidates B and C.
6. The calculation at •⁵ must involve the evaluation of a logarithm within a valid strategy for •⁵ to be awarded.
7. Candidates who take an iterative approach to arrive at $t = 6\cdot5$ gain •² only.
   However, if, in the iterations, $B(t)$ is evaluated for $t = 6\cdot45$ and $t = 6\cdot55$ then award 4/4.

Commonly Observed Responses:

Candidate A	Candidate B
$2 = e^{0\cdot107t}$ •² ✓ $\log_{10} 2 = \log_{10}\left(e^{0\cdot107t}\right)$ •³ ✓ $\log_{10} 2 = 0\cdot107t\log_{10}e$ •⁴ ✓ $t = 6\cdot478\ldots$ •⁵ ✓	$t = 6\cdot48$ hours •⁵ ✓ $t = 6$ hours 48 minutes
Candidate C	**Candidate D**
$\ln(2) = 0\cdot107t$ •⁴ ✓ $t = 6$ hours 48 minutes •⁵ ✗	$400 = 200e^{0\cdot107t}$ •² ✓ $e^{0\cdot107t} = 2$ •³ ^ $t = 6\cdot48$ hours ✓1 •⁴ ✓1 •⁵

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.	(a)		•[1] expression for length in terms of x and y •[2] obtain an expression for y •[3] demonstrate result	•[1] $9x+8y$ •[2] $y=\frac{108}{6x}$ •[3] $\mathrm{L}(x)=9x+8\left(\frac{108}{6x}\right)$ leading to $\mathrm{L}(x)=9x+\frac{144}{x}$	3

Notes:

1. The substitution for y at •[3] must be clearly shown for •[3] to be available.
2. For candidates who omit some, or all, of the internal fencing, only •[2] is available.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		•[4] know to and start to differentiate •[5] complete differentiation •[6] set derivative equal to 0 •[7] obtain for x •[8] verify nature of stationary point •[9] interpret and communicate result	•[4] $\mathrm{L}'(x)=9\ldots$ •[5] $L'(x)=9-\frac{144}{x^2}$ •[6] $9-\frac{144}{x^2}=0$ •[7] $x=4$ •[8] Table of signs for a derivative – see the additional page. •[9] Minimum at $x=4$ **or** •[8] $\mathrm{L}''(x)=\frac{288}{x^3}$ •[9] $\mathrm{L}''(4)>0 \;\therefore$ minimum Do not accept $\frac{d^2y}{dx^2}=\ldots$	6

Question	Generic Scheme	Illustrative Scheme	Max Mark

Notes:

3. For candidates who integrate at the ●^{4} stage ●^{5}, ●^{6}, ●^{7}, ●^{8} and ●^{9} are unavailable.
4. ●^{7}, ●^{8} and ●^{9} are only available for working with a derivative which contains a term with an index ≤ -2.
5. At ●^{5} and ●^{6} accept $-144x^{-2}$.
6. $\sqrt{\frac{144}{9}}$ must be simplified at the ●^{7}, ●^{8} or ●^{9} stage for ●^{7} to be awarded.
7. ●^{9} is not available to candidates who consider a value ≤ 0 in the neighbourhood of 4.
8. A candidate's table of signs must be valid and legitimately lead to a minimum for ●^{9} to be awarded.
9. ●^{9} is not available to candidates who state the minimum exists at a negative value of x.

Commonly Observed Responses:

Table of signs for a derivative

Accept

x	-4^-	-4	-4^+
$L'(x)$	+	0	–
Shape or Slope	/	—	\

x	4^-	4	4^+
$L'(x)$	–	0	+
Shape or Slope	\	—	/

x	$\rightarrow$	-4	$\rightarrow$
$L'(x)$	+	0	–
Shape or Slope	/	—	\

x	$\rightarrow$	4	$\rightarrow$
$L'(x)$	–	0	+
Shape or Slope	\	—	/

Here, for exemplification, tables of signs considering both roots separately have been displayed. However, in this question, it was only expected that candidates would consider the positive root for $\bullet^8$. Do not penalise the consideration of the negative root.

