X100/12/03

NATIONAL 2013

WEDNESDAY, 22 MAY QUALIFICATIONS 2.50 PM - 4.00 PM

MATHEMATICS HIGHER Paper 2

Read carefully

- 1 Calculators may be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.





FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre (-g, -f) and radius $\sqrt{g^2 + f^2 - c}$. The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a}.\mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or
$$\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$$
 where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

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Trigonometric formulae:	$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$
	$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$
	$\sin 2A = 2\sin A \cos A$
	$\cos 2A = \cos^2 A - \sin^2 A$
	$= 2\cos^2 A - 1$
	$= 1 - 2\sin^2 A$

Table of standard derivatives:

f(x)	f'(x)
$\sin ax$ $\cos ax$	$a\cos ax$ - $a\sin ax$

Table of standard integrals:

f(x)	$\int f(x)dx$
sin ax	$-\frac{1}{a}\cos ax + C$
cos ax	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted.

 The first three terms of a sequence are 4, 7 and 16. The sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + c$$
, with $u_1 = 4$.

Find the values of *m* and *c*.

2. The diagram shows rectangle PQRS with P(7, 2) and Q(5, 6).



- (a) Find the equation of QR.
- (b) The line from P with the equation x + 3y = 13 intersects QR at T.



Find the coordinates of T.

(c) Given that T is the midpoint of QR, find the coordinates of R and S.

[Turn over

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- 3. (a) Given that (x-1) is a factor of $x^3 + 3x^2 + x 5$, factorise this cubic fully.
 - (b) Show that the curve with equation

$$y = x^4 + 4x^3 + 2x^2 - 20x + 3$$

has only one stationary point.

Find the *x*-coordinate and determine the nature of this point.

4. The line with equation y = 2x + 3 is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A(0, 3), as shown in the diagram.



The line meets the curve again at B.

Show that B is the point (-3, -3) and find the area enclosed by the line and the curve. **6**

5. Solve the equation

$$\log_5(3-2x) + \log_5(2+x) = 1, \text{ where } x \text{ is a real number.}$$

Marks 4

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6. Given that $\int_0^a 5\sin 3x \, dx = \frac{10}{3}, \quad 0 \le a < \pi,$

calculate the value of *a*.

7. A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions.

The frame of a shelter is to be made of rods of two different lengths:

- *x* metres for top and bottom edges;
- *y* metres for each sloping edge.

Condition 2

Condition 1

The frame is to be covered by a rectangular sheet of material.

The total area of the sheet is 24 m^2 .

(a) Show that the total length, L metres, of the rods used in a shelter is given by

$$L = 3x + \frac{48}{x}.$$

(b) These rods cost $\pounds 8.25$ per metre.

To minimise production costs, the total length of rods used for a frame should be as small as possible.

- (i) Find the value of x for which L is a minimum.
- (ii) Calculate the minimum cost of a frame.
- 8. Solve algebraically the equation

$$\sin 2x = 2\cos^2 x \qquad \text{for } 0 \le x < 2\pi$$

[Turn over for Question 9 on Page six

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9. The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation

$$P_t = P_0 e^{-kt}$$

where:

- P_0 is the initial concentration;
- P_t is the concentration at time t;
- *t* is the time, in days, after the application of the pesticide.
- (a) Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value.

If the half-life of *Xpesto* is 25 days, find the value of *k* to 2 significant figures.

(b) Eighty days after the initial application, what is the percentage decrease in concentration of *Xpesto*?

[END OF QUESTION PAPER]