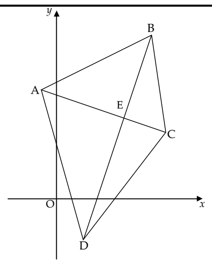
Question	Answer
1	C
2	B
3	D
4	D
5	A
6	C
7	D
8	A
9	B
10	D
11	D
12	C
13	C
14	B
15	B
16	A
17	A
18	C
19	C
20	D

Summary		
	A	4
	B	4
	C	6
	D	6

21 A quadrilateral has vertices A(−1, 8), B(7, 12), C(8, 5) and D(2, − 3) as shown in the diagram.



(a) Find the equation of diagonal BD. 2

(b) The equation of diagonal AC is $x+3y=23$.

Find the coordinates of E, the point of intersection of the diagonals. 3

21 (a)

Generic Scheme			Illustrative Scheme
$\bullet^1$	pd	find gradient of BD	$\bullet^1$ $\frac{15}{5}$ or equivalent
$\bullet^2$	ic	state equation of BD	$\bullet^2$ $y-(-3)=3(x-2)$ **or** $y-12=3(x-7)$

②

Notes

1. There is no need to simplify m_{BD} for $\bullet^1$; however, it must be simplified before $\bullet^2$ can be awarded.
2. If m_{BD} cannot be simplified, due to an error, then $\bullet^2$ is still available.
3. Candidates who determine the equation of AC lose $\bullet^1$ but may still gain $\bullet^2$.
4. Candidates lose $\bullet^1$ and $\bullet^2$ for the equation of any side of the quadrilateral.

Regularly occurring responses

Response 1

Using $y=mx+c$

$y=3x+c$ ✓ $\bullet^1$

$12=3\times7+c$ **or** $-3=3\times2+c$

$c=-9$ ✓ $\bullet^2$

2 marks out of 2

Response 2

$\left.\begin{matrix} m_{\text{AC}}=-\frac{1}{3} \\ m_{\text{BD}}=3 \end{matrix}\right\}$ ✗ $\bullet^1$

Candidate has assumed diagonals are perpendicular - without evidence.

$y-(-3)=3(x-2)$ ✗ $\bullet^2$

1 mark out of 2

21 (b)

Generic Scheme			Illustrative Scheme
$\bullet^3$	ss	start solution of simultaneous equations	$\bullet^3$ e.g. $3x-y=9$ and $x+3y=23$ **or** $3x-9=-\frac{x}{3}+\frac{23}{3}$ **or** $x+3(3x-9)=23$
$\bullet^4$	pd	solve for one variable	$\bullet^4$ $x=5$ **or** $y=6$
$\bullet^5$	pd	solve for second variable	$\bullet^5$ $y=6$ **or** $x=5$

③

Notes

5. Candidates who find the equation of AC in (a), correctly or incorrectly, lose $\bullet^3$, $\bullet^4$ and $\bullet^5$ in (b).
6. Any other incorrect answer from (a) may still gain $\bullet^3$, $\bullet^4$ and $\bullet^5$ as follow through.

Regularly occurring responses

Response 3

$3x-y=3$ and $x+3y=23$ ✗ $\bullet^3$

Subsequent to gaining $\bullet^3$ an error was made in simplifying the equation in (a), but strategy mark still awarded in (b).

$x=3{\cdot}2$ ✗ $\bullet^4$

$y=6{\cdot}6$ ✗ $\bullet^5$

Error going from (a) to (b) is penalised at first pd (or ic) mark.

2 marks out of 3

21 (c) (i) Find the equation of the perpendicular bisector of AB.

(ii) Show that this line passes through E. **5**

21 (c)

Generic Scheme			Illustrative Scheme
$\bullet^6$	ss	know and find midpoint of AB	$\bullet^6$ $(3,10)$
$\bullet^7$	pd	find gradient of AB	$\bullet^7$ $\frac{4}{8}$ or equivalent
$\bullet^8$	ic	interpret perpendicular gradient	$\bullet^8$ $-\frac{8}{4}$ or equivalent **stated, or implied by** $\bullet^9$
$\bullet^9$	ic	state equation of perp. bisector	$\bullet^9$ $y-10=-2(x-3)$ **but not** $y-6=-2(x-5)$
$\bullet^{10}$	ic	justification of point on line	$\bullet^{10}$ when $x=5,\ y=-2\times5+16=6$ **or** $2\times5+6-16=0$

5

Notes

7. Candidates who do not simplify the gradient in (a) and (c) should only be penalised once.
8. $\bullet^9$ is only available as a consequence of using a midpoint and perpendicular gradient.
9. Candidates who use $y-6=-2(x-5)$ at $\bullet^9$ stage, lose $\bullet^9$ and $\bullet^{10}$.
10. Candidates who show that the point of intersection of BD or AC **and** the perpendicular bisector is E gain $\bullet^{10}$.

