**Paper 1 Section A**

| Question | Answer |
|---|---|
| 1 | A |
| 2 | B |
| 3 | B |
| 4 | A |
| 5 | D |
| 6 | C |
| 7 | B |
| 8 | C |
| 9 | A |
| 10 | D |
| 11 | B |
| 12 | C |
| 13 | A |
| 14 | B |
| 15 | C |
| 16 | C |
| 17 | C |
| 18 | D |
| 19 | B |
| 20 | D |

| | | |
|---|---|---|
| **Summary** | A | 4 |
| | B | 6 |
| | C | 6 |
| | D | 4 |

**Paper 1- Section B**

| Question | Generic Scheme | Illustrative Scheme | Max Mark |
|---|---|---|---|
| 21 | Express $2x^2 + 12x + 1$ in the form $a(x+b)^2 + c$. | | |
| | Method 1 | | |
| | •¹ ss identify common factor | •¹ $2(x^2 + 6x\ldots$ **stated or implied by •²** | 3 |
| | •² ss complete the square | •² $2(x+3)^2 \ldots$ | |
| | •³ pd process for $c$ | •³ $2(x+3)^2 - 17$ | |
| | Method 2 | | |
| | •¹ ss expands completed square form | •¹ $ax^2 + 2abx + ab^2 + c$ | |
| | •² ss equates coefficients | •² $a = 2 \quad 2ab = 12 \quad ab^2 + c = 1$ | |
| | •³ pd process for $b$ and $c$ and write in required form | •³ $2(x+3)^2 - 17$ | |

**Notes:**

1. Correct answer without working gains full credit.

**Regularly Occurring Responses:**

**Candidate A**

$2(x^2 + 6x + \frac{1}{2})$ •¹ ✓

$2(x^2 + 6x + 9 - 9 + \frac{1}{2})$ •² ✓

$2(x+3)^2 - 8\frac{1}{2}$ •³ ✗

**Candidate B**

$2x^2 + 12x + 1 = (x+6)^2 - 36 + 1$ •¹ ✗ •² ✗

$= (x+6)^2 - 35$ •³ ✓✗

**Candidate C**

$a(x+b)^2 + c = ax^2 + 2abx + ab^2 + c$ •¹ ✓

$a = 2 \quad 2ab = 12 \quad ab^2 + c = 1$ •² ✓

$b = 3 \quad c = -17$ •³ ✓

•³ awarded as all working relates to completed square form

**Candidate D**

$ax^2 + 2abx + ab^2 + c$ •¹ ✓

$a = 2 \quad 2ab = 12 \quad ab^2 + c = 1$ •² ✓

$b = 3 \quad c = -17$ •³ ✗

•³ is lost as no reference is made to completed square form

**Candidate E**

$ax^2 + 2abx + ab^2 + c$ •¹ ✓

$a = 2 \quad 2ab = 12 \quad b^2 + c = 1$ •² ✗

$a = 2 \quad b = 3 \quad c = -8$

$2(x+3)^2 - 8$ •³ ✓✗

**Candidate F**

$2(x^2 + 12x) + 1$ •¹ ✗

| Question | | Generic Scheme | Illustrative Scheme | Max Mark |
|---|---|---|---|---|
| **22** | | A circle $C_1$ has equation $x^2 + y^2 + 2x + 4y - 27 = 0$. | | |
| | **a** | Write down the centre and calculate the radius of $C_1$. | | |
| | | •¹ ic state centre<br>•² pd find radius | •¹ $(-1, -2)$<br>•² $\sqrt{32}$ | **2** |

**Notes:**

1. Do not penalise candidates who use -1 and -2 for $g$ and $f$ when calculating the radius. However, candidates who use -1 and 2 or 1 and -2 lose •²
2. $\sqrt{32}$ need not be simplified.

| Question | | Generic Scheme | Illustrative Scheme | Max Mark |
|---|---|---|---|---|
| **22** | **b** | The point P(3, 2) lies on the circle $C_1$.<br><br>Find the equation of the tangent at P. | | |
| | | •³ ss find $m$ radius<br>•⁴ ic state $m$ tangent<br>•⁵ ic state equation of tangent | •³ 1<br>•⁴ $-1$<br>•⁵ $y - 2 = -1(x - 3)$ | **3** |

**Notes:**

3. •⁵ is only available as a result of using a perpendicular gradient.

