X100/301

NATIONAL QUALIFICATIONS 2011

WEDNESDAY, 18 MAY 9.00 AM – 10.30 AM

MATHEMATICS HIGHER

Paper 1
(Non-calculator)

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1–20 (40 marks)

Instructions for completion of **Section A** are given on page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks)

1 Full credit will be given only where the solution contains appropriate working.

2 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a}.\mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

[Turn over

SECTION A

ALL questions should be attempted.

1. Given that $\mathbf{p} = \begin{pmatrix} 2 \\ 5 \\ -7 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -4 \\ 2 \\ 0 \end{pmatrix}$, express $2\mathbf{p} - \mathbf{q} - \frac{1}{2}\mathbf{r}$ in component form.

A $\begin{pmatrix} 1 \\ 9 \\ -15 \end{pmatrix}$

B $\begin{pmatrix} 1 \\ 11 \\ -13 \end{pmatrix}$

C $\begin{pmatrix} 5 \\ 9 \\ -13 \end{pmatrix}$

D $\begin{pmatrix} 5 \\ 11 \\ -15 \end{pmatrix}$

2. A line l has equation $3y + 2x = 6$.

What is the gradient of any line parallel to l?

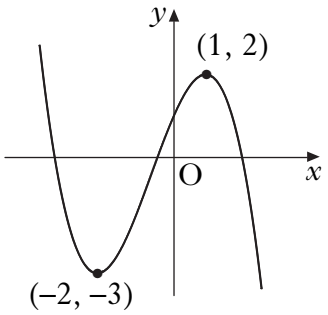
A -2

B $-\frac{2}{3}$

C $\frac{3}{2}$

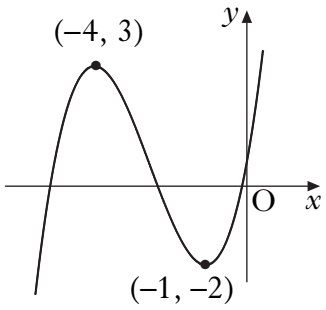
D 2

3. The diagram shows the graph of $y = f(x)$.

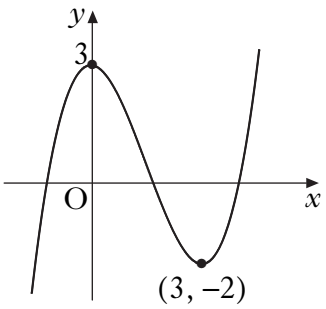


Which of the following shows the graph of $y = f(x + 2) - 1$?

A



B



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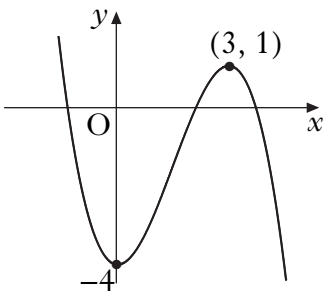

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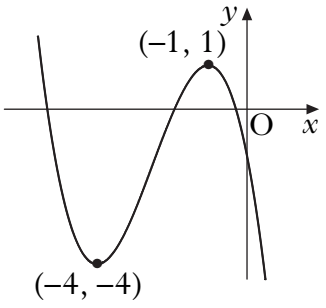


[Turn over

4. A tangent to the curve with equation $y = x^3 - 2x$ is drawn at the point (2, 4).

What is the gradient of this tangent?

A 2

B 3

C 4

D 10

5. If $x^2 - 8x + 7$ is written in the form $(x - p)^2 + q$, what is the value of q?

A -9

B -1

C 7

D 23

6. The point P(2, –3) lies on the circle with centre C as shown.

The gradient of CP is –2.

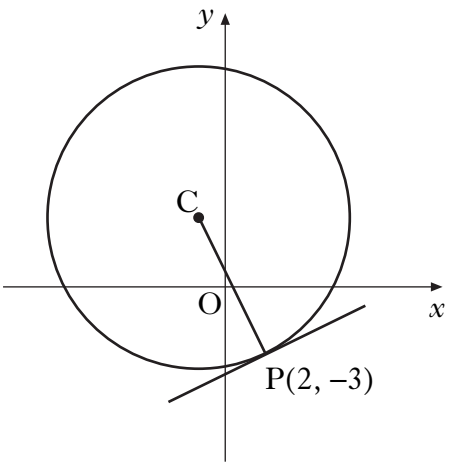
What is the equation of the tangent at P?

A $y + 3 = -2(x - 2)$

B $y - 3 = -2(x + 2)$

C $y + 3 = \frac{1}{2}(x - 2)$

D $y - 3 = \frac{1}{2}(x + 2)$



7. A function f is defined on the set of real numbers by $f(x) = x^3 - x^2 + x + 3$.

What is the remainder when $f(x)$ is divided by $(x - 1)$?