Regularly occurring responses

Response 4

$m_{\text{PERP BISECTOR}}=-2$

$m_{\text{"ME"}}=\ldots=-2$

So perpendicular bisector goes through E ✗ $\bullet^{10}$

There must be reference to the midpoint being a common point to gain this mark.

Response 5

From (i) equation of perpendicular bisector is $y=-2x+16$, using $(3,\ 10)$.

Then in (ii) using $m=-2$ and E(5, 6) leads to $y=-2x+16$. Same equation so E lies on line. ✓ $\bullet^{10}$

Response 6

From (b) $E(3\cdot2,\ 6\cdot6)$

$x=3\cdot2,\ y=\ldots=9\cdot6$, so line does not pass through E. ✗ $\bullet^{10}$

Comment must be consistent with E from (b).

22 A function f is defined on the set of real numbers by $f(x) = (x-2)(x^2+1)$.

(a) Find where the graph of $y = f(x)$ cuts:

(i) the x-axis; (ii) the y-axis. **2**

(b) Find the coordinates of the stationary points on the curve with equation $y = f(x)$ and determine their nature. **8**

Generic Scheme			**Illustrative Scheme**		

22 (a)

$\bullet^1$	ic	interpret x intercept	$\bullet^1$	$(2, 0)$	(minimum response "(i) 2")
$\bullet^2$	ic	interpret y intercept	$\bullet^2$	$(0, -2)$	(minimum response "(ii) -2")

2

Notes

1. Candidates who obtain extra x-axis intercepts lose $\bullet^1$.
2. Candidates who obtain extra y-axis intercepts lose $\bullet^2$.
3. Candidates who interchange intercepts can gain at most one mark.

22 (b)

$\bullet^3$	ic	write in differentiable form	$\bullet^3$	$x^3 - 2x^2 + x - 2$	
$\bullet^4$	ss	know to and start to differentiate	$\bullet^4$	$3x^2 \ldots$ or $\ldots - 4x\ldots$	
$\bullet^5$	pd	complete derivative and equate to 0	$\bullet^5$	$3x^2 - 4x + 1$ **and** $f'(x) = 0$ **or** $3x^2 - 4x + 1 = 0$	
$\bullet^6$	pd	factorise derivative	$\bullet^6$	$(3x-1)(x-1)$	
$\bullet^7$	pd	process for x	$\bullet^7$	$\frac{1}{3}$ and 1	$x = \frac{1}{3}$ and $y = -\frac{50}{27}$
$\bullet^8$	pd	evaluate y-coordinates	$\bullet^8$	$-\frac{50}{27}$ and -2	$x = 1$ and $y = -2$
$\bullet^9$	ic	justify nature of stationary points	$\bullet^9$	$\begin{array}{c\|ccccc} x & \cdots & \frac{1}{3} & \cdots & 1 & \cdots \\ \hline f'(x) & + & 0 & - & 0 & + \end{array}$	Accept a valid expression in lieu of $f'(x)$.
$\bullet^{10}$	ic	interpret and state conclusions	$\bullet^{10}$	max min	

8

This question may be marked vertically. The dotted rectangle shows what is required for $\bullet^{10}$.

Notes

4. $\bullet^5$ is only available if " $=0$ " appears at or before $\bullet^6$ stage.
5. $\bullet^3$, $\bullet^4$ and $\bullet^5$ are the only marks available to candidates who solve $3x^2 - 4x = -1$.
6. At $\bullet^9$ the nature can be determined using the second derivative.
7. $\bullet^9$ is only available if the nature table is consistent with the candidate's derivative.
8. $\bullet^{10}$ is awarded for correct interpretation of the candidate's nature table in words.

