Paper 1 Section A

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5	D
6	A
7	C
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Paper 1- Section B

Question		Generic Scheme			Illustrative Scheme	Max Mark
21	a					
		$\bullet^1$	ss	know to differentiate and one term correct	$\bullet^1$ $=6x....$ or $=....-3x^2$	
		$\bullet^2$	ss	the other term correct and set derivative to 0	$\bullet^2$ $6x-3x^2=0$ stated explicitly	
		$\bullet^3$	pd	solve $\dfrac{dy}{dx}=0$	$\bullet^3$ $x=\;0 \quad 2$	
		$\bullet^4$	pd	evaluate y coordinates	$\bullet^4$ $y=\;0 \quad 4$	
		$\bullet^5$	pd	justify nature of stationary points	$\bullet^5$ use 2nd derivative or nature table	
		$\bullet^6$	ic	interpretation	$\bullet^6$ min. at $(0,0)$ and max. at $(2,4)$	**6**

Notes:

1. $\bullet^2$ is not available for statements such as '$\dfrac{dy}{dx}=0$' with no other working.
2. Accept $3x^2-6x=0$ for $\bullet^2$.
3. For candidates using a nature table, the minimum response for $\bullet^5$ is:

   x values 0 and 2; $\dfrac{dy}{dx}$ or expression $6x-3x^2$; signs and zeroes; shape.

$$\begin{array}{c|ccccc} & & 0 & & 2 & \\ \hline \dfrac{dy}{dx} & - & 0 & + & 0 & - \end{array}$$

4. For candidates who differentiate correctly but then solve $\dfrac{dy}{dx}=0$ incorrectly, $\bullet^4$ may be awarded as a follow through mark. $\bullet^5$ and $\bullet^6$ are not available if a nature table has been used, but may be awarded where candidates have used the 2nd derivative.
5. For candidates who differentiate incorrectly $\bullet^3$ and $\bullet^4$ may be awarded as follow through marks. $\bullet^5$ and $\bullet^6$ are not available if a nature table has been used, but may be awarded where candidates have used the 2nd derivative.
6. At $\bullet^6$ stage accept min at $x=0$ and max at $x=2$.
7. Candidates who find the x-coordinates of the SPs correctly but correctly process only one of these to determine its nature, gain $\bullet^6$ but not $\bullet^5$.

Commonly Observed Responses:

Candidate A

$$\frac{d^2y}{dx^2} = 6 - 6x$$

at $x = 0$, $\frac{d^2y}{dx^2} > 0$, at $x = 2$, $\frac{d^2y}{dx^2} < 0$ $\quad \bullet^5$ ✓

hence minimum SP at $x = 0$, maximum SP at $x = 2$ $\quad \bullet^6$ ✓

Candidate B

$\frac{dy}{dx} = 6x - 3x^2 = 0$ $\quad \bullet^1$ ✓ $\bullet^2$ ✓

$3x(3-x) = 0$

$x = 0, x = 3$ $\quad \bullet^3$ x

$y = 0, y = 0$ $\quad \bullet^4$ ✓

Case (i)

$\frac{d^2y}{dx^2} = 6 - 6x$

$x = 0 \Rightarrow \frac{d^2y}{dx^2} > 0 \Rightarrow$ Minimum SP $\quad \bullet^5$ ✓

$x = 3 \Rightarrow \frac{d^2y}{dx^2} < 0 \Rightarrow$ Maximum SP $\quad \bullet^6$ ✓

Case (ii)

x	$\to$	0	$\to$	3	$\to$
$\frac{dy}{dx}$	$-$	0	?	?	$+$

$\bullet^5$ x $\bullet^6$ x

? inconsistent. Different signs for $6x - 3x^2$ or $3x(3-x)$

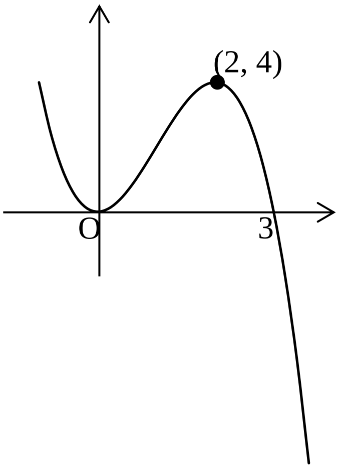
21	b			
$\bullet^7$	pd	find intercepts	$\bullet^7$ $3x^2 - x^3 = 0$ **and** $(3,0)$ or $x = 3$; $(0,0)$ [may appear in part a]	
$\bullet^8$	ic	sketch	$\bullet^8$ sketch	2