**Regularly Occurring Responses:**

| **Candidate A** | | **Candidate B** | |
|---|---|---|---|
| $m_{radius} = 1$ | •³ ✓ | $m_{radius} = 1$ | •³ ✓ |
| ^ | •⁴ ^ | $m_1 m_2 = -1$<br>so $m_2 = 1$ | •⁴ ✗ |
| equation of tangent is $y - 2 = 1(x - 3)$ | •⁵ ✗ | $y - 2 = 1(x - 3)$ | •⁵ ✓̸ |

| Question | | Generic Scheme | Illustrative Scheme | Max Mark |
|---|---|---|---|---|
| 22 | c | A second circle $C_2$ has centre (10, –1).<br><br>The radius of $C_2$ is half of the radius of $C_1$.<br><br>Show that the equation of $C_2$ is $x^2 + y^2 - 20x + 2y + 93 = 0$. | | |

| Generic Scheme | | | Illustrative Scheme | Max Mark |
|---|---|---|---|---|
| $\bullet^6$ | pd | find radius | $\bullet^6$ $\sqrt{8}$ **stated or implied by** $\bullet^7$ | 3 |
| $\bullet^7$ | ic | state equation of circle | $\bullet^7$ $(x-10)^2 + (y+1)^2 = \left(\sqrt{8}\right)^2$ | |
| $\bullet^8$ | pd | expand and complete | $\bullet^8$ $x^2 - 20x + 100 + y^2 + 2y + 1 = 8$ and complete | |

**Accept**

| | |
|---|---|
| $2g = -20,\ 2f = 2$ | Centre $(10, -1)$ |
| $g = -10, f = 1$ | $g = -10,\ f = 1$ |
| $\bullet^6$ Centre $(10, -1)$ | $2g = -20,\ 2f = 2$ |

$\bullet^7$ $r = \sqrt{(-10)^2 + 1^2 - 93} = \sqrt{8}$

$\bullet^8$ $\sqrt{32} = 2\sqrt{8}$

$\frac{1}{2} \times \sqrt{32} = \frac{1}{2} \times 2\sqrt{8} = \sqrt{8} =$ radius of $C_2$

**Regularly Occurring Responses:**

**Candidate C**

$C_2$ centre is (10,-1)

$g = -10,\ f = 1$

$2g = -20,\ 2f = 2$

$x^2 + y^2 - 20x + 2y + \ldots$

$\bullet^6$ ✓ $\bullet^7$ ✗ $\bullet^8$ ✗

**Candidate D**

$x^2 + y^2 - 20x + 2y + 93 = 0$

$2g = -20,\ 2f = 2$

centre (10, -1)

radius = $\sqrt{(-10)^2 + 1^2 - 93} = \sqrt{8}$

$\sqrt{32} = \sqrt{4 \times 8} = 2\sqrt{8}$

so radius of $C_2$ = $\frac{1}{2}$of radius of $C_1$

$\bullet^6$ ✓ $\bullet^7$ ✓ $\bullet^8$ ✓

**Candidate E**

$x^2 + y^2 - 20x + 2y + 93 = 0$

$2g = -20,\ 2f = 2$

centre (10, -1)

radius = $\sqrt{(-10)^2 + 1^2 - 93}$

$= \sqrt{8}$

$\sqrt{32} = 4\sqrt{8}$ ....

$\bullet^6$ ✓ $\bullet^7$ ✓ $\bullet^8$ ✗

**Candidate F**

$x^2 + y^2 - 20x + 2y + 93$

$2g = -20,\ 2f = 2$

centre (10, -1)

radius = $\sqrt{(-10)^2 + 1^2 - 93}$

$= \sqrt{8}$

which is half of $\sqrt{32}$

$\bullet^6$ ✓ $\bullet^7$ ✓ $\bullet^8$ ✗

**Candidate G**

$x^2 + y^2 - 20x + 2y + 93 = 0$

$2g = -20,\ 2f = 2$

centre (10, -1)

