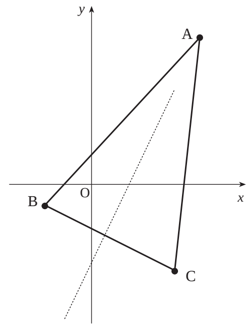
2.01

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A	U1	U2	U3
2.01	a	4	G7	CN		2		2	4			4		
	b	3	G7	CN		1	1	1	3			3		
	c	3	C8	CN		1	2		3			3		

The vertices of triangle ABC are A(7, 9), B(–3, –1) and C(5, –5) as shown in the diagram.
The broken line represents the perpendicular bisector of BC.



(a) Show that the equation of the perpendicular bisector of BC is $y = 2x - 5$. 4

(b) Find the equation of the median from C. 3

(c) Find the coordinates of the point of intersection of the perpendicular bisector of BC and the median from C. 3

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

$\bullet^1$	ss	know and find gradient
$\bullet^2$	ic	interpret perpendicular gradient
$\bullet^3$	ss	know and find midpoint
$\bullet^4$	ic	complete proof
$\bullet^5$	ss	know and find midpoint
$\bullet^6$	pd	calculate gradient
$\bullet^7$	ic	state equation
$\bullet^8$	ss	start to solve sim. equations
$\bullet^9$	pd	find one variable
$\bullet^{10}$	pd	find other variable

Primary Method : Give 1 mark for each •

$\bullet^1$	$m_{BC} = -\frac{1}{2}$ ***stated explicitly***
$\bullet^2$	$m_{\perp} = 2$ *stated / implied by* $\bullet^4$
$\bullet^3$	midpoint of BC = (1, –3)
$\bullet^4$	$y + 3 = 2(x - 1)$ and complete
$\bullet^5$	midpoint of AB = (2,4)
$\bullet^6$	$m_{\text{median}} = -3$
$\bullet^7$	$y + 5 = -3(x - 5)$ *or* $y - 4 = -3(x - 2)$
$\bullet^8$	use $y = 2x - 5$ $y = -3x + 10$
$\bullet^9$	$x = 3$
$\bullet^{10}$	$y = 1$

Notes

In (a)

1 $\bullet^4$ is only available as a consequence of attempting to find and use both a perpendicular gradient and a midpoint.

2 To gain $\bullet^4$ some evidence of completion needs to be shown.
The minimum requirements for this evidence is as shown:

$$y + 3 = 2(x - 1)$$
$$y + 3 = 2x - 2$$
$$y = 2x - 5$$

3 $\bullet^4$ is only available for completion to $y = 2x - 5$ and nothing else.

4 Alternative for $\bullet^4$:
$\bullet^4$ may be obtained by using $y = mx + c$

Notes

In (b)

5 $\bullet^7$ is only available as a consequence of finding the gradient via a midpoint.

6 For candidates who find the equation of the perpendicular bisector of AB, only $\bullet^5$ is available.

In (c)

7 $\bullet^8$ is a strategy mark for juxtaposing the two correctly rearranged equations.

Follow - throughs

Note that from an incorrect equation in (b), full marks are still available in (c). Please follow-through carefully.

Cave

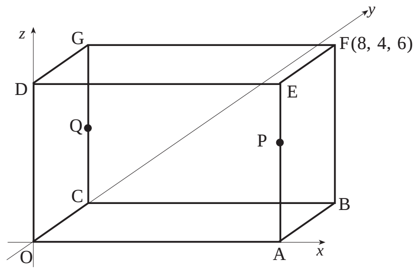
Candidates who find the median, angle bisector or altitude need to show the triangle is isosceles to gain full marks in (a).
For those candidates who do not justify the isosceles triangle, marks may be allocated as shown below:

	Altitude	Median
$\bullet^1$	√	√
$\bullet^2$	√	X
$\bullet^3$	X	√
$\bullet^4$	X	X

2.02

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A		U1	U2	U3
2.02	a	2	G25	CN	8202			2	2						2
	b	2	G25	CN			1	1	2						2
	c	5	G28	CR		1	4		5						5

The diagram shows a cuboid OABC,DEFG.
F is the point (8, 4, 6).
P divides AE in the ratio 2:1.
Q is the midpoint of CG.