Regularly occurring responses

Response 1A

x	0	$\frac{1}{3}$	$\frac{1}{2}$	1	2	
$\frac{dy}{dx}$	1	0	$-\frac{1}{4}$	0	5	✓ $\bullet^9$
		max		min		✓ $\bullet^{10}$

Response 1B

^	$\cdots$	$\frac{1}{3}$	$\cdots$	1	$\cdots$		
$f'(x)$	$+$	0	$-$	0	$+$	✗ $\bullet^9$	
		max		min		✓ $\bullet^{10}$	

x missing

Response 1C

x	$\cdots$	$\frac{1}{3}$	$\cdots$	1	$\cdots$	
slope	/	—	\	—	/	✗ $\bullet^9$
						^ $\bullet^{10}$

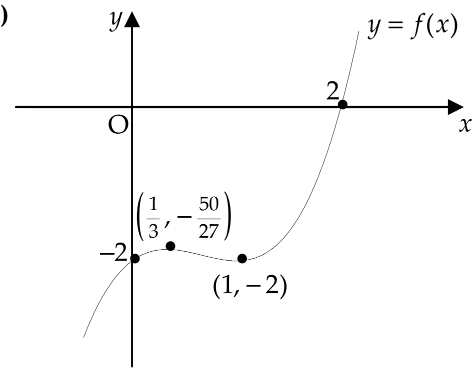
signs or values are necessary

22 (c) On separate diagrams sketch the graphs:

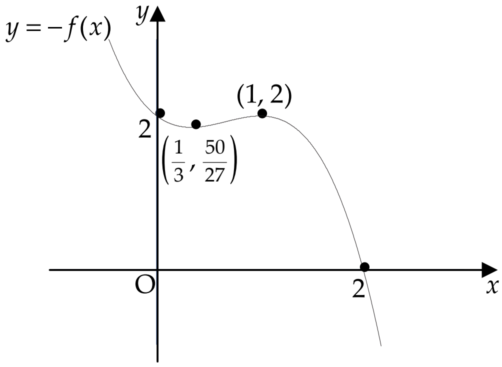
(i) $y = f(x)$; (ii) $y = -f(x)$. **3**

Generic Scheme **Illustrative Scheme**

(c) (i)



(ii)



•[11]	ic	curve showing points from (a) and (b) without annotation	•[11]	sketch
•[12]	ic	**cubic** curve showing **all** intercepts and stationary points annotated	•[12]	sketch
•[13]	ic	curve from (i) reflected in $x-$axis	•[13]	reflected sketch

3

Notes

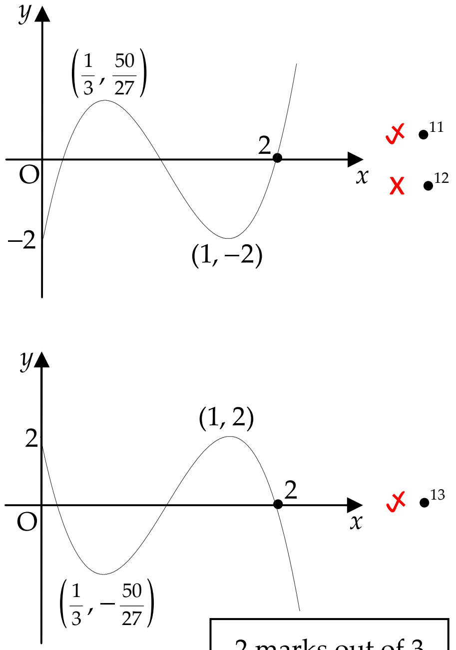
9. •[11] is for any curve consistent with all points found in (a) and (b). Ignore any extra critical points.

10. In (c)(ii), the minimum requirement is the curve from (c)(i) reflected in x-axis showing **at least one** x-intercept unchanged and **at least one** stationary point correctly annotated.

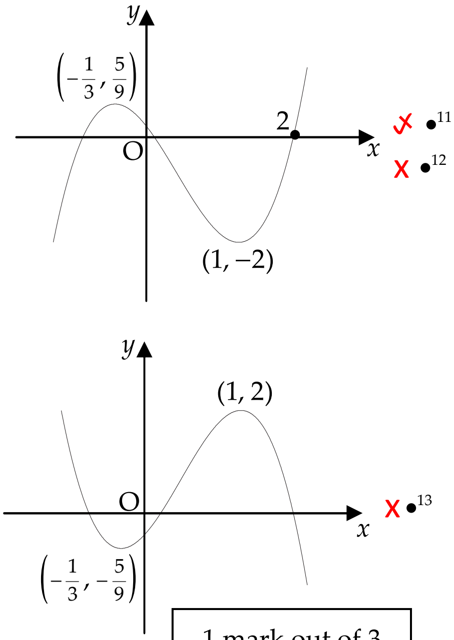
Regularly occurring responses

Follow through from candidate's work in (a) and (b).