radius = $\sqrt{(-10)^2 + 1^2 - 93}$

$= \sqrt{8}$

$\bullet^6$ ✓ $\bullet^7$ ✓ $\bullet^8$ ✗

<table>
<tr><th colspan="2">Question</th><th>Generic Scheme</th><th>Illustrative Scheme</th><th>Max Mark</th></tr>
<tr><td>22</td><td>d</td><td colspan="3">Show that the tangent found in part (b) is also a tangent to circle $C_2$.</td></tr>
<tr><td colspan="2"></td><td>
●9 ss substitute $y = 5 - x$ (or $x = 5 - y$)<br>
●10 pd express in standard quadratic form<br>
●11 ic start proof<br>
●12 ic complete proof
</td><td>
Method 1<br>
Substituting for $y$<br>
●9 $x^2 + (5 - x)^2 - 20x + 2(5 - x) + 93$<br>
●10 $2x^2 - 32x + 128 = 0$<br>
●11 $2(x - 8)^2 = 0$<br>
●12 equal roots $\Rightarrow$ tangent<br>
(or) ●11 $(-32)^2 - 4 \times 2 \times 128$<br>
●12 $b^2 - 4ac = 0$ $\Rightarrow$ tangent<br>
or<br>
Substituting for $x$<br>
●9 $(5 - y)^2 + y^2 - 20(5 - y) + 2y + 93 = 0$<br>
●10 $2y^2 + 12y + 18 = 0$<br>
●11 $2(y + 3)^2 = 0$<br>
●12 equal roots $\Rightarrow$ tangent<br>
(or) ●11 $12^2 - 4 \times 2 \times 18$<br>
●12 $b^2 - 4ac = 0$ $\Rightarrow$ tangent
</td><td rowspan="2">4</td></tr>
<tr><td colspan="2"></td><td>
●9 ss uses perpendicular gradients<br>
●10 pd find equation of radius<br>
●11 ic starts proof<br>
●12 ic completes proof
</td><td>
Method 2<br>
●9 $m$ given line $= -1$, leading to $m_{radius} = 1$<br>
●10 $y + 1 = 1(x - 10)$<br>
●11 $y = -x + 5$<br>
$y = x - 11$<br>
$\Rightarrow x = 8$<br>
$y = -3$<br>
●12 $(8)^2 + (-3)^2 - 20 \times (8) + 2(-3) + 93$ and complete
</td></tr>
<tr><td colspan="5">Notes:</td></tr>
<tr><td colspan="5">
Method 1<br>
4. = 0 must appear at ●9 or ●10 stage to gain ●10.<br>
5. Candidates who arrive at a quadratic equation which does not have equal roots cannot gain ●12 as follow through. ( See General Comments Note 16).<br>
6. Where candidates do not arrive at a quadratic equation in Method 1, marks ●10, ●11 and ●12 are not available.<br>
7. Acceptable communication for ●12, ‘only one answer so implies tangent’, ‘discriminant is 0 so tangent’, ’$x = 8$ twice so tangent’, or equivalent relating to tangency.
</td></tr>
</table>

| Question | Generic Scheme | Illustrative Scheme | Max Mark |
|---|---|---|---|
| **23 a** | The expression $\sqrt{3}\sin x^{\circ} - \cos x^{\circ}$ can be written in the form $k\sin(x-a)^{\circ}$, where $k>0$ and $0 \le a < 360$.<br>Calculate the values of $k$ and $a$. | | |
| | $\bullet^1$ ss use compound angle formula<br>$\bullet^2$ ic compare coefficients<br>$\bullet^3$ pd process for $k$<br>$\bullet^4$ pd process for $a$ | $\bullet^1$ $k\sin x^{o}\cos a^{o} - k\cos x^{o}\sin a^{o}$ **stated explicitly**<br>$\bullet^2$ $k\cos a^{o} = \sqrt{3}$ and $k\sin a^{o} = 1$ **stated explicitly**<br>$\bullet^3$ 2 ( do **not** accept $\sqrt{4}$ )<br>$\bullet^4$ 30 | 4 |