Arrows are taken to mean "in the neighbourhood of"

x	a	-4	b	c	4	d
$L'(x)$	+	0	–	–	0	+
Shape or Slope	/	—	\	\	—	/

Where: $a < -4$, $-4 < b < 0$, $0 < c < 4$, $d > 4$

Do not Accept

x	a	-4	b	4	c
$L'(x)$	+	0	–	0	+
Shape or Slope	/	—	\	—	/

Since the function is discontinuous '$-4 < b < 4$' is not acceptable.

x	$\rightarrow$	-4	$\rightarrow$	4	$\rightarrow$
$L'(x)$	+	0	–	0	+
Shape or Slope	/	—	\	—	/

Since the function is discontinuous '$-4 \rightarrow 4$' is not acceptable.

General Comments

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of $L'(x)$ in the table is an acceptable alternative to writing '+' or '-' signs.
- Acceptable alternatives for $L'(x)$ are: L' , $\frac{dL}{dx}$ or $9-\frac{144}{x^2}$. **DO NOT** accept $\frac{dy}{dx}$ or $f'(x)$.

Question			Generic Scheme	Illustrative Scheme	Max Mark
8.	(a)		•[1] use compound angle formula	•[1] $k\cos x\cos a - k\sin x\sin a$ **stated explicitly**	**4**
			•[2] compare coefficients	•[2] $k\cos a = 5, k\sin a = 2$ **stated explicitly**	
			•[3] process for k	•[3] $k = \sqrt{29}$	
			•[4] process for a and express in required form	•[4] $\sqrt{29}\cos(x + 0\cdot38)$	

Notes:

1. Treat $k\cos x\cos a - \sin x\sin a$ as bad form only if the equations at the •[2] stage both contain k.
2. $\sqrt{29}\cos x\cos a - \sqrt{29}\sin x\sin a$ or $\sqrt{29}(\cos x\cos a - \sin x\sin a)$ is acceptable for •[1] and •[3].
3. Accept $k\cos a = 5, \; -k\sin a = -2$ for •[2].
4. •[2] is not available for $k\cos x = 5, \; k\sin x = 2$, however, •[4] is still available.
5. •[3] is only available for a single value of $k, \; k > 0$.
6. Candidates who work in degrees and do not convert to radian measure do not gain •[4].
7. Candidates may use any form of the wave equation for •[1], •[2] and •[3], however, •[4] is only available if the value of a is interpreted in the form $k\cos(x + a)$.
8. Accept any answer for a that rounds to $0\cdot38$.
9. Evidence for •[4] may not appear until part (b).

Commonly Observed Responses:

Responses with missing information in working:

Candidate A

	•[1] ^
$\sqrt{29}\cos a = 5$	•[2] ✓
$\sqrt{29}\sin a = 2$	•[3] ✓
$\tan a = \frac{2}{5}, \; a = 0\cdot3805...$	
$\sqrt{29}\cos(x + 0\cdot38)$	•[4] ✓

Candidate B

$k\cos x\cos a - k\sin x\sin a$	•[1] ✓
$\cos a = 5$	
$\sin a = 2$	•[2] ✗
	•[3] ✓
$\tan a = \frac{2}{5}$	
$a = 0\cdot38$ (Not consistent with equations at •[2].)	
$\sqrt{29}\cos(x + 0\cdot38)$	•[4] ✗

Responses with the correct expansion of $k\cos(x + a)$ but errors for either •[2] or •[4].

Candidate C

$k\cos a = 5, \; k\sin a = 2$	•[2] ✓
$\tan a = \frac{5}{2}$	•[4] ✗
$a = 1\cdot19$	

Candidate D

$k\cos a = 2, \; k\sin a = 5$	•[2] ✗
$\tan a = \frac{5}{2}, \; a = 1\cdot19$	
$\sqrt{29}\cos(x + 1\cdot19)$	•[4] ✓1

Candidate E

$k\cos a = 5, \; k\sin a = -2$	•[2] ✗
$\tan a = \frac{-2}{5}$	
$\sqrt{29}\cos(x + 5\cdot90)$	•[4] ✓1

Question	Generic Scheme	Illustrative Scheme	Max Mark

Responses with the incorrect labelling; $k\cos A\cos B - k\sin A\sin B$ from formula list.