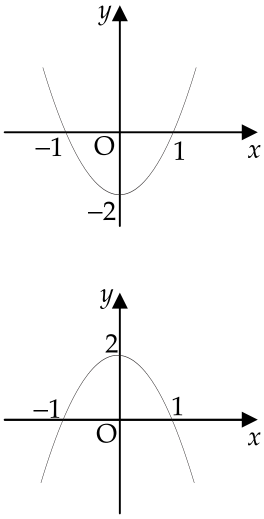
Response 2 **Response 3** **Response 4**



2 marks out of 3



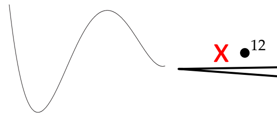
1 mark out of 3



0 marks out of 3

No marks available for a quadratic.

Response 5



Not a cubic due to apparent change of concavity.

23 (a) Solve $\cos 2x^\circ - 3\cos x^\circ + 2 = 0$ for $0 \le x < 360$. **5**

Generic Scheme	**Illustrative Scheme**

23 (a)

Generic Scheme

- $\bullet^1$ ss know to use double angle formula
- $\bullet^2$ ic express as a quadratic in $\cos x^\circ$
- $\bullet^3$ ss start to solve
- $\bullet^4$ pd reduce to equations in $\cos x^\circ$ only
- $\bullet^5$ ic process solutions in given domain

Illustrative Scheme

Method 1 : Using factorisation

$\bullet^1$ $2\cos^2 x^\circ - 1\ldots$ **stated, or implied by** $\bullet^2$

$\bullet^2$ $2\cos^2 x^\circ - 3\cos x^\circ + 1$

$\bullet^3$ $(2\cos x^\circ - 1)(\cos x^\circ - 1)$

$\Big\}$ = 0 must appear at either of these lines to gain $\bullet^2$.

Method 2 : Using quadratic formula

$\bullet^1$ $2\cos^2 x^\circ - 1\ldots$

$\bullet^2$ $2\cos^2 x^\circ - 3\cos x^\circ + 1 = 0$ **stated explicitly**

$\bullet^3$ $\dfrac{-(-3) \pm \sqrt{(-3)^2 - 4\times 2\times 1}}{2\times 2}$

In both methods :

$\bullet^4$ $\cos x^\circ = \frac{1}{2}$ and $\cos x^\circ = 1$

$\bullet^5$ 0, 60 and 300

Candidates who include 360 lose $\bullet^5$

or

$\bullet^4$ $\cos x^\circ = 1$ and $x = 0$

$\bullet^5$ $\cos x^\circ = \frac{1}{2}$ and $x = 60$ or 300

Candidates who include 360 lose $\bullet^4$

⑤

Notes

1. $\bullet^1$ is not available for simply stating that $\cos 2\mathrm{A} = 2\cos^2 \mathrm{A} - 1$ with no further working.
2. In the event of $\cos^2 x - \sin^2 x$ or $1 - 2\sin^2 x$ being substituted for $\cos 2x$, $\bullet^1$ cannot be awarded until the equation reduces to a quadratic in $\cos x$.
3. Substituting $\cos 2\mathrm{A} = 2\cos^2 \mathrm{A} - 1$ or $\cos 2a = 2\cos^2 a - 1$ etc. should be treated as bad form throughout.
4. Candidates may express the quadratic equation obtained at the $\bullet^2$ stage in the form $2c^2 - 3c + 1$ or $2x^2 - 3x + 1$ etc. For candidates who do not solve a trigonometric quadratic equation at $\bullet^5$, $\cos x$ must appear explicitly to gain $\bullet^4$.
5. $\bullet^4$ and $\bullet^5$ are only available as a consequence of solving a quadratic equation.
6. Any attempt to solve $ax^2 + bx = c$ loses $\bullet^3$, $\bullet^4$ and $\bullet^5$.
7. $\bullet^5$ is not available to candidates who work in radian measure and do not convert their answers into degree measure.

