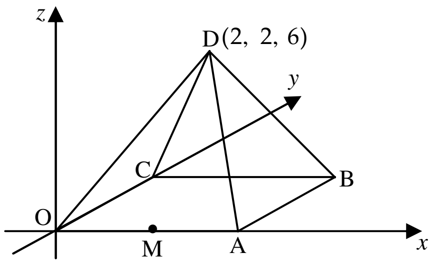
1 D,OABC is a square based pyramid as shown in the diagram below.

O is the origin, D is the point $(2,\ 2,\ 6)$ and $OA = 4$ units.

M is the mid-point of OA.

(a) State the coordinates of B. **1**

(b) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. **3**



Throughout this question, coordinates written as components and vice versa are treated as bad form.

Generic Scheme			Illustrative Scheme
1 (a)			
•1	ic	state coordinates of B	•1 $(4,\ 4,\ 0)$
			1
1 (b)			
•2	pd	state components of $\overrightarrow{DB}$	•2 $\begin{pmatrix}2\\2\\-6\end{pmatrix}$
•3	ic	state coordinates of M	•3 $(2,\ 0,\ 0)$ **stated, or implied by •4**
•4	pd	state components of $\overrightarrow{DM}$	•4 $\begin{pmatrix}0\\-2\\-6\end{pmatrix}$
			3

Regularly occurring responses

Response 1A (Transcription error for D)

$\overrightarrow{DB} = \begin{pmatrix}4\\4\\0\end{pmatrix} - \begin{pmatrix}2\\6\\6\end{pmatrix} = \begin{pmatrix}2\\-2\\-6\end{pmatrix}$ ✗ •2

$\overrightarrow{DM} = \begin{pmatrix}2\\0\\0\end{pmatrix} - \begin{pmatrix}2\\6\\6\end{pmatrix} = \begin{pmatrix}0\\-6\\-6\end{pmatrix}$ ✓ •3 ✓̸ •4

2 marks out of 3

Response 1B (Transcription error for D)

$\overrightarrow{DB} = \begin{pmatrix}2\\-2\\-6\end{pmatrix}$ and $\overrightarrow{DM} = \begin{pmatrix}0\\-6\\-6\end{pmatrix}$ with no working.

0 marks out of 3

Response 2A

$\overrightarrow{DB} = \mathbf{d} + \mathbf{b} = \begin{pmatrix}6\\6\\6\end{pmatrix}$ ✗ •2

$\overrightarrow{DM} = \mathbf{d} + \mathbf{m} = \begin{pmatrix}4\\2\\6\end{pmatrix}$ ✓ •3 ✓̸ •4

2 marks out of 3

Response 2B

$\overrightarrow{DB} = \begin{pmatrix}6\\6\\6\end{pmatrix}$ and $\overrightarrow{DM} = \begin{pmatrix}4\\2\\6\end{pmatrix}$ with no working.

0 marks out of 3

1 (c)	Find the size of angle BDM.	5

	Generic Scheme		Illustrative Scheme
1 (c)			
$\bullet^5$	ss	know to use scalar product	$\bullet^5$ $\cos B\hat{D}M = \dfrac{\overrightarrow{DB}.\overrightarrow{DM}}{\lvert\overrightarrow{DB}\rvert\lvert\overrightarrow{DM}\rvert}$ **stated, or implied by $\bullet^9$**
$\bullet^6$	pd	find scalar product	$\bullet^6$ $\overrightarrow{DB}.\overrightarrow{DM} = 32$
$\bullet^7$	pd	find magnitude of a vector	$\bullet^7$ $\lvert\overrightarrow{DB}\rvert = \sqrt{44}$
$\bullet^8$	pd	find magnitude of a vector	$\bullet^8$ $\lvert\overrightarrow{DM}\rvert = \sqrt{40}$
$\bullet^9$	pd	evaluate angle BDM	$\bullet^9$ $40\cdot3^\circ$ or $0\cdot703$ rads

5

Notes

1. $\bullet^5$ is not available to candidates who evaluate the wrong angle.
2. If candidates do not attempt $\bullet^9$, then $\bullet^5$ is only available if the formula quoted relates to the labelling in the question.
3. $\bullet^9$ should be awarded to any answer which rounds to 40° or $0\cdot7$ rads.
4. In the event that both magnitudes are equal **or** there is only one non-zero component, $\bullet^8$ is not available.

Regularly occurring responses

Response 3A

$\cos B\hat{O}M = \dfrac{\overrightarrow{OB}.\overrightarrow{OM}}{\lvert\overrightarrow{OB}\rvert\lvert\overrightarrow{OM}\rvert}$ ✗ $\bullet^5$

$\overrightarrow{OB}.\overrightarrow{OM} = 8$ ✓ $\bullet^6$

$\lvert\overrightarrow{OB}\rvert = \sqrt{32}$ ✓ $\bullet^7$

$\lvert\overrightarrow{OM}\rvert = 2$ ✗ $\bullet^8$

45° ✓ $\bullet^9$

3 marks out of 5

Response 3B

$\cos B\hat{O}D = \dfrac{\overrightarrow{OB}.\overrightarrow{OD}}{\lvert\overrightarrow{OB}\rvert\lvert\overrightarrow{OD}\rvert}$ ✗ $\bullet^5$

$\overrightarrow{OB}.\overrightarrow{OD} = 16$ ✓ $\bullet^6$

$\lvert\overrightarrow{OB}\rvert = \sqrt{32}$ ✓ $\bullet^7$

$\lvert\overrightarrow{OD}\rvert = \sqrt{44}$ ✓ $\bullet^8$

$64\cdot8^\circ$ ✓ $\bullet^9$

4 marks out of 5

Response 3C

$\cos D\hat{B}M = \dfrac{\overrightarrow{BD}.\overrightarrow{BM}}{\lvert\overrightarrow{BD}\rvert\lvert\overrightarrow{BM}\rvert}$ ✗ $\bullet^5$