(a) State the coordinates of P and Q. 2

(b) Write down the components of $\overrightarrow{PQ}$ and $\overrightarrow{PA}$. 2

(c) Find the size of angle QPA. 5

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

$\bullet^1$	ic	interpret ratio
$\bullet^2$	ic	interpret ratio
$\bullet^3$	pd	process vectors
$\bullet^4$	ic	interpret diagram
$\bullet^5$	ss	know to use scalar product
$\bullet^6$	pd	find scalar product
$\bullet^7$	pd	find magnitude of vector
$\bullet^8$	pd	find magnitude of vector
$\bullet^9$	pd	evaluate angle

Primary Method : Give 1 mark for each •

$\bullet^1$ P = (8, 0, 4)

$\bullet^2$ Q = (0, 4, 3)

$\bullet^3$ $\overrightarrow{PQ} = \begin{pmatrix} -8 \\ 4 \\ -1 \end{pmatrix}$

$\bullet^4$ $\overrightarrow{PA} = \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix}$

$\bullet^5$ $\cos QPA = \dfrac{\overrightarrow{PQ}.\overrightarrow{PA}}{|\overrightarrow{PQ}||\overrightarrow{PA}|}$ *stated / implied by* $\bullet^9$

$\bullet^6$ $\overrightarrow{PQ}.\overrightarrow{PA} = 4$

$\bullet^7$ $|\overrightarrow{PQ}| = \sqrt{81}$

$\bullet^8$ $|\overrightarrow{PA}| = \sqrt{16}$

$\bullet^9$ $83{\cdot}6°$, 1.459 *radians*, 92.9 *gradians*

Notes

1 Treat coordinates written as column vectors as bad form.

2 Treat column vectors written as coordinates as bad form.

3 For candidates who do not attempt $\bullet^9$, the formula quoted at $\bullet^5$ must relate to the labelling in order for $\bullet^5$ to be awarded.

4 Candidates who evaluate $\hat{POQ}$ correctly gain 4/5 marks in (c) (74° or 75°)

Exemplar 1

$\bullet^3,\bullet^4$ X, X $\overrightarrow{OA} = \begin{pmatrix} 8 \\ 0 \\ 0 \end{pmatrix}$ $\overrightarrow{OQ} = \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix}$

$\bullet^5$ X $\cos AOQ = \dfrac{\overrightarrow{OA}.\overrightarrow{OQ}}{|\overrightarrow{OA}||\overrightarrow{OQ}|}$

$\bullet^6$ $\surd$ $\overrightarrow{OA}.\overrightarrow{OQ} = 0$

$\bullet^7$ $\surd$ $|\overrightarrow{OA}| = \sqrt{64}$

$\bullet^8$ $\surd$ $|\overrightarrow{OQ}| = \sqrt{25}$

$\bullet^9$ $\surd$ 90°

Exemplar 2

$\bullet^3,\bullet^4$ X, X $\overrightarrow{OA} = \begin{pmatrix} 8 \\ 0 \\ 0 \end{pmatrix}$ $\overrightarrow{OQ} = \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix}$

$\bullet^6$ $\surd$ $\overrightarrow{OA}.\overrightarrow{OQ} = 0$

$\bullet^9$ $\surd$ 90°

Alternative for $\bullet^5$ to $\bullet^8$

$\bullet^5$ $\cos QPA = \dfrac{PA^2 + PQ^2 - QA^2}{2PA \times PQ}$

$\bullet^6$ $|\overrightarrow{PA}| = \sqrt{16}$

$\bullet^7$ $|\overrightarrow{PQ}| = \sqrt{81}$

$\bullet^8$ $|\overrightarrow{QA}| = \sqrt{89}$

2.03

qu	part	2	code	calc	source	ss	pd	ic	C	B	A		U1	U2	U3
2.03	a	2	T4	CN	8203			2	2				2		
	b	4	T13	CR		1	2	1	4						4
	c	2	C20	CN			1	1	1	1					2

(*a*) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$.
Write down the value of p.

(ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$.
Write down the value of q. 2

(*b*) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. 4

(*c*) Hence find $f'(x) + g'(x)$ as a single trigonometric expression. 2

Diagram 1

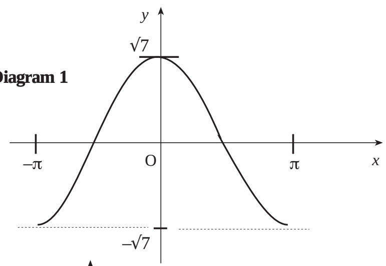
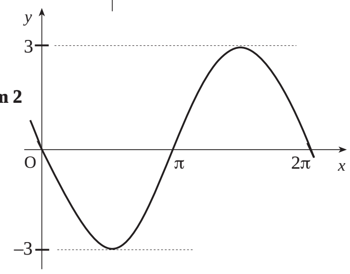


Diagram 2



The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

$\bullet^1$	ic	interpret graph
$\bullet^2$	ic	interpret graph
$\bullet^3$	ss	expand
$\bullet^4$	ic	compare coefficients
$\bullet^5$	pd	process "k"
$\bullet^6$	pd	process "a"
$\bullet^7$	ss	state equation
$\bullet^8$	pd	differentiate

Primary Method : Give 1 mark for each •

$\bullet^1$	$p = \sqrt{7}$
$\bullet^2$	$q = -3$
$\bullet^3$	$k\cos x\cos a - k\sin x\sin a$ ***stated explicitly***
$\bullet^4$	$k\cos a = \sqrt{7}$ ***and*** $k\sin a = 3$ ***stated explicitly***
$\bullet^5$	$k = 4$
$\bullet^6$	$a \approx 0{\cdot}848$
$\bullet^7$	$4\cos(x + 0{\cdot}848)$
$\bullet^8$	$-4\sin(x + 0{\cdot}848)$

Notes

In (a)

1 For $\bullet^1$ accept $p = 2.6$ leading to $k = 4.0$, $a = 0.86$ in (b).

In (b)

2 $k(\cos x\cos a - \sin x\sin a)$ is acceptable for $\bullet^3$.

3 Treat $k\cos x\cos a - \sin x\sin a$ as bad form only if the equations at the $\bullet^4$ stage both contain k.

4 $4(\cos x\cos a - \sin x\sin a)$ is acceptable for $\bullet^3$ and $\bullet^5$.

5 $k = \sqrt{16}$ does not earn $\bullet^5$.

6 No justification is needed for $\bullet^5$.

7 Candidates may use any form of wave equation as long as their final answer is in the form $k\cos(x + a)$. If not, then $\bullet^6$ is not available.

Notes

8 Candidates who use degrees throughout this question lose $\bullet^6$, $\bullet^7$ and $\bullet^8$.

Common Error 1

(sic)

$q = 3 \quad \Rightarrow k = 4,\ \tan a = -\frac{3}{\sqrt{7}}$

$\Rightarrow a = 5.44$ *or* -0.85

$\bullet^2 X$, $\bullet^3 \surd$, $\bullet^4 \surd$, $\bullet^5 \surd$, $\bullet^6 \surd$

Common Error 2

(sic)

$q = 3 \quad \Rightarrow k = 4,\ \tan a = -\frac{3}{\sqrt{7}}$

$\Rightarrow a = 0.85$

$\bullet^2 X$, $\bullet^3 \surd$, $\bullet^4 \surd$, $\bullet^5 \surd$, $\bullet^6 X$

Note that $\bullet^6$ is not awarded as it is not consistent with previous working.

Alternative Method (for $\bullet^7$ and $\bullet^8$**)**

If :

$f'(x) + g'(x) = -\sqrt{7}\sin x - 3\cos x$ ....

then $\bullet^7$ is only available once the candidate has reached e.g.

"choose $k\sin(x + a)$

$\Rightarrow k\sin a = -3,\ k\cos a = -7$."