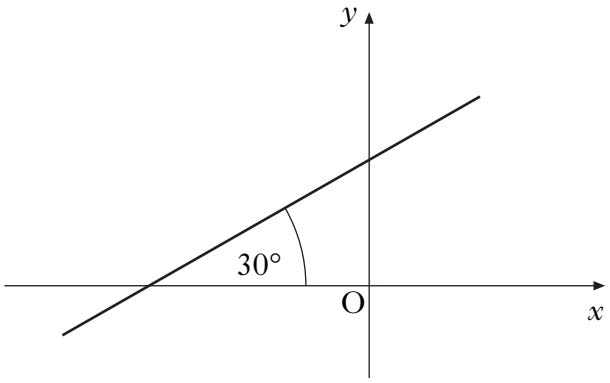
A 0

B 2

C 3

D 4

8. A line makes an angle of 30° with the positive direction of the x-axis as shown.



What is the gradient of the line?

A $\frac{1}{\sqrt{3}}$

B $\frac{1}{\sqrt{2}}$

C $\frac{1}{2}$

D $\frac{\sqrt{3}}{2}$

9. The discriminant of a quadratic equation is 23.

Here are two statements about this quadratic equation:

(1) the roots are real;

(2) the roots are rational.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

[Turn over

10. Solve $2\cos x = \sqrt{3}$ for x, where $0 \leq x < 2\pi$.

A $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

B $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

C $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

D $\frac{\pi}{6}$ and $\frac{11\pi}{6}$

11. Find $\int \left(4x^{\frac{1}{2}} + x^{-3} \right) dx$, where $x > 0$.

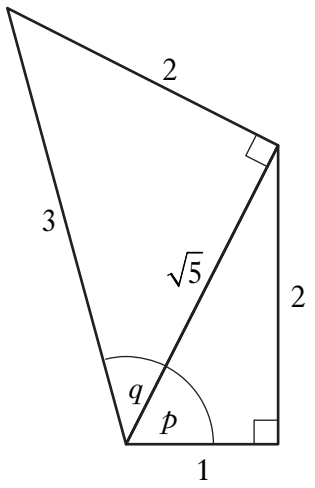
A $2x^{-\frac{1}{2}} - 3x^{-4} + c$

B $2x^{-\frac{1}{2}} - \frac{1}{2}x^{-2} + c$

C $\frac{8}{3}x^{\frac{3}{2}} - 3x^{-4} + c$

D $\frac{8}{3}x^{\frac{3}{2}} - \frac{1}{2}x^{-2} + c$

12. The diagram shows two right-angled triangles with sides and angles as given.



What is the value of $\sin(p + q)$?

A $\frac{2}{\sqrt{5}} + \frac{2}{3}$

B $\frac{2}{\sqrt{5}} + \frac{\sqrt{5}}{3}$

C $\frac{2}{3} + \frac{2}{3\sqrt{5}}$

D $\frac{4}{3\sqrt{5}} + \frac{1}{3}$

13. Given that $f(x) = 4 \sin 3x$, find $f'(0)$.

A 0

B 1

C 12

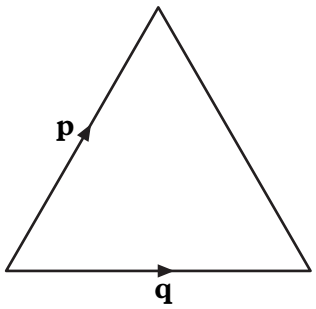
D 36

[Turn over

14. An equilateral triangle of side 3 units is shown.

The vectors **p** and **q** are as represented in the diagram.

What is the value of **p . q**?



A 9

B $\frac{9}{2}$

C $\frac{9}{\sqrt{2}}$

D 0

15. Given that the points S(–4, 5, 1), T(–16, –4, 16) and U(–24, –10, 26) are collinear, calculate the ratio in which T divides SU.

A 2 : 3

B 3 : 2

C 2 : 5

D 3 : 5

16. Find $\int \frac{1}{3x^4}\,dx$, where $x \neq 0$.

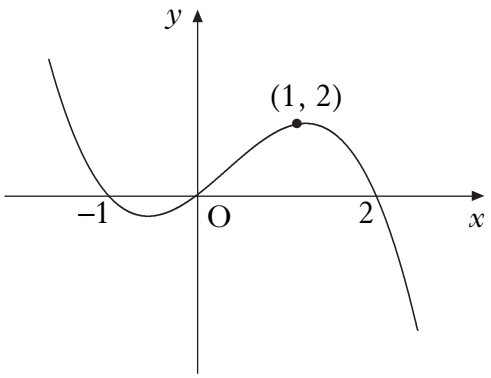
A $-\frac{1}{9x^3} + c$

B $-\frac{1}{x^3} + c$

C $\frac{1}{x^3} + c$

D $\frac{1}{12x^3} + c$

17. The diagram shows the graph of a cubic.



What is the equation of this cubic?