$\overrightarrow{BD}.\overrightarrow{BM} = 12$ ✓ $\bullet^6$

$\lvert\overrightarrow{BD}\rvert = \sqrt{44}$ ✓ $\bullet^7$

$\lvert\overrightarrow{BM}\rvert = \sqrt{20}$ ✓ $\bullet^8$

$66\cdot1^\circ$ ✓ $\bullet^9$

4 marks out of 5

Response 4

$\cos B\hat{D}M = \dfrac{\overrightarrow{BD}.\overrightarrow{DM}}{\lvert\overrightarrow{BD}\rvert\lvert\overrightarrow{DM}\rvert}$ ✗ $\bullet^5$

$\overrightarrow{BD}.\overrightarrow{DM} = -32$ ✓ $\bullet^6$

$\lvert\overrightarrow{BD}\rvert = \sqrt{44}$ ✓ $\bullet^7$

$\lvert\overrightarrow{DM}\rvert = \sqrt{40}$ ✓ $\bullet^8$

$139\cdot7^\circ$ ✓ $\bullet^9$

4 marks out of 5

Response 5A

(Scalar Product is 0)

In 1(b) $\overrightarrow{DB} = \begin{pmatrix} 2 \\ -2 \\ -6 \end{pmatrix}$, $\overrightarrow{DM} = \begin{pmatrix} 0 \\ 6 \\ -2 \end{pmatrix}$

$\cos B\hat{D}M = \dfrac{\overrightarrow{DB}.\overrightarrow{DM}}{\lvert\overrightarrow{DB}\rvert\lvert\overrightarrow{DM}\rvert}$ ✓ $\bullet^5$

$\overrightarrow{DB}.\overrightarrow{DM} = 0$ ✓ $\bullet^6$

$\lvert\overrightarrow{DB}\rvert = \sqrt{44}$ ✓ $\bullet^7$

$\lvert\overrightarrow{DM}\rvert = \sqrt{40}$ ✓ $\bullet^8$

90° ✓ $\bullet^9$

5 marks out of 5

Response 5B

In 1(b) $\overrightarrow{DB} = \begin{pmatrix} 2 \\ -2 \\ -6 \end{pmatrix}$, $\overrightarrow{DM} = \begin{pmatrix} 0 \\ 6 \\ -2 \end{pmatrix}$

$\cos B\hat{D}M = \dfrac{\overrightarrow{DB}.\overrightarrow{DM}}{\lvert\overrightarrow{DB}\rvert\lvert\overrightarrow{DM}\rvert}$ ✓ $\bullet^5$

$\overrightarrow{DB}.\overrightarrow{DM} = 0$ ✓ $\bullet^6$

so perpendicular ✓ $\bullet^9$

^ $\bullet^7$ ^ $\bullet^8$

3 marks out of 5

Response 6 (Cosine rule)

$\cos B\hat{D}M = \dfrac{DB^2 + DM^2 - BM^2}{2 \times DB \times DM}$ ✓ $\bullet^5$

$DB = \sqrt{44}$ ✓ $\bullet^6$

$DM = \sqrt{40}$ ✓ $\bullet^7$

$BM = \sqrt{20}$ ✓ $\bullet^8$

$40\cdot3^\circ$ ✓ $\bullet^9$

5 marks out of 5

2 Functions f, g and h are defined on the set of real numbers by

- $f(x) = x^3 - 1$ • $g(x) = 3x + 1$ • $h(x) = 4x - 5$

(a) Find $g(f(x))$. **2**

(b) Show that $g(f(x)) + x\,h(x) = 3x^3 + 4x^2 - 5x - 2$. **1**

Generic Scheme			**Illustrative Scheme**	
2 (a)				
$\bullet^1$	ic	interpret notation	$\bullet^1$ $g(x^3 - 1)$	**stated, or implied by** $\bullet^2$
$\bullet^2$	ic	complete process	$\bullet^2$ $3(x^3 - 1) + 1$	

Notes

1. $3x^3 - 2$ without working gains only 1 mark.
2. $f(g(x))$ loses $\bullet^1$ but will gain $\bullet^2$ for $(3x+1)^3 - 1$.
3. $f(x) \times g(x)$ loses both $\bullet^1$ and $\bullet^2$.

2

Generic Scheme			**Illustrative Scheme**	
2 (b)				
$\bullet^3$	ic	substitute and complete	$\bullet^3$ $3(x^3 - 1) + 1 + x(4x - 5)$ $= 3x^3 + 4x^2 - 5x - 2$	**stated explicitly**
			or $3(x^3 - 1) + 1 + 4x^2 - 5x$ $= 3x^3 + 4x^2 - 5x - 2$	**stated explicitly**
			or $3x^3 - 2 + x(4x - 5)$ $= 3x^3 + 4x^2 - 5x - 2$	**stated explicitly**
			or $3x^3 - 2 + 4x^2 - 5x$ $= 3x^3 + 4x^2 - 5x - 2$	**stated explicitly**

1

Regularly occurring responses

CAVE : Watch out for erroneous working leading to the required cubic.