$\bullet^8$ is available for evaluating k and a.

2.04

qu	part	mk	code	calc	source	ss		ic	C	B	A		U1	U2	U3
2.04	a	2	G9	CN	8204			2	2					2	
	b	4	G14	CN		1	1	2	2	2				4	
	c	5	G12	CN		1	4			5				5	

(a) Write down the centre and calculate the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$. 2

(b) A second circle has equation $(x-4)^2 + (y-6)^2 = 26$.
Find the distance between the centres of these two circles and hence show that the circles intersect. 4

(c) The line with equation $y = 4 - x$ is a common chord passing through the points of intersection of the two circles.
Find the coordinates of the points of intersection of the two circles. 5

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

$\bullet^1$	ic	state centre of circle
$\bullet^2$	ic	find radius of circle
$\bullet^3$	ic	state centre and radius
$\bullet^4$	pd	find distance between centres
$\bullet^5$	ss	find sum of radii
$\bullet^6$	ic	interpret result
$\bullet^7$	ss	know to and substitute
$\bullet^8$	pd	start process
$\bullet^9$	pd	write in standard form
$\bullet^{10}$	pd	solve for x
$\bullet^{11}$	pd	solve for y

Primary Method : Give 1 mark for each •

$\bullet^1$ $(-4,-2)$

$\bullet^2$ $\sqrt{58}$ (≈ 7.6)

$\bullet^3$ $(4,6)$ ***and*** $\sqrt{26}$ (≈ 5.1) *s / i* $\bullet^4$ *and* $\bullet^5$

$\bullet^4$ $d_{centres} = \sqrt{128}$ *accept* 11.3

$\bullet^5$ $\sqrt{58} + \sqrt{26}$ *accept* 12.7

$\bullet^6$ compare 12.7 and 11.3

$\bullet^7$ $x^2 + (4-x)^2 + \ldots$

$\bullet^8$ $x^2 + 16 - 8x + x^2 + \ldots$

$\bullet^9$ $2x^2 - 4x - 6 = 0$

		$\bullet^{10}$	$\bullet^{11}$
$\bullet^{10}$	x	3	-1
$\bullet^{11}$	y	1	5

Notes

In (a)

1 If a linear equation is obtained at the $\bullet^9$ stage, then $\bullet^9$, $\bullet^{10}$ and $\bullet^{11}$ are not available.

2 Solving the circles simultaneously to obtain the equation of the common chord gains no marks.

3 The comment given at the $\bullet^6$ stage must be consistent with previous working.

alt. for $\bullet^7$ to $\bullet^{11}$:

$\bullet^7$ $(4-y)^2 + \ldots$

$\bullet^8$ $y^2 - 8y + 16 + y^2 + \ldots$

$\bullet^9$ $y^2 - 6y + 5 = 0$

		$\bullet^{10}$	$\bullet^{11}$
$\bullet^{10}$	y	1	5
$\bullet^{11}$	x	3	-1

2.05

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A	U1	U2	U3
2.05		5	T10	CR		1	4			5			5	

Solve the equation $\cos 2x° + 2\sin x° = \sin^2 x°$ in the interval $0 \le x < 360$. 5

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

- $\bullet^1$ ss use double angle formula
- $\bullet^2$ pd obtains standard form (i.e. " ……… = 0")
- $\bullet^3$ pd factorise
- $\bullet^4$ pd process factors
- $\bullet^5$ pd completes solutions

Primary Method : Give 1 mark for each •

- $\bullet^1$ $\cos 2x = 1 - 2\sin^2 x$
- $\bullet^2$ $3\sin^2 x - 2\sin x - 1 = 0$
- $\bullet^3$ $(3\sin x + 1)(\sin x - 1) = 0$

	$\bullet^4$	$\bullet^5$
$\bullet^4$	$\sin x = -\frac{1}{3}$	$\sin x = 1$
$\bullet^5$	$199.5°, 340.5°$	$90°$

Notes

1 $\bullet^1$ is not available for $1 - 2\sin^2 A$ with no further working.

2 $\bullet^2$ is only available for the three terms shown written in any correct order.