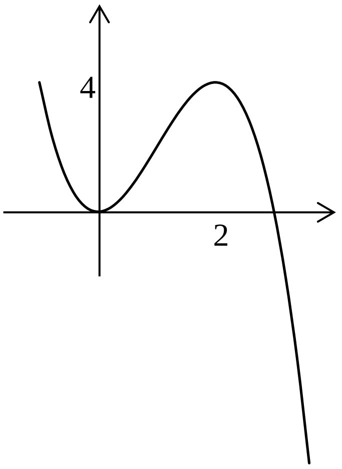
Notes:

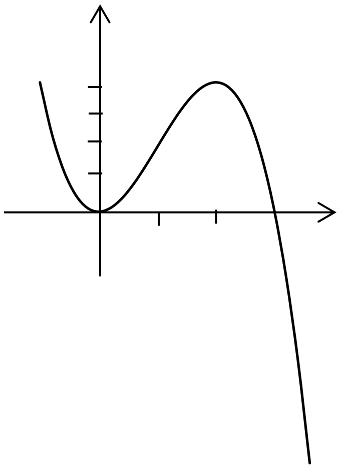
8. $\bullet^7$ accept $3x^2 - x^3 = 0$ **and** correctly annotated diagram with 0, 3 and no other intercepts marked on sketch.
9. The minimum required for $\bullet^8$ is a cubic curve, consistent with the SPs found in part (a) and appropriate number of x intercepts appearing on their sketch. It must be possible to determine the coordinates of the SPs from the sketch.

Commonly Observed Responses:

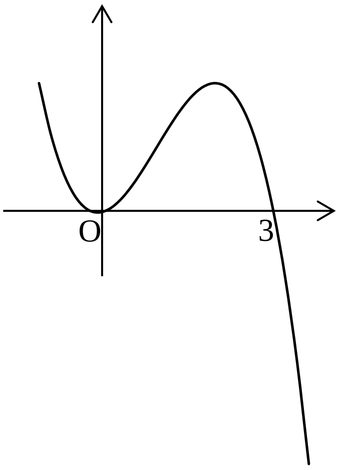
The following are acceptable for •[8]

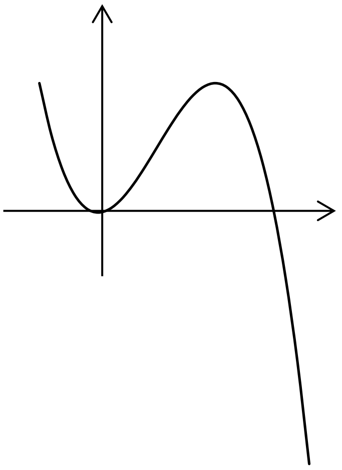


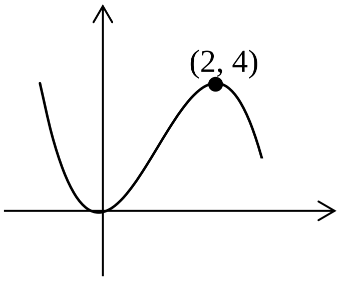


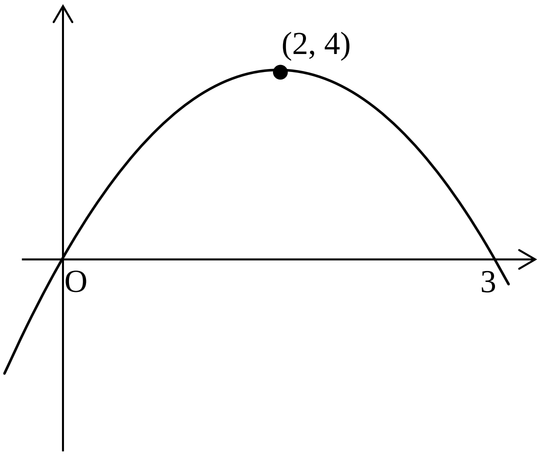


Do not accept the following for •[8]