Response 1 $3x^3 - 2 + x\underline{(4x + 5)} = 3x^3 + 4x^2 - 5x - 2$ ✗ $\bullet^3$

Response 2 $\underline{3x^3 - 4} + x(4x - 5) = 3x^3 + 4x^2 - 5x - 2$ ✗ $\bullet^3$

Response 3 From (a) $(3x+1)^3 - 1$

In (b) $\underline{3x^3 + 3} - 1 + x(4x - 5) = 3x^3 + 2 + 4x^2 - 5x$ ✗ $\bullet^3$

$= 3x^3 + 4x^2 - 5x - 2$

Response 4A From (a) $g(f(x)) = 3x^3 - 2$

In (b) $xh(x) = 4x^2 - 5x$

^ $3x^3 + 4x^2 - 5x - 2$ ✗ $\bullet^3$

Response 4B From (a) $g(f(x)) = 3x^3 - 2$

In (b) $\underline{3x^2 - 2} + 4x^2 - 5x$ ✗ $\bullet^3$

$= 3x^3 + 4x^2 - 5x - 2$

As the form of the answer was given in the question, this mark is not available.

Note : $\bullet^3$ is not available to candidates who leave their answer as $3x^3 - 2 + 4x^2 - 5x$.

2	(c)	(i) Show that $(x-1)$ is a factor of $3x^3+4x^2-5x-2$.	
		(ii) Factorise $3x^3+4x^2-5x-2$ fully.	**5**
	(d)	Hence solve $g(f(x))+x\,h(x)=0$.	**1**

Generic Scheme	**Illustrative Scheme**

2 (c)

Method 1 : Using synthetic division

Generic Scheme			Illustrative Scheme
•4	ss	know to use $x=1$	•4 $\begin{array}{r\|rrrr} 1 & 3 & 4 & -5 & -2 \\ \hline \end{array}$
•5	pd	complete evaluation	•5 $\begin{array}{r\|rrrr} 1 & 3 & 4 & -5 & -2 \\ & & 3 & 7 & 2 \\ \hline & 3 & 7 & 2 & 0 \end{array}$
•6	ic	state conclusion	•6 "remainder is zero so $(x-1)$ is a factor", accept "$(x-1)$ is a factor"
•7	ic	find quadratic factor	•7 $3x^2+7x+2$ **stated, or implied by •8**
•8	pd	factorise completely	•8 $(x-1)(3x+1)(x+2)$ **stated explicitly**

If **only** the word 'factor' appears, it must be linked to the 0 in the table. The link could be as little as 'so', '$\therefore$', '$\rightarrow$', '$\Rightarrow$' or 'hence'. The word 'factor' **only**, with no link, does not gain •6.

Method 2 : Using substitution and inspection

•4 know to use $x=1$

•5 $3+4-5-2=0$

•6 $(x-1)$ is a factor

•7 $(x-1)(3x^2+7x+2)$ **stated, or implied by •8**

•8 $(x-1)(3x+1)(x+2)$ **stated explicitly**

5

Notes

4. •6 is only available as a consequence of the evidence for •4 and •5.
5. Communication at •6 must be consistent with working at •5.
   i.e. candidate's working must arrive legitimately at zero before •6 is awarded.
   If the remainder is not 0 then an appropriate statement would be '$(x-1)$ is not a factor'.
6. Unacceptable statements : $x=1$ is a factor, $(x+1)$ is a factor, $x=1$ is a root, $(x-1)$ is a root etc.
7. •9 cannot be awarded for solving $3x^3+4x^2-5x-2=0$ in (c).

2 (d)

Generic Scheme			Illustrative Scheme
•9	ic	interpret and solve equation in (d)	•9 $-2,\ -\frac{1}{3}$ and 1

These must appear explicitly here at (d).

1

Notes

8. From (c) $(x-1)(3x+1)(x+2)$ leading to $x=1,\ x=-\frac{1}{3}$ and $x=-2$ then $(1,\ 0)$, $\left(-\frac{1}{3},\ 0\right)$ and $(-2,\ 0)$ gains •9.

   However, $(x-1)(3x+1)(x+2)$ leading to $(1,\ 0)$, $\left(-\frac{1}{3},\ 0\right)$ and $(-2,\ 0)$ **only** does not gain •9.

9. From (c) $(3x+1)(x+2)$ only, leading to $x=-\frac{1}{3},\ x=-2$ does not gain •9 as equation solved is not a cubic.

3	(a)	A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$.	
		Write down the values of u_1 and u_2.	**1**
	(b)	A second sequence is given by $4,\ 5,\ 7,\ 11,\ \ldots\ .$	
		It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$.	
		Find the values of p and q.	**3**

Generic Scheme			Illustrative Scheme	
3 (a)				
$\bullet^1$	pd	find terms of sequence	$\bullet^1$ $u_1 = 8$ and $u_2 = -4$	Accept "8 and -4"
				1

3 (b)

$\bullet^2$	ic	interpret sequence	$\bullet^2$ e.g. $4p + q = 5$ **and** $5p + q = 7$
$\bullet^3$	ss	solve for one variable	$\bullet^3$ $p = 2$ or $q = -3$
$\bullet^4$	pd	state second variable	$\bullet^4$ $q = -3$ or $p = 2$
			3

Notes

1. Candidates may use $7p + q = 11$ as one of their equations at $\bullet^2$.
2. Treat equations like $p4 + q = 5$ or $p(4) + q = 5$ as bad form.
3. Candidates should not be penalised for using $u_{n+1} = pu_n + q$.

Regularly occurring responses

Response 1A (No working)

$p = 2$ and $q = -3$

or $v_{n+1} = 2v_n - 3$

1 mark out of 3

Response 1B (Only one equation)

$4p + q = 5$

$p = 2$ and $q = -3$

1 mark out of 3

Response 1C (By verification)

$p = 2$ and $q = -3$ (ex nihilo)

$v_2 = 8 - 3 = 5$

and $v_3 = 10 - 3 = 7$

2 marks out of 3

3 (c) Either the sequence in (*a*) or the sequence in (*b*) has a limit.