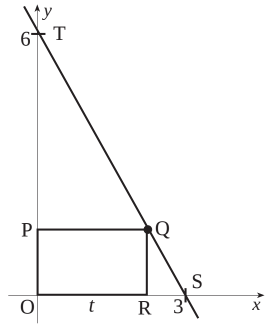
3 The "=0" has to appear at least once "en route" to $\bullet^3$.

4 $\bullet^4$ and $\bullet^5$ are only available for solving a quadratic equation.

2.06

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A		U1	U2	U3
2.06		3	G3	CN	8206	1		2			3		3		
		6	C11	CN		2	2	2		6			6		

In the diagram Q lies on the line joining (0, 6) and (3, 0).
OPQR is a rectangle, where P and R lie on the axes and OR $= t$.



(a) Show that QR $= 6 - 2t$. 3

(b) Find the coordinates of Q for which the rectangle has a maximum area. 6

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

$\bullet^1$	ss	know and use e.g. similar triangles, trigonometry or gradient
$\bullet^2$	ic	establish equation
$\bullet^3$	ic	find a length
$\bullet^4$	ss	know how and find area
$\bullet^5$	ss	set derivative of the area function to zero
$\bullet^6$	pd	differentiate
$\bullet^7$	pd	solve
$\bullet^8$	ic	justify stationary point
$\bullet^9$	ic	state coordinates

Primary Method : Give 1 mark for each •

$\bullet^1$	ΔOST, RSQ are similar $s\,/\,i$ by $\bullet^2$
$\bullet^2$	$\frac{\text{QR}}{6} = \frac{3-t}{3}$ or equivalent
$\bullet^3$	$\text{QR} = 6 - 2t$
$\bullet^4$	$A(t) = t(6-2t)$
$\bullet^5$	$A'(t) = 0$
$\bullet^6$	$6 - 4t$
$\bullet^7$	$t = \frac{3}{2}$
$\bullet^8$	*e.g. nature table*
$\bullet^9$	$\text{Q} = \left(\frac{3}{2}, 3\right)$

Notes

1. "$y = 6 - 2x$" appearing *ex nihilo* can be awarded neither $\bullet^1$ nor $\bullet^2$.
   $\bullet^3$ is still available with some justification *e.g.* OR $= t$ gives $y = 6 - 2t$.
2. The "=0" has to appear at least once before the $\bullet^7$ stage for $\bullet^5$ to be awarded.
3. Do not penalise the use of $\frac{dy}{dx}$ in lieu of $A'(t)$ for instance in the nature table.
4. The minimum requirements for the nature table are shown on the right.
   Of course other methods may be used to justify the nature of the stationary point(s).

Variation 1:

$\bullet^1$ $\tan\text{'S'} = \frac{6}{3}$

$\bullet^2$ $\tan\text{'S'} = \frac{\text{QR}}{3-t}$ *and equate*

Variation 2 :

$\bullet^1$ $\surd\, m_{\text{line}} = -2$ $\quad s\,/\,i$ *by* $\bullet^2$

$\bullet^2$ $\surd$equation of line : $y = -2x + 6$

Variation 3

$\bullet^1$ $\surd \quad m_{\text{line}} = -2$

$\bullet^2$ $\surd$equation of line : $y = 6 - 2x$

Variation 4

$\bullet^1$ X (*nothing stated*)

$\bullet^2$ Xequation of line : $y = 6 - 2x$

Alternative Method : (for $\bullet^5$ *to* $\bullet^8$**)**

$\bullet^5$	strategy to find roots $\Rightarrow$ t.p.s
$\bullet^6$	$t = 0,\ t = 3$
$\bullet^7$	max *t.p.* since coeff of "t^2" < 0
$\bullet^8$	turning pt at $t = \frac{3}{2}$