A $y = -x(x + 1)(x - 2)$

B $y = -x(x - 1)(x + 2)$

C $y = x(x + 1)(x - 2)$

D $y = x(x - 1)(x + 2)$

18. If $f(x) = (x - 3)(x + 5)$, for what values of x is the graph of $y = f(x)$ above the x-axis?

A $-5 < x < 3$

B $-3 < x < 5$

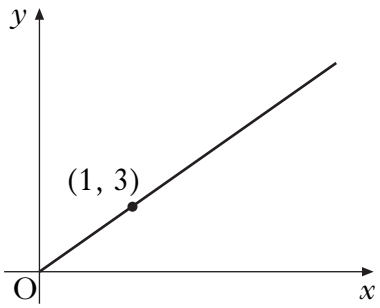
C $x < -5, x > 3$

D $x < -3, x > 5$

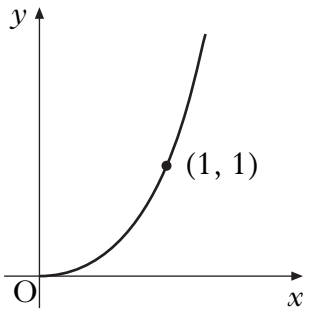
[Turn over

19. Which of the following diagrams represents the graph with equation $\log_3 y = x$?

A



B



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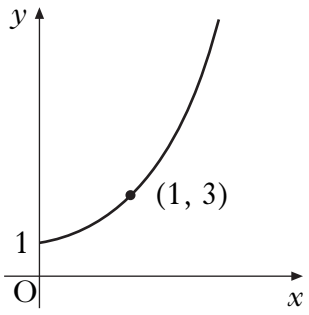
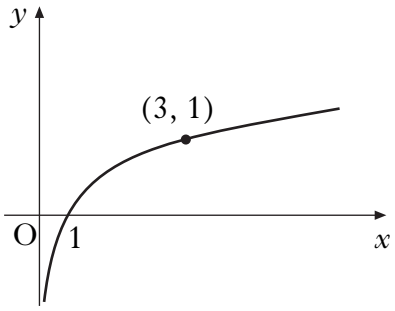


D



20. On a suitable domain, D, a function g is defined by $g(x) = \sin^2 \sqrt{x-2}$.

Which of the following gives the real values of x in D and the corresponding values of $g(x)$?

A $x \geq 0$ and $-1 \leq g(x) \leq 1$

B $x \geq 0$ and $0 \leq g(x) \leq 1$

C $x \geq 2$ and $-1 \leq g(x) \leq 1$

D $x \geq 2$ and $0 \leq g(x) \leq 1$

[*END OF SECTION A*]

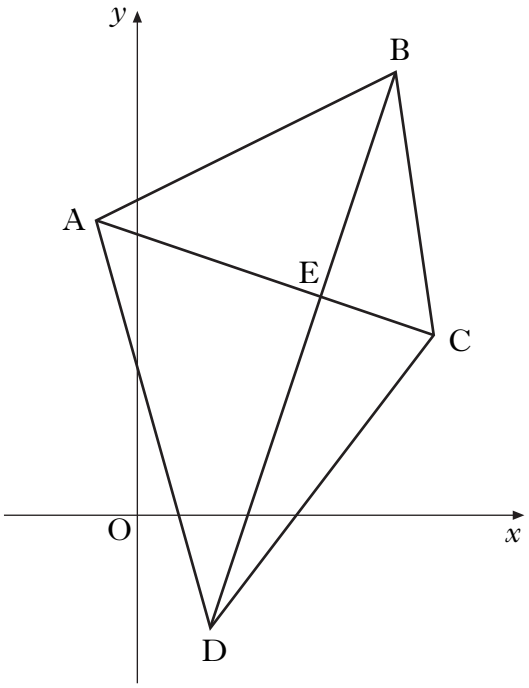
[Turn over for SECTION B

Marks

SECTION B

ALL questions should be attempted.

21. A quadrilateral has vertices A(–1, 8), B(7, 12), C(8, 5) and D(2, –3) as shown in the diagram.



(*a*) Find the equation of diagonal BD. **2**

(*b*) The equation of diagonal AC is $x + 3y = 23$.
Find the coordinates of E, the point of intersection of the diagonals. **3**

(*c*) (i) Find the equation of the perpendicular bisector of AB.

(ii) Show that this line passes through E. **5**

Marks

22. A function f is defined on the set of real numbers by $f(x) = (x - 2)(x^2 + 1)$.

(*a*) Find where the graph of $y = f(x)$ cuts:

(i) the x-axis;

(ii) the y-axis. **2**

(*b*) Find the coordinates of the stationary points on the curve with equation $y = f(x)$ and determine their nature. **8**

(*c*) On separate diagrams sketch the graphs of:

(i) $y = f(x)$;

(ii) $y = -f(x)$. **3**

23. (*a*) Solve $\cos 2x° - 3\cos x° + 2 = 0$ for $0 \leq x < 360$. **5**

(*b*) Hence solve $\cos 4x° - 3\cos 2x° + 2 = 0$ for $0 \leq x < 360$. **2**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]