**Notes:**

1. Treat $k\sin x^{o}\cos a^{o}$ - $\cos x^{o}\sin a^{o}$ as bad form only if the equations at the $\bullet^2$ stage both contain $k$.
2. $2\sin x^{o}\cos a^{o}$ - $2\cos x^{o}\sin a^{o}$ or $2(\sin x^{o}\cos a^{o}$ - $\cos x^{o}\sin a^{o})$ is acceptable for $\bullet^1$ and $\bullet^3$.
3. Accept $k\cos a^{o} = \sqrt{3}$ and $-k\sin a^{o} = -1$ for $\bullet^2$.
4. $\bullet^2$ is not available for $k\cos x^{o} = \sqrt{3}$ and $k\sin x^{o} = 1$, however, $\bullet^4$ is still available.
5. $\bullet^3$ is only available for a single value of $k$, $k > 0$.
6. $\bullet^4$ is only available for a single value of $a$ expressed in degrees.
7. Candidates who identify and use any form of the wave equation may gain $\bullet^1$, $\bullet^2$ and $\bullet^3$, however, $\bullet^4$ is only available if the value of $a$ is interpreted for the form $k\sin(x-a)^{o}$.
8. Do not penalise omission of degree sign at $\bullet^1$ or $\bullet^2$.

**Regularly Occurring Responses:**

Response 1: Missing information in working.

| Candidate A | Candidate B | Candidate C |
|---|---|---|
| $\bullet^1$ ✗<br>$2\cos a = \sqrt{3}$<br>$-2\sin a = -1$ $\bullet^2$ ✓<br>$\tan a = \frac{1}{\sqrt{3}}$ $\bullet^3$ ✓<br>$a = 30$ $\bullet^4$ ✓ | $\bullet^1$ ✗<br>$\cos a = \sqrt{3}$<br>$\sin a = 1$ $\bullet^2$ ✗<br>$\bullet^3$ ✗<br>$\tan a = \frac{1}{\sqrt{3}}$<br>$a = 30$ $\bullet^4$ ✗<br>Not consistent with evidence at $\bullet^2$ | $k\sin x^{o}\cos a^{o}$ - $k\cos x^{o}\sin a^{o}$ $\bullet^1$ ✓<br>$k\cos a = \sqrt{3}$, $k\sin a = 1$ $\bullet^2$ ✓<br>$k = 2$ or $-2$ $\bullet^3$ ✗<br>$\tan a = \frac{1}{\sqrt{3}}$ $\bullet^4$ ✗<br>$a = 30$ or 210<br>However candidates who then write $\sqrt{3}\sin x^{o} - \cos x^{o} = 2\sin(x\text{-}30)^{o}$ would gain $\bullet^3$ and $\bullet^4$ |

Response 2: Labelling incorrect, sin (A – B) = sin A cos B – cos A sin B from formula list.

| Candidate D | Candidate E | Candidate F |
|---|---|---|
| $k\sin A\cos B - k\cos A\sin B$ $\bullet^1$ ✗<br>$k\cos a = \sqrt{3}$<br>$k\sin a = 1$ $\bullet^2$ ✓<br>$\tan a = \frac{1}{\sqrt{3}}$<br>$a = 30$ $\bullet^4$ ✓ | $k\sin A\cos B - k\cos A\sin B$ $\bullet^1$ ✗<br>$k\cos x = \sqrt{3}$<br>$k\sin x = 1$ $\bullet^2$ ✗<br>$\tan x = \frac{1}{\sqrt{3}}$<br>$x = 30$ $\bullet^4$ ✓ | $k\sin A\cos B - k\cos A\sin B$ $\bullet^1$ ✗<br>$k\cos B = \sqrt{3}$<br>$k\sin B = 1$ $\bullet^2$ ✓<br>$\tan B = \frac{1}{\sqrt{3}}$<br>$B = 30$ $\bullet^4$ ✓ |

| Question | | Generic Scheme | | Illustrative Scheme | | Max Mark |
|---|---|---|---|---|---|---|
| 23 | b | Determine the maximum value of $4 + 5\cos x^{\circ} - 5\sqrt{3}\sin x^{\circ}$, where $0 \leq x < 360$. | | | | |
| $\bullet^5$ | ic | interpret expression | $\bullet^5$ | $4 - 5 \times 2\sin(x - 30)^{\circ}$ | | |
| $\bullet^6$ | pd | state maximum | $\bullet^6$ | 14 | | 2 |

**Notes:**

9. A solution using calculus gains no marks unless angles are converted to radian measure before differentiating.
10. 'Maximum = 14' with no working gains no marks.
11. $\bullet^5$ is awarded for demonstrating a clear link between the expression in (b) and the wave in part (a)
12. Candidates who start afresh, and use any form of the wave function to arrive at $4 \pm 10\cos(\ldots)$ or $4 \pm 10\sin(\ldots)$ correctly, can gain both $\bullet^5$ and $\bullet^6$.
13. $\bullet^6$ is only available if, at the $\bullet^5$ stage, the candidate's answer in (a) is multiplied by an integer $k$, $k \neq \pm 1$.
14. Candidates who equate the given expression to 0 and attempt to solve gain 0 marks.