(i) Calculate this limit.

(ii) Why does the other sequence not have a limit? **3**

Generic Scheme			Illustrative Scheme
3 (c)			
•5	ss	know how to find valid limit	•5 $l = -\frac{1}{2}l$ or $l = \frac{0}{1-\left(-\frac{1}{2}\right)}$
•6	pd	calculate a valid limit only	•6 $l = 0$
•7	ic	state reason	•7 outside interval $-1 < p < 1$

3

Using $l = \frac{b}{1-a}$, $a = -\frac{1}{2}$ and $b = 0$, without substituting, and stating $l = 0$, gains •5 but not •6.

Notes

4. Just stating that $l = al + b$ or $l = \frac{b}{1-a}$ is not sufficient for •5.
5. Any calculations based on formulae masquerading as a limit rule cannot gain •5 and •6.
6. For candidates who use '$b = 0$', •6 is only available to those who simplify $\frac{0}{\dots}$ to 0.
7. Accept $2 > 1$ or $p > 1$ for •7. This may be expressed in words.
8. Candidates who use a without reference to p or 2 cannot gain •7.

Regularly occurring responses

Response 2

$l = \frac{0}{\frac{1}{2}}$ ✗ •5

$l = 0$ ✗ •6

See note 5

0 marks out of 3

Response 3A

From (b) $p = \frac{3}{4}$ and $q = 2$

In (c) $l = \frac{3}{4}l + 2$ and $l = -\frac{1}{2}l$ ✓ •5

$l = 8$ $l = 0$ ✓ •6

Both limits exist as $-1 < \frac{3}{4} < 1$ and $-1 < -\frac{1}{2} < 1$ ✓ •7

3 marks out of 3

Response 3B

From (b) $p = \frac{3}{4}$ and $q = 2$

In (c) $l = \frac{3}{4}l + 2$ and $l = -\frac{1}{2}l$ ✓ •5

$l = 8$ $l = 0$ ✓ •6

Impossible ✗ •7

2 marks out of 3

Response 4A

$l = \frac{0}{1-\left(-\frac{1}{2}\right)}$ ✓ •5 and $l = \frac{-3}{1-2}$

$l = 0$ $\underline{l = 3}$ ✗ •6

First has limit because $-1 < -\frac{1}{2} < 1$ ✗ •7

1 mark out of 3

Response 4B

$l = \frac{0}{1-\left(-\frac{1}{2}\right)}$ ✓ •5 and $l = \frac{-3}{1-2}$

$l = 0$ $\underline{l = 3}$ ✗ •6

Second sequence has no limit as $-1 < 2 < 1$ not true ✓ •7

2 marks out of 3

Response 5A

$l = -\frac{1}{2}l$ and $l = 2l - 3$ ✓ •5

$l = 0$ $l = 3$ ✓ •6

1st has limit because $-1 < 0 < 1$ ✗ •7

2 marks out of 3

Response 5B

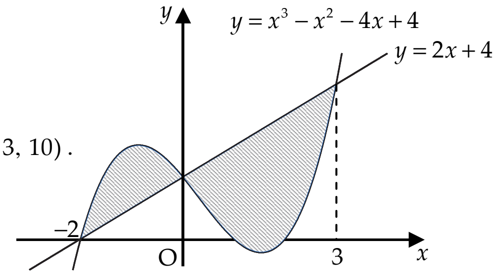
$l = -\frac{1}{2}l$ and $l = 2l - 3$ ✓ •5

$l = 0$ $l = 3$ ✓ •6

Second sequence has no limit because 2 is not between -1 and 1 ✓ •7

3 marks out of 3

4	The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$. The curve and the line intersect at the points $(-2,\ 0),\ (0,\ 4)$ and $(3,\ 10)$. Calculate the total shaded area.	10

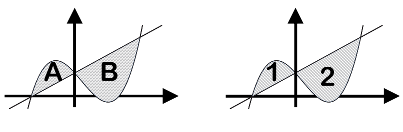


Generic Scheme			Illustrative Scheme	
4				
•[1]	ss	know to integrate	•[1] $\int \ldots$ **or** attempt integration	
•[2]	ic	know to deal with areas on each side of $y-$axis	•[2] Evidence of attempting to interpret the diagram to left of $y-$axis separately from diagram to the right.	
•[3]	ic	interpret limits of one area	•[3] $\int_{-2}^{0}$	e.g. $\int_{0}^{3}$ with no other
•[4]	ic	use "upper − lower"	•[4] $(x^3 - x^2 - 4x + 4) - (2x + 4)$	$(2x+4) - (x^3 - x^2 - 4x + 4)$
•[5]	pd	integrate	•[5] $\frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2$	$3x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$
•[6]	ic	substitute in limits	•[6] $-\left(\frac{1}{4}(-2)^4 - \frac{1}{3}(-2)^3 - 3(-2)^2\right)$	$\left(3(3)^2 + \frac{1}{3}(3)^3 - \frac{1}{4}(3)^4\right)$
			Evidence for •[6] may be implied by •[7], but •[7] must be consistent with •[5].	
•[7]	pd	evaluate the area on one side	•[7] $\frac{16}{3}$	$\frac{63}{4}$
•[8]	ss	interpret integrand with limits of the other area	•[8] $\int_{0}^{3} (2x+4) - (x^3 - x^2 - 4x + 4)\,dx$	$\int_{-2}^{0} (x^3 - x^2 - 4x + 4) - (2x+4)\,dx$
•[9]	pd	evaluate the area on the other side	•[9] $\frac{63}{4}$	$\frac{16}{3}$
•[10]	ic	state total area	•[10] $21\frac{1}{12}$ or $\frac{253}{12}$ or $21 \cdot 1$	$21\frac{1}{12}$ or $\frac{253}{12}$ or $21 \cdot 1$

10

Notes

1. The evidence for •[2] may not appear until •[8] stage.
2. The evidence for •[2] may appear in a diagram e.g.



3. Where a candidate differentiates at •[5], then •[5], •[6], •[7] and •[9] are not available.
4. Candidates who substitute at •[6], without integrating at •[5] lose •[5], •[6] and •[7]. However •[8], •[9] and •[10] are still available.

