

X100/301

NATIONAL
QUALIFICATIONS
2009

THURSDAY, 21 MAY
9.00 AM – 10.30 AM

MATHEMATICS
HIGHER
Paper 1
(Non-calculator)

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1–20 (40 marks)

Instructions for completion of **Section A** are given on page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks)

- 1 Full credit will be given only where the solution contains appropriate working.
- 2 Answers obtained by readings from scale drawings will not receive any credit.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

[Turn over]

SECTION A

ALL questions should be attempted.

1. A sequence is defined by $u_{n+1} = 3u_n + 4$ with $u_1 = 2$.

What is the value of u_3 ?

- A 34
- B 21
- C 18
- D 13

2. A circle has equation $x^2 + y^2 + 8x + 6y - 75 = 0$.

What is the radius of this circle?

- A 5
- B 10
- C $\sqrt{75}$
- D $\sqrt{175}$

3. Triangle PQR has vertices at P(-3, -2), Q(-1, 4) and R(3, 6).

PS is a median. What is the gradient of PS?

- A -2
- B $-\frac{7}{4}$
- C 1
- D $\frac{7}{4}$

4. A curve has equation $y = 5x^3 - 12x$.

What is the gradient of the tangent at the point (1, -7)?

- A -7
- B -5
- C 3
- D 5

5. Here are two statements about the points S(2, 3) and T(5, -1):

(1) The length of ST = 5 units;

(2) The gradient of ST = $\frac{4}{3}$.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

6. A sequence is generated by the recurrence relation $u_{n+1} = 0.7u_n + 10$.

What is the limit of this sequence as $n \rightarrow \infty$?

A $\frac{100}{3}$

B $\frac{100}{7}$

C $\frac{17}{100}$

D $\frac{3}{10}$

7. If the exact value of $\cos x$ is $\frac{1}{\sqrt{5}}$, find the exact value of $\cos 2x$.

A $-\frac{3}{5}$

B $-\frac{2}{\sqrt{5}}$

C $\frac{2}{\sqrt{5}}$

D $\frac{3}{5}$

[Turn over

8. What is the derivative of $\frac{1}{4x^3}$, $x \neq 0$?

A $\frac{1}{12x^2}$

B $-\frac{1}{12x^2}$

C $\frac{4}{x^4}$

D $-\frac{3}{4x^4}$

9. The line with equation $y = 2x$ intersects the circle with equation $x^2 + y^2 = 5$ at the points J and K.

What are the x -coordinates of J and K?

A $x_J = 1, x_K = -1$

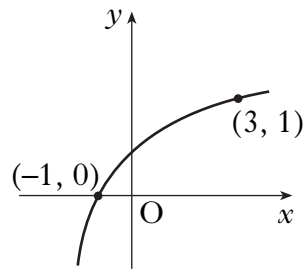
B $x_J = 2, x_K = -2$

C $x_J = 1, x_K = -2$

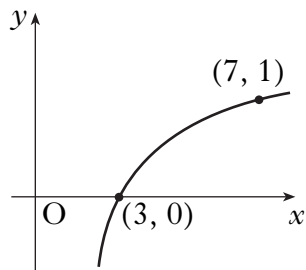
D $x_J = -1, x_K = 2$

10. Which of the following graphs has equation $y = \log_5(x - 2)$?

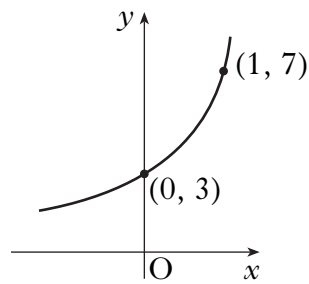
A



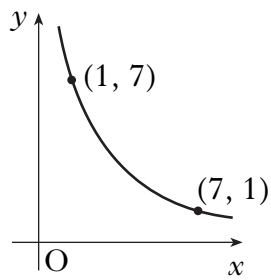
B



C



D



[Turn over

11. How many solutions does the equation

$$(4 \sin x - \sqrt{5})(\sin x + 1) = 0$$

have in the interval $0 \leq x < 2\pi$?

- A 4
- B 3
- C 2
- D 1

12. A function f is given by $f(x) = 2x^2 - x - 9$.

Which of the following describes the nature of the roots of $f(x) = 0$?

- A No real roots
- B Equal roots
- C Real distinct roots
- D Rational distinct roots

13. k and a are given by

$$k \sin a^\circ = 1$$

$$k \cos a^\circ = \sqrt{3}$$

where $k > 0$ and $0 \leq a < 90$.

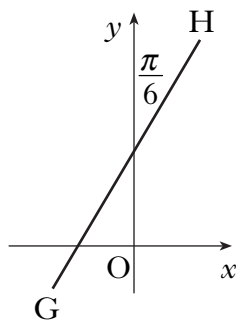
What are the values of k and a ?

	k	a
A	2	60
B	2	30
C	$\sqrt{10}$	60
D	$\sqrt{10}$	30

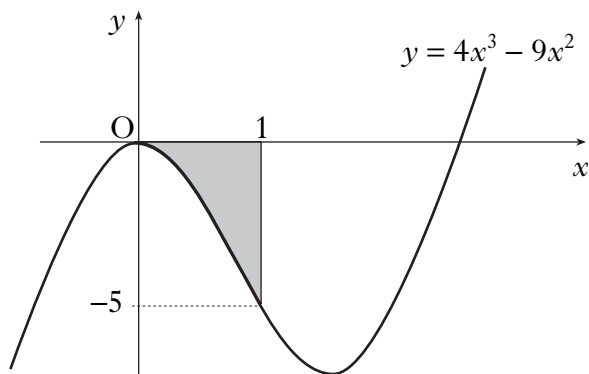
14. If $f(x) = 2\sin\left(3x - \frac{\pi}{2}\right) + 5$, what is the range of values of $f(x)$?