**Regularly Occurring Responses:**

| Candidate J | | Candidate K | |
|---|---|---|---|
| $4 - 5 \times 2\sin(x - 60)^{0}$ | $\bullet^5$ ✓ | $4 + 2\sin(x - 30)^{0}$ | $\bullet^5$ ✗ |
| | | Max 2 + 4 | |
| Max = 14 | $\bullet^6$ ✓ | Max = 6 | $\bullet^6$ ✓ |

| Question | | | Generic Scheme | | Illustrative Scheme | Max Mark |
|---|---|---|---|---|---|---|
| 24 | a | i | Show that the points A( –7, –8, 1), T(3, 2, 5) and B(18, 17, 11) are collinear. | | | |
| 24 | a | ii | Find the ratio in which T divides AB. | | | |
| •[1] | ss | | use vector approach | •[1] | $\overrightarrow{AT} = \begin{pmatrix}10\\10\\4\end{pmatrix}$ or $\overrightarrow{TB} = \begin{pmatrix}15\\15\\6\end{pmatrix}$ | 4 |
| •[2] | ic | | compare two vectors | •[2] | $\overrightarrow{TB}$ or $\overrightarrow{AT}$ **and** $\overrightarrow{AT} = \frac{2}{3}\overrightarrow{TB}$ or equivalent | |
| •[3] | ic | | complete proof | •[3] | $\overrightarrow{AT}$ and $\overrightarrow{TB}$ are parallel and since there is a common point A, B and T are collinear | |
| •[4] | ic | | state ratio | •[4] | 2:3 **stated explicitly** (see Note 4) | |

**Notes:**

1. Any appropriate combination of vectors is acceptable.
2. •[3] can only be awarded if a candidate has stated, common point, parallel (common direction) and collinear.
3. Treat $\begin{pmatrix}10\\10\\4\end{pmatrix}$ written as $(10, 10, 4)$ as bad form.
4. Accept $1:\frac{3}{2}$ or $\frac{2}{3}:1$
5. •[3] requires evidence of vectors being parallel, simply stating parallel is insufficient.

**Regularly Occurring Responses:**

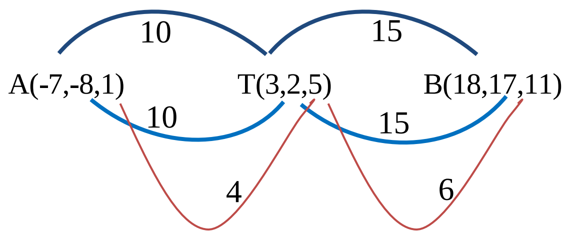
| Candidate A | Candidate B | Candidate C |
|---|---|---|
| $\overrightarrow{AT} = 2\begin{pmatrix}5\\5\\2\end{pmatrix}$ $\overrightarrow{TB} = 3\begin{pmatrix}5\\5\\2\end{pmatrix}$ •[2] ✓ | $\overrightarrow{AT} = \begin{pmatrix}10\\10\\4\end{pmatrix}$ or $\overrightarrow{TB} = \begin{pmatrix}15\\15\\6\end{pmatrix}$ •[1] ✓ | $\overrightarrow{AT} = \begin{pmatrix}10\\10\\4\end{pmatrix}$ *or* $\overrightarrow{TB} = \begin{pmatrix}15\\15\\6\end{pmatrix}$ •[1] ✓ |
| | $\overrightarrow{TB} = \frac{2}{3}\overrightarrow{AT}$ •[2] ✗ | $\overrightarrow{TB} = \frac{2}{3}\overrightarrow{AT}$ •[2] ✗ |
| | TB and AT are parallel, T is a common point so A, T and B are collinear. •[3] ✓ | TB and AT are parallel, T is a common point so A,T and B are collinear. •[3] ✓ |
| | AT:TB = 2:3 •[4] ✗ | AT:TB = 3:2 •[4] ✓ |

**Candidate D**

$\overrightarrow{AT} = \begin{pmatrix}10\\10\\4\end{pmatrix}$ or $\overrightarrow{TB} = \begin{pmatrix}15\\15\\6\end{pmatrix}$ •[1] ✓