Regularly occurring responses

General comment to markers

In this question you should scan the entire response before starting to mark. Where errors occur in the integration/evaluation, use $\bullet^3$ to $\bullet^7$ to mark the better solution and $\bullet^8$ and $\bullet^9$ to mark the poorer solution.

A tabular approach to allocating marks is particularly useful in questions like this, where a candidate's response is spread over several pages, or contains working which appears randomly set out. Response 1 indicates the approach to take here.

Response 1

$$\int_{-2}^{3} \text{upper} - \text{lower}$$

$$\int_{-2}^{3} x^3 - x^2 - 6x\,dx$$

$$\tfrac{1}{4}x^4 - \tfrac{1}{3}x^3 - 3x^2$$

$$\left(\frac{3^4}{4} - \frac{3^3}{3} - 3.3^2\right) - \left(\frac{-2^4}{4} - \frac{-2^3}{3} - 3.-2^2\right)$$

$$-\frac{125}{12}$$

$\bullet^1$ ✓
$\bullet^2$ ✗
$\bullet^3$ ✗
$\bullet^4$ ✗
$\bullet^5$ ✓
$\bullet^6$ ✓
$\bullet^7$ ✗
$\bullet^8$ ✗
$\bullet^9$ ✗
$\bullet^{10}$ ✗

Response 2

$$\int_{-2}^{0} \ldots = \frac{16}{3}$$

$$\int_{0}^{3} 6x + x^2 - x^3\,dx = \frac{63}{4}$$

$$\int_{1}^{2} x^3 - x^2 - 4x + 4\,dx = -\frac{7}{12}$$

$$\text{Total area} = \frac{16}{3} + \frac{63}{4} + \frac{7}{12} = \frac{260}{12}$$

$\bullet^1$ ✓
$\bullet^2$ ✓
$\bullet^3$ ✓
$\bullet^4$ ✓
$\bullet^5$ ✓
$\bullet^6$ ✓
$\bullet^7$ ✓
$\bullet^8$ ✗
$\bullet^9$ ✗
$\bullet^{10}$ ✓̸

The appearance of this integral is sufficient to lose $\bullet^8$ and $\bullet^9$.

Response 3

$$\int_{1}^{2} x^3 - x^2 - 4x + 4\,dx \quad ✓ \bullet^1$$

$$\tfrac{1}{4}x^4 - \tfrac{1}{3}x^3 - 2x^2 + 4x \quad ✓ \bullet^5$$

$$\left(\frac{2^4}{4} - \frac{2^3}{3} - 2.2^2 + 4.2\right) - \left(\frac{1}{4} - \frac{1}{3} - 2 + 4\right) \quad ✓ \bullet^6$$

$$-\frac{7}{12} \quad ✗ \bullet^7$$

Candidates who evaluate an integral and obtain a negative answer must deal with this correctly.
The minimum evidence would be

$$\text{e.g.} \int_{-2}^{0} \ldots = -\frac{16}{3}$$

$$A = \frac{16}{3} \text{ or Area} = \frac{16}{3}$$

N.B. If due to an error the evaluation is negative it must be dealt with correctly. The responses below illustrate what is required under this circumstance. If both integrals lead to negative values only $\bullet^7$ **or** $\bullet^9$ is lost.

Response 4A

$$\int_{0}^{3} \ldots \frac{63}{4}$$

$$\int_{-2}^{0} 6x + x^2 - x^3\,dx \quad ✗ \bullet^8$$

$$= \ldots$$

$$= -\frac{16}{3} \quad ✗ \bullet^9$$

$$\text{Area} = \frac{63}{4} + -\frac{16}{3} \quad ✗ \bullet^{10}$$

$$= \frac{125}{12}$$

Response 4B

$$\int_{0}^{3} \ldots \frac{63}{4}$$

$$\int_{-2}^{0} 6x + x^2 - x^3\,dx \quad ✗ \bullet^8$$

$$= \ldots$$

$$= -\frac{16}{3} \quad ✗ \bullet^9$$

$$\text{Area} = \frac{63}{4} + \frac{16}{3} = \frac{253}{12} \quad ✓̸ \bullet^{10}$$

Response 4C

$$\int_{0}^{3} \ldots \frac{63}{4}$$

$$\int_{-2}^{0} 6x + x^2 - x^3\,dx \quad ✗ \bullet^8$$

$$= \ldots$$

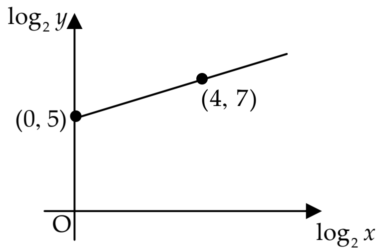
$$= -\frac{16}{3} \quad \text{can't be negative}$$

$$= \frac{16}{3} \quad ✗ \bullet^9$$

$$\text{Area} = \frac{63}{4} + \frac{16}{3} = \frac{253}{12} \quad ✓̸ \bullet^{10}$$

5 Variables x and y are related by the equation $y = kx^n$.

The graph of $\log_2 y$ against $\log_2 x$ is a straight line through the points $(0, 5)$ and $(4, 7)$, as shown in the diagram.