Regularly occurring responses

Response 1

(Reading $\cos 2x^\circ$ as $\cos^2 x^\circ$)

$\underline{\cos^2 x^\circ} - 3\cos x^\circ + 2 = 0$ ✗ $\bullet^1$ ✗ $\bullet^2$

$(\cos x^\circ - 2)(\cos x^\circ - 1) = 0$ ✓̸ $\bullet^3$

$\cos x^\circ = 2$ or $\cos x^\circ = 1$ ✓̸ $\bullet^4$

no solution $x = 0$ ✓̸ $\bullet^5$

2 marks out of 5

Response 2A

(See note 6 above)

$2\cos^2 x^\circ - 1 - 3\cos x^\circ + 2 = 0$ ✓ $\bullet^1$

$2\cos^2 x^\circ - 3\cos x^\circ = -1$ ✓̸ $\bullet^2$

$\cos x^\circ(2\cos x^\circ - 3) = -1$ ✓̸ $\bullet^3$

$\cos x^\circ = -1$ or $\cos x^\circ = 1$ ✗ $\bullet^4$

$x = 180$ $x = 0$ ✓̸ $\bullet^5$

1 mark out of 5

Response 2B

(See note 6 above)

$2\cos^2 x^\circ - 1 - 3\cos x^\circ + 2 = 0$ ✓ $\bullet^1$

$2\cos^2 x^\circ - 3\cos x^\circ + 1 = 0$ ✓ $\bullet^2$

$2\cos^2 x^\circ - 3\cos x^\circ = -1$

$\cos x^\circ(2\cos x^\circ - 3) = -1$ ✓̸ $\bullet^3$

$\cos x^\circ = -1$ or $\cos x^\circ = 1$ ✗ $\bullet^4$

$x = 180$ $x = 0$ ✓̸ $\bullet^5$

2 marks out of 5

23	(b)	Hence solve $\cos 4x^\circ - 3\cos 2x^\circ + 2 = 0$ for $0 \le x < 360$.	2

Generic Scheme			Illustrative Scheme
23 (b)			
$\bullet^6$	ic	interpret relationship with (a)	$\bullet^6$ $2x = 0$ and 60 and 300
$\bullet^7$	ic	interpret periodicity	$\bullet^7$ 0, 30, 150, 180, 210 and 330

②

Notes

8. Do not penalise the inclusion of 360 in (b).

9. Ignore extra answers, correct or incorrect, outside the given interval, but penalise incorrect answers within the interval.

10. Do not penalise candidates who use radians in (b) if they have already been penalised in (a).

11. Candidates who go back to 'first principles' for (b) can only gain $\bullet^6$ and $\bullet^7$ for a correct method leading to valid solutions as stated in the Illustrative Scheme.

Regularly occurring responses

Response 3A

From (a) $x = 0,\ 60,\ 300$

(b) $\cos 4x^\circ - 3\cos 2x^\circ + 2 = 0$

$2(\cos 2x^\circ - 3\cos x^\circ + 1) = 0$ ✗ $\bullet^6$

$x = 0,\ 30,\ 150,\ 180,\ 210,\ 330$ ✓̸ $\bullet^7$

1 mark out of 2

Response 3B

From (a) $x = 0,\ 60,\ 300$

(b) ^ $\bullet^6$

$x = 0,\ 30,\ 150,\ 180,\ 210,\ 330$ ✓̸ $\bullet^7$

1 mark out of 2

Response 4A

From (a) $x = 0,\ 60,\ 300$

(b) $x \div 2 = 0,\ 30,\ 150$ ^ $\bullet^6$ ✗ $\bullet^7$

0 marks out of 2

Response 4B

From (a) $x = 0,\ 60,\ 300$

(b) $x \div 2 = 0,\ 30,\ 150,\ 180,\ 210,\ 330$ ^ $\bullet^6$ ✓̸ $\bullet^7$

1 mark out of 2

Response 5

From (a) $x = 0,\ 60,\ 300$

(b) $\cos(2.2x^\circ) - 3\cos 2x^\circ + 2 = 0$ ✓ $\bullet^6$

$x = 0,\ 30,\ 150,\ 180,\ 210,\ 330$ ✓ $\bullet^7$

2 marks out of 2

Response 6

From (a) $x = 0,\ 60,\ 300$

(b) period $\div 2$ ✓ $\bullet^6$

✓ $\bullet^7$

so $x = 0,\ 30,\ 150,\ 180,\ 210,\ 330,\ \underline{360,\ 570}$

2 marks out of 2

Response 7

From (a) $x = 0,\ 60,\ 300$

(b) $2x$ repeats every 180 ^ $\bullet^6$

$x = 0,\ 60,\ 300,\ 0+180,\ 60+180$

$= 0,\ 60,\ 180,\ 240,\ 300$ ✗ $\bullet^7$

0 marks out of 2

Response 8 (Wrong angles from (a))

e.g. $x = 0,\ 30,\ 330$

(b) $2x = 0,\ 30,\ 330$ ✓̸ $\bullet^6$

$x = 0,\ 15,\ 165,\ 180,\ 195,\ 345$ ✓̸ $\bullet^7$

2 marks out of 2