

X100/302

NATIONAL
QUALIFICATIONS
2010

FRIDAY, 21 MAY
10.50 AM – 12.00 NOON

MATHEMATICS
HIGHER
Paper 2

Read Carefully

- 1 **Calculators may be used in this paper.**
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

ALL questions should be attempted.

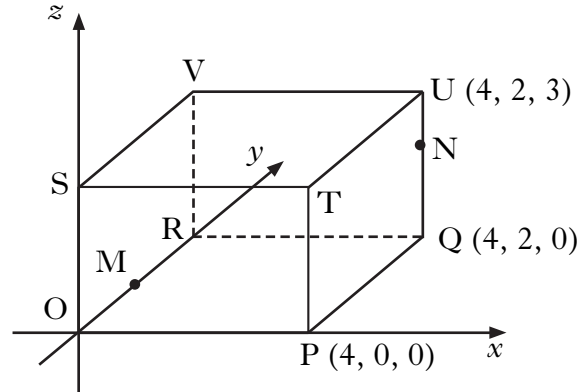
1. The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

P is the point (4, 0, 0),

Q is (4, 2, 0) and U is (4, 2, 3).

M is the midpoint of OR.

N is the point on UQ such that $UN = \frac{1}{3}UQ$.

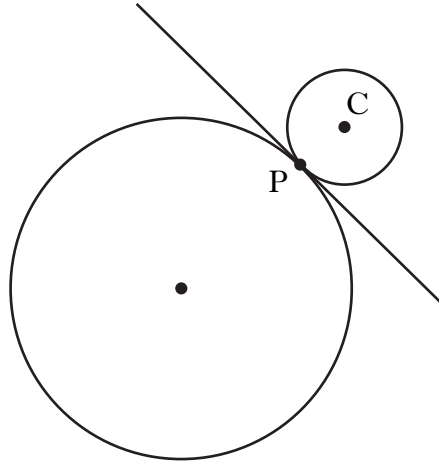


- (a) State the coordinates of M and N. 2
- (b) Express $\overrightarrow{VM}$ and $\overrightarrow{VN}$ in component form. 2
- (c) Calculate the size of angle MVN. 5
2. (a) $12 \cos x^\circ - 5 \sin x^\circ$ can be expressed in the form $k \cos(x + a)^\circ$, where $k > 0$ and $0 \leq a < 360$.
Calculate the values of k and a . 4
- (b) (i) Hence state the maximum and minimum values of $12 \cos x^\circ - 5 \sin x^\circ$.
(ii) Determine the values of x , in the interval $0 \leq x < 360$, at which these maximum and minimum values occur. 3

[Turn over

3. (a) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$.
- (ii) Find the coordinates of the point of contact, P.
- (b) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (a) and a second smaller circle with centre C.

5



The line $y = 3 - x$ is a common tangent at the point P.

The radius of the larger circle is three times the radius of the smaller circle.

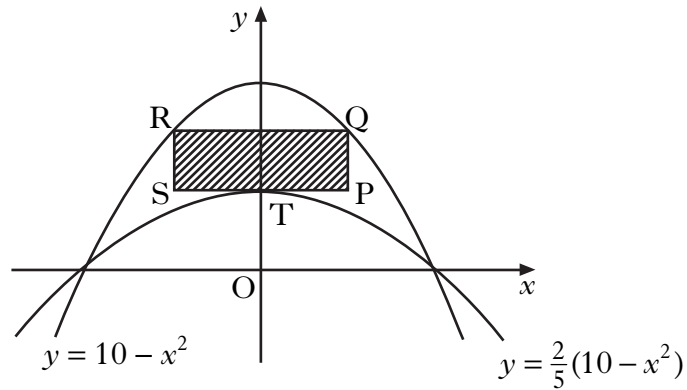
Find the equation of the smaller circle.

6

4. Solve $2 \cos 2x - 5 \cos x - 4 = 0$ for $0 \leq x < 2\pi$.

5

5. The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.



A rectangle PQRS is placed between the two parabolas as shown, so that:

- Q and R lie on the upper parabola;
- RQ and SP are parallel to the x -axis;
- T, the turning point of the lower parabola, lies on SP.

(a) (i) If $TP = x$ units, find an expression for the length of PQ.

(ii) Hence show that the area, A , of rectangle PQRS is given by

$$A(x) = 12x - 2x^3.$$

3

(b) Find the maximum area of this rectangle.

6

[Turn over for Questions 6 and 7 on Page six]

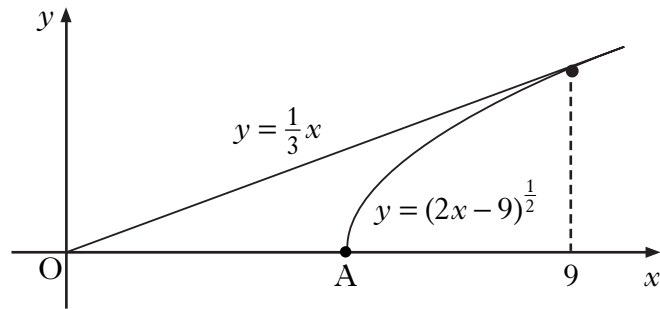
6. (a) A curve has equation $y = (2x - 9)^{\frac{1}{2}}$.

Show that the equation of the tangent to this curve at the point where $x = 9$ is $y = \frac{1}{3}x$.

5

- (b) Diagram 1 shows part of the curve and the tangent.

The curve cuts the x -axis at the point A.

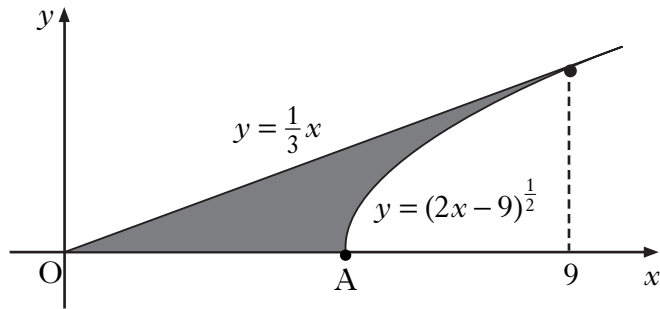


Find the coordinates of point A.

1

- (c) Calculate the shaded area shown in diagram 2.

7



7. (a) Given that $\log_4 x = P$, show that $\log_{16} x = \frac{1}{2}P$.

3

- (b) Solve $\log_3 x + \log_9 x = 12$.

3

[END OF QUESTION PAPER]