Question		Generic Scheme	Illustrative Scheme	Max Mark
22	a			
		•1 ss know to use $x=-1$ and obtain an equation	•1 $6(-1)^3+7(-1)^2+a(-1)+b=0$	4
		•2 ss know to use $x=2$ and obtain an equation	•2 $6(2)^3+7(2)^2+a(2)+b=72$	
		•3 pd process equations to find one value	•3 $a=-1$ or $b=-2$	
		•4 pd find the other value	•4 $b=-2$ or $a=-1$	
			Alternative Method for •1 and •2 •1 $\begin{array}{r\|cccc} -1 & 6 & 7 & a & b \\ & & -6 & -1 & -a+1 \\ \hline & 6 & 1 & a-1 & b-a+1=0 \end{array}$ •2 $\begin{array}{r\|cccc} 2 & 6 & 7 & a & b \\ & & 12 & 38 & 2a+76 \\ \hline & 6 & 19 & a+38 & 2a+b+76=72 \end{array}$	

Note:

1. An incorrect value at •3 should be followed through for the possible award of •4. However, if the equations are such that no solution exists, then •3 and •4 are not available.

Commonly Observed Responses:

Candidate A

•1 x

$$\begin{array}{r|cccc} 1 & 6 & 7 & a & b \\ & & 6 & 13 & a+13 \\ \hline & 6 & 13 & a+13 & a+b+13=0 \end{array}$$

•2 ✓ repeated error

$$\begin{array}{r|cccc} -2 & 6 & 7 & a & b \\ & & -12 & 10 & -2a-20 \\ \hline & 6 & -5 & a+10 & -2a+b-20=72 \end{array}$$

Solving to get $a=-35,\ b=22$ •3 ✓ •4 ✓

Leading to, in part (b), $\Rightarrow 6x^3+7x^2-35x+22=(x-1)(6x^2+13x-22)$

•5 ✓ •6 ✓ •7 ^

Question		Generic Scheme		Illustrative Scheme	Max Mark
22	b				
$\bullet^5$	ss	substitute for a and b and know to divide by $x+1$	$\bullet^5$	$(6x^3+7x^2-x-2)\div(x+1)$ Stated or implied by $\bullet^6$	
$\bullet^6$	pd	obtain quadratic factor	$\bullet^6$	$(x+1)(6x^2+x-2)$	
$\bullet^7$	pd	complete factorisation	$\bullet^7$	$(x+1)(3x+2)(2x-1)$	3

Notes:

2. For candidates who substitute $a=-1$ into the correct quotient from part (a), $\bullet^5$, $\bullet^6$ and $\bullet^7$ are available.
3. Candidates who use incorrect values obtained in part (a) may gain $\bullet^5$, $\bullet^6$ and $\bullet^7$
4. Where the quadratic factor obtained is irreducible, candidates must clearly demonstrate that $b^2-4ac<0$ to gain $\bullet^7$.
5. Do not penalise the inclusion of '$=0$' or for solving for x.
6. Candidates who use values, ex nihilo, for a and b can gain $\bullet^5$, if division is correct, but $\bullet^6$ and $\bullet^7$ are only available if $(x+1)$ is a factor of the resulting expression.

Commonly Observed Responses:

Candidate B

22a no solution

22b $a=-4, b=-5$ ex nihilo

$(6x^3+7x^2-4x-5)\div(x+1)$ $\bullet^5$ ✓

$$\begin{array}{r|rrrr} -1 & 6 & 7 & -4 & -5 \\ & & -6 & -1 & 5 \\ \hline & 6 & 1 & -5 & 0 \end{array}$$

$(x+1)(6x^2+x-5)$ $\bullet^6$ ✓

$(x+1)(6x-5)(x+1)$ $\bullet^7$ ✓

Candidate C

22a no solution

22b $a=2, b=3$ ex nihilo

$(6x^3+7x^2+2x+3)\div(x+1)$ $\bullet^5$ ✓

$$\begin{array}{r|rrrr} -1 & 6 & 7 & 2 & 3 \\ & & -6 & -1 & -1 \\ \hline & 6 & 1 & 1 & 2 \end{array}$$

$\Rightarrow$ $(x+1)$ is not a factor

$\bullet^6$ and $\bullet^7$ are not available

Candidate D

22a no solution

22b $a=4, b=3$ ex nihilo

$(6x^3+7x^2+4x+3)\div(x+1)$ $\bullet^5$ ✓

$$\begin{array}{r|rrrr} -1 & 6 & 7 & 4 & 3 \\ & & -6 & -1 & -3 \\ \hline & 6 & 1 & 3 & 0 \end{array}$$