Find the values of k and n. **5**

Generic Scheme			**Illustrative Scheme**	
5				
Method 1			Method 1	
•¹	ss	introduce logarithms to $y = kx^n$	•¹ $\log_2 y = \log_2 kx^n$	**stated explicitly**
•²	ic	use laws of logarithms	•² $\log_2 y = n\log_2 x + \log_2 k$	**stated explicitly**
•³	ic	interpret intercept	•³ $\log_2 k = 5$ **or** $\log_2 y = 5$	Accept without working
•⁴	ic	solve for k	•⁴ $k = 32$ or 2^5	
•⁵	ic	interpret gradient	•⁵ $n = \frac{1}{2}$	Accept without working

Generic Scheme			**Illustrative Scheme**	
Method 2			Method 2	
•¹	ss	state linear equation	•¹ $\log_2 y = \frac{1}{2}\log_2 x + 5$	•¹ $\log_2 y = \frac{1}{2}\log_2 x + 5$
•²	ic	introduce logarithms	•² $\ldots + 5\log_2 2$ or $\ldots + \log_2 2^5$	•² $\log_2 y = \log_2 x^{\frac{1}{2}} + 5$
•³	ic	use laws of logarithms	•³ $\log_2 y = \log_2 x^{\frac{1}{2}} + \ldots$	•³ $\log_2\left(\dfrac{y}{x^{\frac{1}{2}}}\right) = 5$
•⁴	ic	use laws of logarithms	•⁴ $\log_2 y = \log_2 2^5 x^{\frac{1}{2}}$	•⁴ $\dfrac{y}{x^{\frac{1}{2}}} = 2^5$
•⁵	ic	interpret result	•⁵ $y = 2^5 x^{\frac{1}{2}}$	•⁵ $y = 2^5 x^{\frac{1}{2}}$

Generic Scheme			**Illustrative Scheme**
Method 3			Method 3
•¹	ic	interpret point on log. graph	•¹ $\log_2 x = 4$ **and** $\log_2 y = 7$
•²	ic	convert from log. to exp. form	•² $x = 2^4$ **and** $y = 2^7$
•³	ic	interpret point and convert	•³ $\begin{cases} \log_2 x = 0 \textbf{ and } \log_2 y = 5 \\ x = 1 \textbf{ and } y = 2^5 \end{cases}$
•⁴	ss	know to substitute points	•⁴ $2^7 = k \times \left(2^4\right)^n$ **and** $2^5 = k$ (from $2^5 = k.1^n$)
•⁵	ic	interpret result	•⁵ $n = \frac{1}{2}$

5

Notes

1. Omission of base 2 is treated as bad form at the •¹ and •² stage.
2. Gradient $(m) = \frac{1}{2}$ is not sufficient for •⁵.
3. Throughout this question accept 32 in lieu of 2^5.
4. Markers should not pick and choose within methods. Use the method which gives the candidate the highest mark.

Regularly occurring responses

Response 1A

With no working

$k = 32$ ✓ $\bullet^3$

$n = \frac{1}{2}$ ✓ $\bullet^5$

2 marks out of 5

Response 1B

With no working

$k = \frac{1}{2}$ X $\bullet^3$

$n = 32$ X $\bullet^5$

0 marks out of 5

Response 2 (Method 1)

$\log_2 k = 5$ ✓ $\bullet^3$

$k = 32$ ✓ $\bullet^4$

$n = \frac{1}{2}$ ✓ $\bullet^5$

3 marks out of 5

Response 3 (Variation of Method 2 and Response 1A)

$\log_2 y = \frac{1}{2}\log_2 x + 5$ ✓ $\bullet^1$

$\log_2 y = \log_2 \sqrt{x} + 5$ ✓ $\bullet^2$

$y = \sqrt{x} + 5$

$k = 1,$

$n = \frac{1}{2}$ ✓ $\bullet^5$

3 marks out of 5

Response 4 (Variation of Method 2 and Response 1A)

$y = \frac{1}{2}x + 5$

$\log_2 y = \frac{1}{2}\log_2 x + 5$ ✓ $\bullet^1$

$\log_2 y - \log_2 x^{\frac{1}{2}} = 5$ ✓ $\bullet^2$

$\frac{y}{\sqrt{x}} = 5$ X

$y = 5\sqrt{x}$

$k = 5$ X

$n = \frac{1}{2}$ ✓ $\bullet^5$

3 marks out of 5

6	(a)	The expression $3\sin x - 5\cos x$ can be written in the form $R\sin(x+a)$ where $R>0$ and $0 \le a < 2\pi$.	
		Calculate the values of R and a.	4

Generic Scheme | **Illustrative Scheme**

6 (a)

$\bullet^1$	ss	use compound angle formula	$\bullet^1$ $R\sin x\cos a + R\cos x\sin a$		**stated explicitly**
$\bullet^2$	ic	compare coefficients	$\bullet^2$ $R\cos a = 3$ and $R\sin a = -5$		**stated explicitly**
$\bullet^3$	pd	process R	$\bullet^3$ $\sqrt{34}$	(Accept $5\cdot 8$)	with or without working
$\bullet^4$	pd	process a	$\bullet^4$ $5\cdot 253$	(Accept $5\cdot 3$)	must be consistent with $\bullet^2$