Nature Table
minimum requirements for •^{8}

$\bullet^8$

		$\frac{3}{2}$	
A'	$+$	0	$-$
	$\cdot^{\cdot^{\cdot}}$	$\cdots$	$\ddots$

2.07

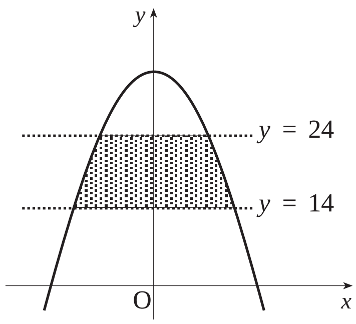
qu	part	mk	code	calc	source	ss	pd	ic	C	B	A	U1	U2	U3
2.07		8	C19	CN		3	4	1			8		8	

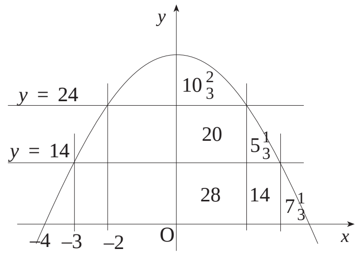
The parabola shown in the diagram has equation

$$y = 32 - 2x^2.$$

The shaded area lies between the lines $y = 14$ and $y = 24$.

Calculate the shaded area. 8





The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

Generic Marking Scheme

•1	ic	interpret limits
•2	pd	find both x-values
•3	ss	know to integrate
•4	pd	integrate
•5	ic	state limits
•6	pd	evaluate limits
•7	ss	select "what to add to what"
•8	pd	completes a valid strategy

Primary Method : Give 1 mark for each •

•1	$32 - 2x^2 = 24$ ***or*** 14
•2	$x = 2$ ***and*** 3
•3	$\int (32 - 2x^2)\ dx$
•4	$32x - \frac{2}{3}x^3$
•5	$\left[\ldots\right]_2^3$
•6	$19\frac{1}{3}$
•7	*e.g.* $19\frac{1}{3} - 14 + 20$ and then double $s\,/\,i$ by •8
•8	$50\frac{2}{3}$

Notes

1 For $\int_{14}^{24} (32 - 2x^2)dx \quad = \left[32x - \frac{2}{3}x^3\right]$ may be awarded •3 and •4 *ONLY*.

2 For integrating "along the y - axis"

•1 strategy: choose to integrate along y-axis

•2 $x = \sqrt{\left(16 - \frac{1}{2}y\right)}$

•3 $\int \left(16 - \frac{1}{2}y\right)^{\frac{1}{2}}\ dy$

•4 $-2.\frac{2}{3}\left(16 - \frac{1}{2}y\right)^{\frac{3}{2}}$

•5 $\left[\ldots\right]_{14}^{24}$

•6 $-\frac{4}{3}\left(4^{\frac{3}{2}} - 9^{\frac{3}{2}}\right)$

•7 $2\times\ldots\ldots\ldots$

•8 $50\frac{2}{3}$

Exemplar 1 (•3 *to* •8)

•3 $\int (32 - 2x^2 - 14)\ dx$

•4 $18x - \frac{2}{3}x^3$

•5 $\left[\ldots\right]_{-3}^3$

•6 72

•7 *e.g.* $72 - \int_{-2}^{2} (32 - 2x^2 - 24)\ dx$

•8 $50\frac{2}{3}$

or

•5 $\left[\ldots\right]_0^3$

•6 36

•7 *e.g.* $2\times\left[36 - \int_0^2 (32 - 2x^2 - 24)\ dx\right]$

Variations (•3 *to* •6)

The following are examples of sound opening integrals which will lead to the area after one more integral at most.

$\int_0^2 (32 - 2x^2)dx \quad = \ldots\ldots = 58\frac{2}{3}$

$\int_0^3 (32 - 2x^2)dx \quad = \ldots\ldots = 78$

$\int_2^3 (32 - 2x^2)dx \quad = \ldots\ldots = 19\frac{1}{3}$

$\int_0^2 (32 - 2x^2 - 24)dx \quad = \ldots\ldots = 10\frac{2}{3}$

$\int_0^3 (32 - 2x^2 - 14)dx \quad = \ldots\ldots = 36$

$\int_2^3 (32 - 2x^2 - 14)dx \quad = \ldots\ldots = 5\frac{1}{3}$