$(x+1)(6x^2+x+3)$ $\bullet^6$ ✓

$b^2-4ac=1-72=-71$

$-71<0$ so does not factorise $\bullet^7$ ✓

Question		Generic Scheme	Illustrative Scheme	Max Mark
23	a			
		•1 ss substitute $3x-5$	•1 $x^2+(3x-5)^2+2x-4(3x-5)-15=0$	
		•2 pd express in standard quadratic form	•2 $10x^2-40x+30=0$	
		•3 pd find x-coordinates	•3 $x=1$ $x=3$	
		•4 pd find y-coordinates	•4 $y=-2$ $y=4$ •3 •4	4

Notes:

1. '$=0$' must appear at •1 or •2 for mark •2 to be awarded.
2. If $x=\frac{1}{3}(y+5)$ is substituted at •1 then $10y^2-20y-80=0$ is obtained at •2.
3. **Special Case:** In cases where $x=1$ **and** $x=3$ do not appear as a result of •1 and •2, but are substituted into the equation of the line to obtain the y values, **if** the candidate then checks that both points lie on the circle, $3/4$ marks are awarded. If, in addition, the candidate makes a statement to the effect that a line can only cut a circle in, at most, 2 points, then $4/4$ marks are awarded. Otherwise, $0/4$ marks.
4. •3 and •4 are not available for any attempt to solve a quadratic equation of the form $ax^2+bx=c$

Commonly Observed Responses:

Candidate A

$x^2+(3x-5)^2+2x-4(3x-5)-15=0$	•1 ✓
$10x^2-40x+40=0$	•2 ✗
$x=2$ and $y=1$	•3 ✓ •4 ^

Question		Generic Scheme	Illustrative Scheme	Max Mark
23	b			
		•5 ss state centre	•5 $(-1,2)$	
		•6 pd calculate gradients	•6 $m=-2$, $m=\frac{1}{2}$	
		•7 ic communicate result	•7 demonstrates $m_1\times m_2=-2\times\frac{1}{2}=-1$ $\Rightarrow$ PT is perpendicular to QT [or other appropriate statement]	
			Alternative Method	
		•5 ss state centre	•5 $(-1,2)$	
		•6 pd calculate vectors	•6 eg $\begin{pmatrix}-2\\4\end{pmatrix}$ and $\begin{pmatrix}-4\\-2\end{pmatrix}$	
		•7 ic communicate result	•7 $\begin{pmatrix}-2\\4\end{pmatrix}.\begin{pmatrix}-4\\-2\end{pmatrix}=-2\times-4+4\times-2=0$ $\Rightarrow$ PT is perpendicular to QT [or other appropriate statement]	3

Notes:

4. Other valid strategies:
   a Converse of Pythagoras' Theorem:
   - •[6] process lengths, $\text{PT} = \text{QT} = \sqrt{20}$, $\text{PQ} = \sqrt{40}$
   - •[7] apply converse and communicate result clearly.

   b Cosine Rule:
   - •[6] process lengths, •[7] apply cosine rule to obtain angle 90° and communicate result clearly.

Commonly Observed Responses:

Candidate B		**Candidate C**	
$\text{T}(-1,2)$	•[5] ✓	$\text{T}(-1,2)$	•[5] ✓
$m = \frac{1}{2}, m = -2$	•[6] ✓	$m_1 = \frac{1}{2}, m_2 = -2$	•[6] ✓
$m_1 \times m_2 = -1$	•[7] ^	$m_1 \times m_2 = -1$	•[7] ✓
No link between required condition and gradients found.		Required condition is linked to gradients found.	

23 c

•[8] ss knows to find and states centre •[9] pd calculate radius •[10] ic state equation of circle	•[8] centre (2, 1) •[9] radius = $\sqrt{10}$ •[10] $(x-2)^2 + (y-1)^2 = 10$	**3**
	Alternative Method	
•[8] ss substitute points into general equation of circle	$x^2 + y^2 + 2gx + 2fy + c = 0$ •[8] $25 + 6g + 8f + c = 0$ $5 + 2g - 4f + c = 0$ $5 - 2g + 4f + c = 0$	
•[9] pd find f or g or c •[10] ic state values of f, g and c	•[9] $f = -1$, or $g = -2$, or $c = -5$ •[10] $f = -1, g = -2, c = -5$	