Candidate F	**Candidate G**	**Candidate H**
$k\cos A\cos B - k\sin A\sin B$ $\bullet^1$ ✗	$k\cos A\cos B - k\sin A\sin B$ $\bullet^1$ ✗	$k\cos A\cos B - k\sin A\sin B$ $\bullet^1$ ✗
$k\cos a = 5,\ k\sin a = 2$ $\bullet^2$ ✓	$k\cos x = 5,\ k\sin x = 2$ $\bullet^2$ ✗	$k\cos B = 5,\ k\sin B = 2$ $\bullet^2$ ✓1
$\tan a = \frac{2}{5},\ a = 0\cdot3805...$	$\tan x = \frac{2}{5},\ x = 0\cdot3805...$	$\tan B = \frac{2}{5},\ B = 0\cdot3805...$
$\sqrt{29}\cos\left(x + 0\cdot38\right)$ $\bullet^3$ ✓ $\bullet^4$ ✓	$\sqrt{29}\cos\left(x + 0\cdot38\right)$ $\bullet^3$ ✓ $\bullet^4$ ✓1	$\sqrt{29}\cos\left(x + 0\cdot38\right)$ $\bullet^3$ ✓ $\bullet^4$ ✓1

Question	Generic Scheme	Illustrative Scheme	Max Mark
(b)	$\bullet^5$ equate to 12 and simplify constant terms	$\bullet^5$ $5\cos x - 2\sin x = 2$ or $5\cos x - 2\sin x - 2 = 0$	4
	$\bullet^6$ use result of part (a) and rearrange	$\bullet^6$ $\cos\left(x + 0\cdot3805\ldots\right) = \frac{2}{\sqrt{29}}$	
	$\bullet^7$ solve for $x + a$	$\bullet^7$ $1\cdot1902\ldots,$ ($\bullet^7$) $5\cdot0928\ldots$ ($\bullet^8$)	
	$\bullet^8$ solve for x	$\bullet^8$ $0\cdot8097\ldots,$ $4\cdot712\ldots$	

Notes:

10. The values of x may be given in radians or degrees.
11. Do not penalise candidates who attribute the values of x to the wrong points.
12. Accept any answers, in degrees or radians, that round correct to one decimal place.
13. $\bullet^4$ is unavailable for candidates who give their answer in degrees in part (a) and in part (b). $\bullet^4$ is unavailable for candidates who give their answer in degrees in part (a) and radians in part (b). $\bullet^8$ is unavailable for candidates who give their answer in radians in part (a) and degrees in part (b).

Conversion Table:

Degrees	Radians
$21\cdot8$	$0\cdot3805...$
$46\cdot4$	$0\cdot8097...$
$68\cdot2$	$1\cdot190...$
270	$4\cdot712...$ or $\frac{3\pi}{2}$
$291\cdot8$	$5\cdot0928...$

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
9.			•1 write in integrable form	•1 $2x^{\frac{1}{2}}+x^{-\frac{1}{2}}$	4
			•2 integrate one term	•2 $\frac{4}{3}x^{\frac{3}{2}}$ or $2x^{\frac{1}{2}}$	
			•3 complete integration	•3 $2x^{\frac{1}{2}}+c$ or $\frac{4}{3}x^{\frac{3}{2}}+c$	
			•4 state expression for $f(x)$	•4 $f(x)=\frac{4}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}-2$	

Notes:

1. For candidates who do not attempt to write $f'(x)$ as the sum of two integrable terms, award 0/4.
2. •2 and •3 are only available for integrating terms involving fractional indices.
3. The term integrated at •3 must have an index of opposite sign to that of the term integrated at •2.
4. For candidates who differentiate one term, only •1 and •2 are available.
5. For candidates who differentiate both terms, only •1 is available.
6. For •4 . accept ' $f(x)=\frac{4}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+c,\ c=-2$ '
7. Candidates must simplify coefficients in **<u>their</u>** final line of working for the last mark available for that line of working to be awarded.

Commonly Observed Responses:

Candidate A		Candidate B	
$f'(x)=2x+x^{-\frac{1}{2}}$	•1 ✗	$f'(x)=2x+x^{-\frac{1}{2}}$	•1 ✗
$x^2+2x^{\frac{1}{2}}+c$	•2 ✓ •3 ☑2	$x^2+2x^{\frac{1}{2}}$	•2 ✓ •3 ✗
$f(x)=x^2+2x^{\frac{1}{2}}-47$	•4 ☑1	$f(x)=x^2+2x^{\frac{1}{2}}$	•4 ^