4

Notes

1. Treat as bad form the use of k instead of R.
2. Treat $R\sin x\cos a + \cos x\sin a$ as bad form only if the equations at the $\bullet^2$ stage both contain R.
3. $\sqrt{34}\sin x\cos a + \sqrt{34}\cos x\sin a$ or $\sqrt{34}(\sin x\cos a + \cos x\sin a)$ is acceptable for $\bullet^1$ and $\bullet^3$.
4. $\bullet^2$ is not available for $R\cos x = 3$ and $R\sin x = -5$, however, $\bullet^4$ is still available.
5. $\bullet^4$ is only available for a single value of a.
6. Candidates who work in degrees and don't convert to radian measure lose $\bullet^4$. Do not accept $\frac{301\pi}{180}$ or $\frac{5\pi}{3}$.
7. Candidates may use any form of the wave equation for $\bullet^1$, $\bullet^2$ and $\bullet^3$, however, $\bullet^4$ is only available if the value of a is interpreted for the form $R\sin(x+a)$.

Regularly occurring responses

For $\bullet^2$ and $\bullet^4$

Response 1A

$R\cos a = 3 \quad R\sin a = 5$ ✗ $\bullet^2$

$\tan a = \frac{5}{3}$

$a = 1\cdot 03$ ✗ $\bullet^4$

Response 1B

$R\cos a = 3 \quad R\sin a = 5$ ✗ $\bullet^2$

$\tan a = \frac{3}{5}$

$a = 0\cdot 54$ ✗ $\bullet^4$

Response 1C

$R\cos a = 3 \quad R\sin a = -5$ ✓ $\bullet^2$

$\tan a = -\frac{3}{5}$

$a = 5\cdot 74$ ✗ $\bullet^4$

Response 2

$R\sin(x-a) = R\sin x\cos a - R\cos x\sin a$ ✗ $\bullet^1$

$R\cos a = 3 \quad R\sin a = 5$ ✗ $\bullet^2$

$R = \sqrt{34}$ ✓ $\bullet^3$

$a = 1\cdot 03$ ✗ $\bullet^4$

See note 7

3 marks out of 4

Response 3

$k\sin x\cos a + k\cos x\sin a$ ✓ $\bullet^1$

$\cos a = 3 \quad \sin a = -5$ ✗ $\bullet^2$

$R = \sqrt{34}$ ✗ $\bullet^3$

$a = 5\cdot 3$ ✗ $\bullet^4$

Not consistent with working at $\bullet^2$

2 marks out of 4

6 (b) Hence find the value of t, where $0 \le t \le 2$, for which

$$\int_0^t (3\cos x + 5\sin x)\,dx = 3$$

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Generic Scheme			Illustrative Scheme	
6 (b)				
$\bullet^5$	pd	integrate given expression	$\bullet^5$	$3\sin x - 5\cos x$
$\bullet^6$	ic	substitute limits	$\bullet^6$	$(3\sin t - 5\cos t) - (3\sin 0 - 5\cos 0)$
$\bullet^7$	pd	process limits	$\bullet^7$	$3\sin t - 5\cos t + 5$
$\bullet^8$	ss	know to use wave equation	$\bullet^8$	$\sqrt{34}\sin(t+5\cdot3)+5$
$\bullet^9$	ic	write in standard format	$\bullet^9$	$\sin(t+5\cdot3) = -\frac{2}{\sqrt{34}}$
$\bullet^{10}$	ss	start to solve equation	$\bullet^{10}$	$t+5\cdot3 = 3\cdot5$ and $5\cdot9$
$\bullet^{11}$	pd	complete and state solution	$\bullet^{11}$	$t = 0\cdot6$

$\bullet^5$ to $\bullet^{11}$ are available to candidates who chose to write this integrand as new wave function.

7

Notes

8. The inclusion of "+ c" at $\bullet^5$ stage should be treated as bad form.
9. For those candidates who use a as $5\cdot253$ or $5\cdot25...$, follow through their working for $\bullet^8$ to $\bullet^{11}$.
10. Candidates who use degree measure in (a) lose $\bullet^4$ and if they continue to do so in (b), only $\bullet^5$, $\bullet^6$, $\bullet^7$ and $\bullet^8$ are available (see also response 6A and 6B below.)

Regularly occurring responses

Response 4 (No integration)

$\int_0^t 3\cos x + 5\sin x\,dx = \sqrt{34}\sin(x+5\cdot3)$

lose $\bullet^5$, $\bullet^6$, $\bullet^7$ and $\bullet^8$

then

$\sin(x+5\cdot3) = \frac{3}{\sqrt{34}}$ ✗ $\bullet^9$

$x+5\cdot3 = 0\cdot5,\ 2\cdot6,\ 6\cdot8$ ✗ $\bullet^{10}$

$x = 1\cdot5$ ✗ $\bullet^{11}$

Needs to be in terms of t

Response 5

$\dots 3\sin x - 5\cos x$ ✓ $\bullet^5$

$3\sin t - 5\cos t - 0$ ✗ $\bullet^6$ ✗ $\bullet^7$

$\sqrt{34}\sin(t+5\cdot3)$ ✗ $\bullet^8$

$\sin(t+5\cdot3) = \frac{3}{\sqrt{34}}$ ✗ $\bullet^9$

$t+5\cdot3 = 0\cdot5,\ 2\cdot6,\ 6\cdot8$ ✗ $\bullet^{10}$

$t = 1\cdot5$ ✗ $\bullet^{11}$

Response 6A (Misreading question)

$\int \sqrt{34}\sin(x+5\cdot3)\,dx$ ✗ $\bullet^5$ $\bullet^8$

$= -\sqrt{34}\cos(t+5\cdot3) = 3$ ✗ $\bullet^6$ ✗ $\bullet^7$

$\cos(t+5\cdot3) = -\frac{3}{\sqrt{34}}$ ✗ $\bullet^9$

$t+5\cdot3 = 2\cdot1,\ 4\cdot2,\ 8\cdot4$ ✗ $\bullet^{10}$

$t = 3\cdot1$ i.e. no solution ✗ $\bullet^{11}$

3 marks out of 7

If a is left in degrees no marks are available.