Notes:

5. $(\sqrt{10})^2$ must be simplified to gain •[10]
6. For candidates who find P and Q correctly in part (a), award •[8] if centre $(2,1)$ appears without working.
7. For the mid-point of PQ being (2,1), •[8] is available unless subsequent working indicates that this is not the intended centre.
8. •[9] is only available as a result of PQ being a diameter, or using a valid strategy to find the centre eg midpoint of PQ or point of intersection of the perpendicular bisectors of PT and TQ. •[10] is still available.
9. Where an incorrect centre or an incorrect radius appear ex nihilo •[10] is not available.

Question		Generic Scheme	Illustrative Scheme	Max Mark
24				
			Method 1	
		$\bullet^1$ ss take $\log_9$ of both sides of the equation	$\bullet^1$ $\log_9 y = \log_9 ka^x$	
		$\bullet^2$ pd apply laws of logarithms	$\bullet^2$ $\log_9 y = \log_9 k + \log_9 a^x$	
		$\bullet^3$ pd apply laws of logarithms	$\bullet^3$ $\log_9 y = \log_9 k + x\log_9 a$	
		$\bullet^4$ pd find k	$\bullet^4$ $\log_9 k = 2, k = 81$ or $k = 9^2 = 81$	
		$\bullet^5$ pd find a	$\bullet^5$ $\log_9 a = \frac{1}{2}, a = 3$ or $a = 9^{\frac{1}{2}} = 3$	5
			Method 2	
		$\bullet^1$ ss know to use equation of the line	$\bullet^1$ $\log_9 y = \frac{1}{2}x + 2$	
		$\bullet^2$ pd write in exponential form	$\bullet^2$ $y = 9^{\frac{1}{2}x+2}$	
		$\bullet^3$ pd apply laws of indices	$\bullet^3$ $y = 9^{\frac{1}{2}x}9^2$	
		$\bullet^4$ pd find k	$\bullet^4$ $k = 81$	
		$\bullet^5$ pd find a	$\bullet^5$ $a=3$	

Notes:

1. Candidates who start with $\bullet^3$ $\log_9 y = \log_9 k + x\log_9 a$ also gain $\bullet^1$ and $\bullet^2$.
2. In Method 1, base 9 must appear by $\bullet^4$ stage, for $\bullet^1$ to be awarded.
3. For $k = 81$ and $a = 3$ with spurious or no working, $\bullet^4$ and $\bullet^5$ are not available.

Commonly Observed Responses:

Candidate A

$\log y = \log ka^x$	$\bullet^1$ ✗	See Note 2
$\log y = \log k + \log a^x$	$\bullet^2$ ✓	
$\log y = \log k + x\log a$	$\bullet^3$ ✓	
$k = 81$	$\bullet^4$ ✓✗	No evidence of which base is being used.
$a = 3$	$\bullet^5$ ✓	Answers at both $\bullet^4$ and $\bullet^5$ are consistent with using base 9.

Candidate B : A combination of Method 1 and Method 2.

M2 $\quad \log_9 y = \frac{1}{2}x + 2$ $\quad \bullet^1$ ✓

M1 $\quad \log_9 y = \log_9 ka^x$

$\Rightarrow \log_9 y = x\log_9 a + \log 9k$ $\quad \bullet^2$ ✓ $\bullet^3$ ✓

equating gradients and intercepts

$\log_9 a = \frac{1}{2}$

$a = 9^{\frac{1}{2}} = 3$ $\quad \bullet^4$ ✓

$\log_9 k = 2$ $\quad \bullet^5$ ✓

$k = 9^2 = 81$

Candidate C

at $(0,2)$ $\log_9 y = 2$

$\Rightarrow y = 9^2 = 81$ $\quad \bullet^1$ ✓

Substitute into equation

$81 = ka^0$

$\Rightarrow k = 81$ $\quad \bullet^2$ ✓

at $(6,5)$ $\log_9 y = 5$

$\Rightarrow y = 9^5$ $\quad \bullet^3$ ✓

substitute into equation

$9^5 = ka^6$

$\Rightarrow 9^5 = 81a^6$ $\quad \bullet^4$ ✓

$\Rightarrow a^6 = 9^3 = 3^6$

$\Rightarrow a = 3$ $\quad \bullet^5$ ✓