Candidate C		Candidate D	
$f'(x)=2x^{\frac{1}{2}}+1$	•1 ✗	$f'(x)=\frac{2x+1}{x^{\frac{1}{2}}}$	•1 ^
$\frac{4}{3}x^{\frac{3}{2}}+x+c$	•2 ✓ •3 ☑2	$\frac{x^2+x}{2x^{\frac{3}{2}}}+c$	•2 ✗ •3 ✗ •4 ✗ See Note 1
$f(x)=\frac{4}{3}x^{\frac{3}{2}}+x-5$	•4 ☑1	$f(x)=\frac{x^2+x}{2x^{\frac{3}{2}}}+\frac{115}{3}$	

Question	Generic Scheme	Illustrative Scheme	Max Mark

Candidate E

$f'(x) = 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}$ •1 ✓

$= \dfrac{2x^{\frac{3}{2}}}{\frac{3}{2}} + \dfrac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$ •2 ✓

•3 ✓2 •4 ^

Question			Generic Scheme	Illustrative Scheme	Max Mark
10.	(a)		•1 Start to differentiate •2 Complete differentiation	•1 $\frac{1}{2}\left(x^2+7\right)^{-\frac{1}{2}}$..... •2 ....$\times\ 2x$	**2**

Notes:

1. On this occasion there is no requirement to simplify coefficients.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		•3 link to (a) and integrate	•3 $4\left(x^2+7\right)^{\frac{1}{2}}\ (+\ c)$	**1**

Notes:

2. A candidate's answer at •3 must be consistent with earlier working.

Commonly Observed Responses:

Candidate A

$\int 4x(x^2+7)^{\frac{-1}{2}}\,dx$

$= \dfrac{4x(x^2+7)^{\frac{1}{2}}}{\frac{1}{2}\times 2x} + c$

$= \dfrac{4x(x^2+7)^{\frac{1}{2}}}{x} + c$

$= 4(x^2+7)^{\frac{1}{2}} + c$ •3 ✗

Question			Generic Scheme	Illustrative Scheme	Max Mark
11.	(a)		●[1] substitute for $\sin 2x$ and $\tan x$	●[1] $(2\sin x\cos x)\times\frac{\sin x}{\cos x}$	**4**
			●[2] simplify	●[2] $2\sin^2 x$	
			●[3] use an appropriate substitution	●[3] $2\left(1-\cos^2 x\right)$ or $1-\left(1-2\sin^2 x\right)$	
			●[4] simplify and communicate result	●[4] $1-\cos 2x=1-\cos 2x$ or $2\sin^2 x=2\sin^2 x$ $\therefore$ Identity shown	

Notes:

1. ●[1] is not available to candidates who simply quote $\sin 2x=2\sin x\cos x$ and $\tan x=\frac{\sin x}{\cos x}$ without substituting into the identity.
2. ●[4] is not available to candidates who work throughout with A in place of x.
3. ●[3] is not available to candidates who simply quote $\cos 2x=1-2\sin^2 x$ without substituting into the identity.
4. On this occasion, at ●[4] do not penalise the omission of 'LHS = RHS' or a similar statement.

Commonly Observed Responses:

Candidate A		Candidate B	
$\sin 2x\tan x=1-\cos 2x$		$\sin 2x\tan x=1-\cos 2x$	●[1] ^ ●[2] ^
$2\sin x\cos x\times\frac{\sin x}{\cos x}=1-\cos 2x$	●[1] ✓	$\sin 2x\tan x=1-(1-2\sin^2 x)$	
$2\sin^2 x=1-\cos 2x$	●[2] ✓	$\sin 2x\tan x=2\sin^2 x$	●[3] ✓
$2\sin^2 x-1=-\cos 2x$	●[3] ✗ ●[4] ✗	$\tan x=\frac{2\sin^2 x}{2\sin x\cos x}$	●[4] ✗
$-(1-2\sin^2 x)=-\cos 2x$		$\tan x=\tan x$	
$-\cos 2x=-\cos 2x$			
In proving the identity, candidates must work with both sides independently. ie in each line of working the LHS must be equivalent to the left hand side of the line above.			

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		●[5] link to (a) and substitute ●[6] differentiate	●[5] $f(x)=1-\cos 2x$ or $f(x)=2\sin^2 x$ ●[6] $f'(x)=2\sin 2x$ or $f'(x)=4\sin x\cos x$	2
Notes:					
Commonly Observed Responses:					

[END OF MARKING INSTRUCTIONS]