$\overrightarrow{TB} = \frac{2}{3}\overrightarrow{AT}$ •[2] ✗

TB and AT are parallel. T is a common point so A, T and B are collinear. •[3] ✓




•[4] ✓

10:15 = 10:15 = 4:6 = 2:3

| Question | | Generic Scheme | | Illustrative Scheme | Max Mark |
|---|---|---|---|---|---|
| 24 | b | The point C lies on the $x$-axis.<br><br>If TB and TC are perpendicular, find the coordinates of C. | | | |
| | | **Method 1** | | **Method 1** | |
| $\bullet^5$ | ic | interpret C | $\bullet^5$ | $(c, 0, 0)$ | |
| $\bullet^6$ | pd | use vector approach | $\bullet^6$ | $\overrightarrow{TC} = \begin{pmatrix} c-3 \\ -2 \\ -5 \end{pmatrix}$ | |
| $\bullet^7$ | ss | know to use scalar product equal to 0 | $\bullet^7$ | $\overrightarrow{TB}.\overrightarrow{TC} = 0$ | |
| $\bullet^8$ | pd | start to solve | $\bullet^8$ | $15(c-3) + 15 \times (-2) + 6 \times (-5) \ldots$ | |
| $\bullet^9$ | pd | complete | $\bullet^9$ | $c = 7$ | 5 |
| | | **Method 2** | | **Method 2** | |
| $\bullet^5$ | ic | interpret C | $\bullet^5$ | $(c, 0, 0)$ | |
| $\bullet^6$ | pd | use vector approach | $\bullet^6$ | $\overrightarrow{TC} = \begin{pmatrix} c-3 \\ -2 \\ -5 \end{pmatrix}$ | |
| $\bullet^7$ | ss | know to use Pythagoras and calculate $\lvert\overrightarrow{TC}\rvert$ or $\lvert\overrightarrow{TB}\rvert$ | $\bullet^7$ | $\lvert\overrightarrow{TC}\rvert = \sqrt{(c-3)^2 + 4 + 25}$ | |
| $\bullet^8$ | pd | calculate the other two lengths | $\bullet^8$ | $\lvert\overrightarrow{TB}\rvert = \sqrt{486}$ and<br>$\lvert\overrightarrow{BC}\rvert = \sqrt{(c-18)^2 + 289 + 121}$ | |
| $\bullet^9$ | pd | complete | $\bullet^9$ | $c = 7$ | |

**Notes:**

6. In Method 1, = 0 must appear at $\bullet^7$ or $\bullet^8$ for $\bullet^9$ to be available.
7. In Method 1, candidates who use $\overrightarrow{TB}.\overrightarrow{TC} = -1$ can gain a maximum of 4 marks.
8. C must appear in coordinate form at $\bullet^5$ or $\bullet^9$ for $\bullet^5$ to be awarded.
9. If C has more than one non-zero coordinate $\bullet^9$ is not available.
10. $\bullet^8$ is only available for expressions with an unknown.

**Regularly Occurring Responses:**

**Candidate E**

$C = (c, \quad 0, \quad 0)$ $\bullet^5$ ✓

$\overrightarrow{TC} = \begin{pmatrix} c-3 \\ -2 \\ -5 \end{pmatrix}$ $\bullet^6$ ✓

$\overrightarrow{TB}.\overrightarrow{TC} = -1$ $\bullet^7$ ✗

$15(c-3) + 15(-2) + 6(-5) = -1$ $\bullet^8$ ✓

$c = \frac{104}{15}$ $\bullet^9$ ✓

**Candidate F**

$15(c-3) + 15 \times (-2) + 6 \times (-5) = 0$

$c = 7$

$(7, 0, 0)$ **Gains full marks**

**Candidate G**

$\overrightarrow{TC} = c - \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$ $\bullet^6$ ^

$\overrightarrow{TB}.\overrightarrow{TC} = 15(c-3) + 15(c-2) + 6(c-5)$ $\bullet^8$ ✗

It is not clear at $\bullet^6$ what is meant by '$c$' so $\bullet^8$ cannot be awarded as follow through.

**However**

$\overrightarrow{TC} = \begin{pmatrix} x-3 \\ y-2 \\ z-5 \end{pmatrix}$ $\bullet^6$ ✓