- A $-1 \leq f(x) \leq 11$
- B $2 \leq f(x) \leq 8$
- C $3 \leq f(x) \leq 7$
- D $-3 \leq f(x) \leq 7$

15. The line GH makes an angle of $\frac{\pi}{6}$ radians with the y -axis, as shown in the diagram.
What is the gradient of GH?



- A $\sqrt{3}$
 B $\frac{1}{2}$
 C $\frac{1}{\sqrt{2}}$
 D $\frac{\sqrt{3}}{2}$
16. The graph of $y = 4x^3 - 9x^2$ is shown in the diagram.
Which of the following gives the area of the shaded section?



- A $\left[x^4 - 3x^3 \right]_{-5}^0$
 B $-\left[x^4 - 3x^3 \right]_0^1$
 C $\left[12x^2 - 18x \right]_{-5}^0$
 D $-\left[12x^2 - 18x \right]_0^1$

17. The vector $\mathbf{u}$ has components $\begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}$.

Which of the following is a unit vector parallel to $\mathbf{u}$?

- A $-\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{k}$
B $-3\mathbf{i} + 4\mathbf{k}$
C $-\frac{3}{\sqrt{7}}\mathbf{i} + \frac{4}{\sqrt{7}}\mathbf{k}$
D $-\frac{1}{3}\mathbf{i} + \frac{1}{4}\mathbf{k}$

18. Given that $f(x) = (4 - 3x^2)^{-\frac{1}{2}}$ on a suitable domain, find $f'(x)$.

- A $-3x(4 - 3x^2)^{-\frac{1}{2}}$
B $-\frac{1}{2}(4 - 6x)^{-\frac{3}{2}}$
C $2(4 - 3x^3)^{\frac{1}{2}}$
D $3x(4 - 3x^2)^{-\frac{3}{2}}$

19. For what values of x is $6 + x - x^2 < 0$?

- A $x > 3$ only
B $x < -2$ only
C $x < -2, x > 3$
D $-3 < x < 2$

20. $A = 2\pi r^2 + 6\pi r$.

What is the rate of change of A with respect to r when $r = 2$?

- A 10π
B 12π
C 14π
D 20π

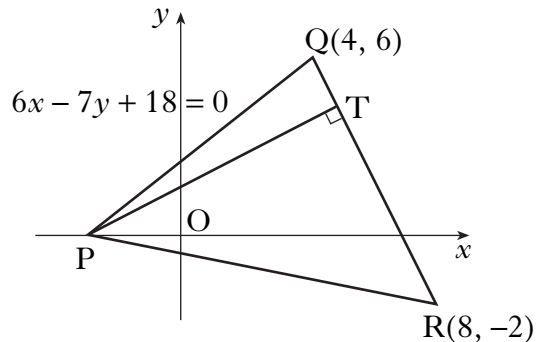
[END OF SECTION A]

SECTION B

ALL questions should be attempted.

Marks

- 21.** Triangle PQR has vertex P on the x -axis, as shown in the diagram.
Q and R are the points (4, 6) and (8, -2) respectively.
The equation of PQ is $6x - 7y + 18 = 0$.



- (a) State the coordinates of P.
- (b) Find the equation of the altitude of the triangle from P.
- (c) The altitude from P meets the line QR at T. Find the coordinates of T.

1

3

4

- 22.** D, E and F have coordinates (10, -8, -15), (1, -2, -3) and (-2, 0, 1) respectively.

- (a) (i) Show that D, E and F are collinear.
(ii) Find the ratio in which E divides DF.

4

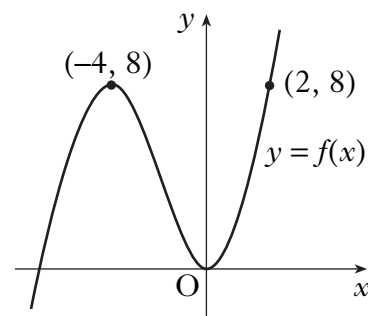
- (b) G has coordinates (k , 1, 0).

Given that DE is perpendicular to GE, find the value of k .

4

- 23.** The diagram shows a sketch of the function $y = f(x)$.

- (a) Copy the diagram and on it sketch the graph of $y = f(2x)$.
- (b) On a separate diagram sketch the graph of $y = 1 - f(2x)$.



2

3

[Turn over for Question 24 on Page twelve]

24. (a) Using the fact that $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$, find the exact value of $\sin\left(\frac{7\pi}{12}\right)$.

3

(b) Show that $\sin(A + B) + \sin(A - B) = 2\sin A \cos B$.

2

(c) (i) Express $\frac{\pi}{12}$ in terms of $\frac{\pi}{3}$ and $\frac{\pi}{4}$.

(ii) Hence or otherwise find the exact value of $\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)$.

4

[END OF SECTION B]

[END OF QUESTION PAPER]