Response 6B (Misreading question)

$\int \sqrt{34}\sin(x+5\cdot3)\,dx$ ✗ $\bullet^5$ $\bullet^8$

$= \left[-\sqrt{34}\cos(x+5\cdot3)\right]_0^t$

$= -\sqrt{34}\cos(t+5\cdot3) + \sqrt{34}\cos 5\cdot3$ ✗ $\bullet^6$

$-\sqrt{34}\cos(t+5\cdot3) + 3\cdot2$ ✗ $\bullet^7$

$\cos(t+5\cdot3) = \frac{-0\cdot2}{-\sqrt{34}}$ ✗ $\bullet^9$

$t+5\cdot3 = 1\cdot5,\ 4\cdot7,\ 7\cdot8$ ✗ $\bullet^{10}$

$t = 2\cdot5$ i.e. no solution ✗ $\bullet^{11}$

5 marks out of 7

7	Circle C_1 has equation $(x+1)^2+(y-1)^2=121$. A circle C_2 with equation $x^2+y^2-4x+6y+p=0$ is drawn inside C_1. The circles have no points of contact. What is the range of values of p?	9

Generic Scheme | **Illustrative Scheme**

7

$\bullet^1$	ic	state centre of C_1	$\bullet^1$	$(-1,\ 1)$	
$\bullet^2$	ic	state radius of C_1	$\bullet^2$	11	Do not accept $\sqrt{121}$
$\bullet^3$	ic	state centre of C_2	$\bullet^3$	$(2,\ -3)$	
$\bullet^4$	pd	find radius of C_2 in terms of p	$\bullet^4$	$\sqrt{13-p}$	Accept c in lieu of p
$\bullet^5$	ic	interpret upper bound for p	$\bullet^5$	$p<13$	
$\bullet^6$	ic	find distance between centres (d)	$\bullet^6$	5	**stated explicitly**
$\bullet^7$	ss	identify relevant relationship	$\bullet^7$	$\sqrt{13-p}<6$ or $r_2+d<11$ or $r_2<6$	
$\bullet^8$	ic	develop relationship by squaring	$\bullet^8$	$13-p<36$	
$\bullet^9$	pd	find lower bound for p	$\bullet^9$	$p>-23$	

9

Notes

1. Treat as bad form the use of c in lieu of p.
2. The evidence for $\bullet^7$ must involve an inequality, but may be in words.
3. Treat $\sqrt{13}-p$ as bad form as long as it is clear that the candidate is using $\sqrt{13-p}$.
4. Candidates who are only working with an equation lose both $\bullet^7$ and $\bullet^9$, however, $\bullet^8$ may still be available.
5. $\bullet^9$ is only available to candidates who solve an inequation involving a negative coefficient of p.

Regularly occurring responses

Response 1A

Marks 1 to 3 gained

^ $\bullet^5$ ^ $\bullet^6$

$\sqrt{-2^2+3^2-p}<11$

$\sqrt{13-p}<11$ ✗ $\bullet^7$ ✓ $\bullet^4$

$13-p<121$ ✗ $\bullet^8$

$p>-108$ ✗ $\bullet^9$

Response 1B

$C_1=(-1,\ 1)$ ✓ $\bullet_1$ $C_2=(2,-3)$ ✓ $\bullet^3$

$r_1=11$ ✓ $\bullet^2$ $r_2=\sqrt{13+p}$ ✗ $\bullet^4$

$d=5$ ✓ $\bullet^6$

$\sqrt{13+p}<11$ ✗ $\bullet^7$

$13+p<121$ ✗ $\bullet^8$

$p<108$ ✗ $\bullet^9$

Response 2

For marks 7 to 9

$\sqrt{13-p}<6$ ✓ $\bullet^7$

$\sqrt{13}-\sqrt{p}<6$ ✗ $\bullet^8$

$\underline{169-p<36}$

$-p<-133$

$p>133$ ✗ $\bullet^9$

Penalise the use of $\le$ and/or $\ge$ once only.

Response 3 (see note 4)

$\sqrt{13-p}=0$

$p=13$ ✗ $\bullet^5$

$\sqrt{13-p}=6$ ✗ $\bullet^7$

$13-p=36$ ✗ $\bullet^8$

$p>-23$ ✗ $\bullet^9$

Response 4

$\sqrt{13-p}\ge 0$

$p\le 13$ ✗ $\bullet^5$

$\sqrt{13-p}\le 6$ ✗ $\bullet^7$

$p\ge -23$ ✗ $\bullet^9$

Response 5

$0<\sqrt{13-p}<6$ ✓ $\bullet^7$

$0<13-p<36$ ✓ $\bullet^8$

$-13<-p<23$

so $p<13$ ✓ $\bullet^5$ and $p>-23$ ✓ $\bullet^9$

or $-23<p$ ✓ $\bullet^9$ <13 ✓ $\bullet^5$

Regularly occurring responses

Response 6

$(x-2)^2+(y+3)^2=13-p$

$13-p<121$ $\bullet^4$ $\bullet^7$

$p>-108$ $\bullet^9$